2.882
Complexity

April 20, 2005

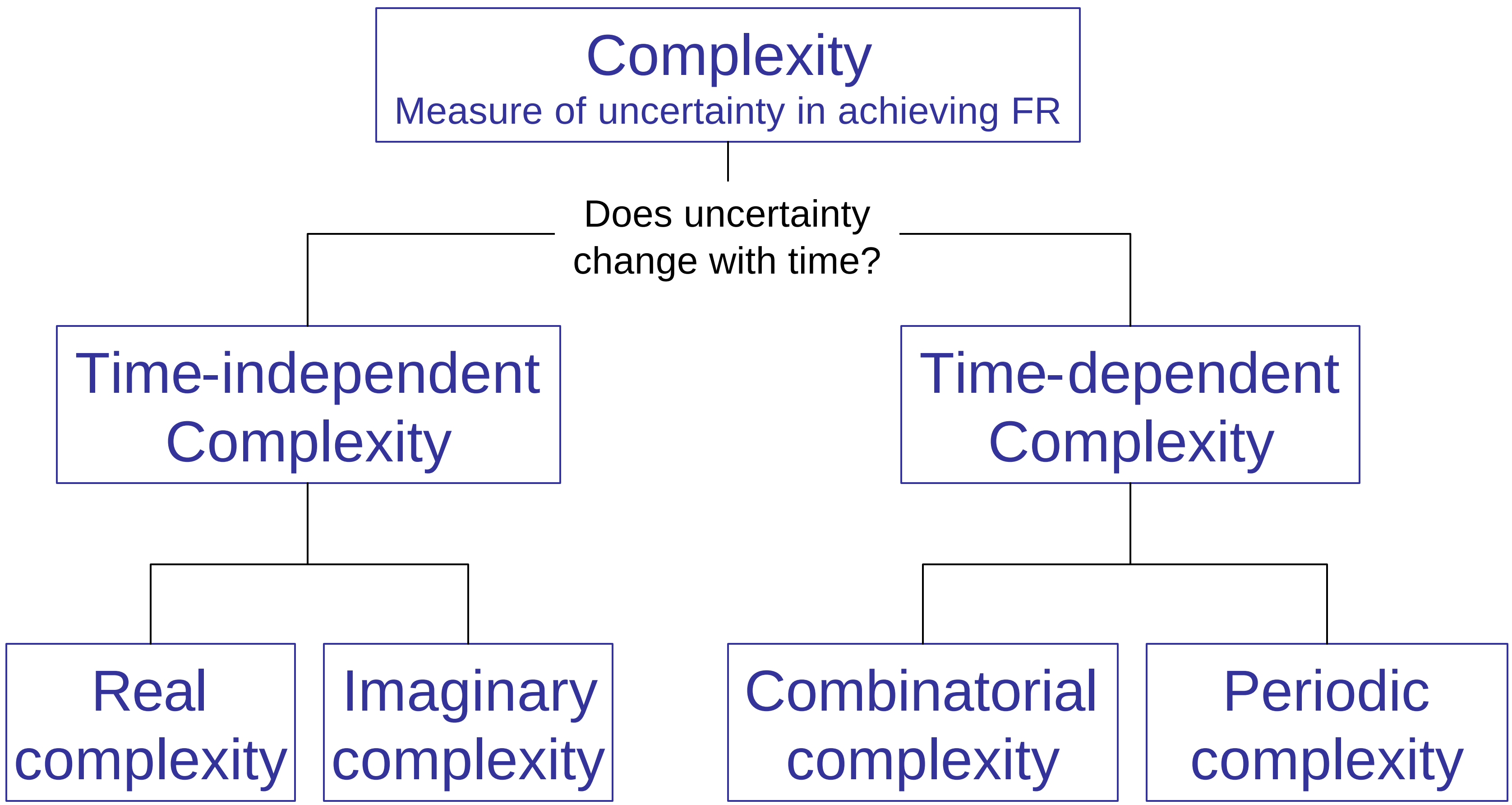
Complexity in AD

- Complexity

  "Measure of uncertainty in achieving the desired functional requirements of a system"

  - Difficulty
  - Relativity
  - Information
  - Ignorance

Four types of complexity in AD



Complexity: A measure of *uncertainty* in achieving the desired set of *FRs* of a system

- Time-independent real complexity

  “Measure of uncertainty when the *probability of achieving the functional requirements is less than 1.0* (because the common range is not identical to the system range)”

- Time-independent imaginary complexity

  “Uncertainty that arises because of the *designer’s lack of knowledge and understanding* of a specific design itself”

- Time-dependent combinatorial complexity

  “Time-dependent combinatorial complexity arises because in many situations, *future events cannot be predicted a priori*. … This type of time-dependent complexity will be defined as time-dependent combinatorial complexity.”

- Time-dependent periodic complexity

  “Consider the problem of scheduling airline flights. … it is periodic and thus *uncertainties created during the prior period are irrelevant*. … This type of time-dependent complexity will be defined as time-dependent periodic complexity.”

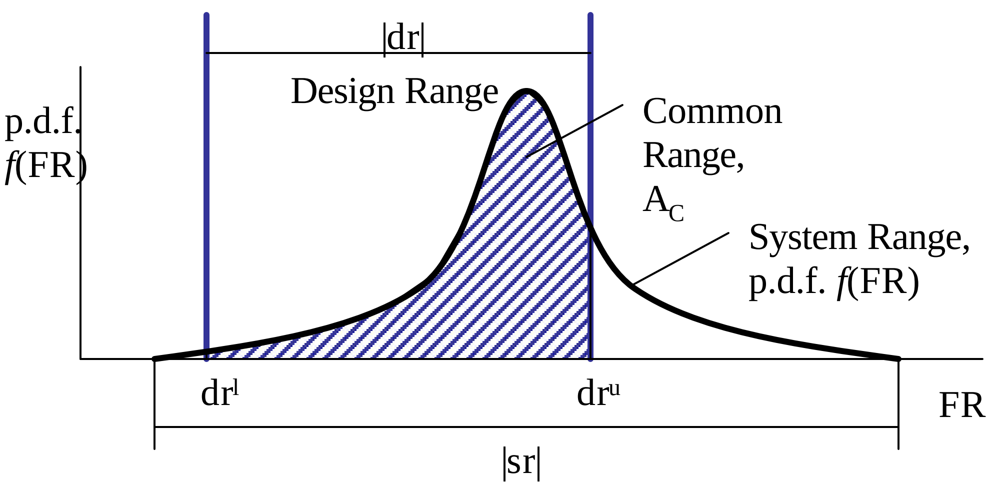
Time-independent Real Complexity

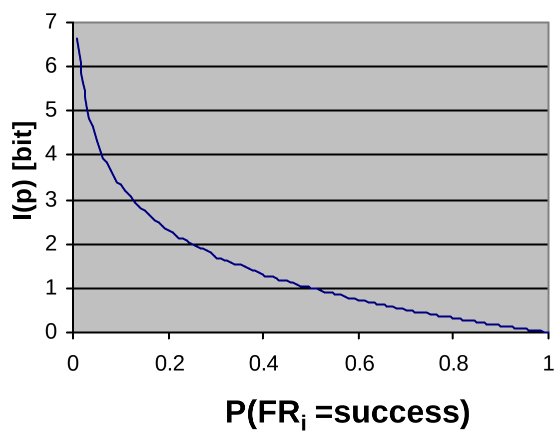
- Time-independent real complexity
  - caused by system range's being outside of the design range.
  - Real complexity ~ Information content
  - Take u_i as a random variable

    $u_i =$ 1 (success) with $P(\mathrm{FR_i} =$ success)

    0 (failure) $1-P(\mathrm{FR_i} =$ success)
  - Information content:

    $I(u_i = 1) \equiv -\log_2 P(\mathrm{FR_i} = \text{success})$





Time-independent Imaginary complexity

- Imaginary complexity ~ Ignorance
- Ignorance causes complexity.
- Types of ignorance
  - Functional requirement
  - Knowledge required to synthesize(or identify) design parameters
  - Ignorance about the interactions between FRs and DPs
- p (probability of selecting a right sequence)
  - For uncoupled design, $p = 1$
  - For decoupled design, $p = z/n!$
  - For coupled design, $p = 0$

Time-dependent complexity

- Time-dependency
  - Complexity ≡ Uncertainty in achieving a set of FR
  - Complexity is time-dependent if
    1) uncertainty (probabilistic) is time-dependent

       Time-varying system range
    2) behavior of FR is time-dependent

       FR = **FR**(t)
- Combinatorial / Periodic complexity
  - Uncertainty increases indefinitely : combinatorial complexity
  - Uncertainty in one period is irrelevant to the next period : periodic complexity

Origins of complexity and reduction

Time-independent

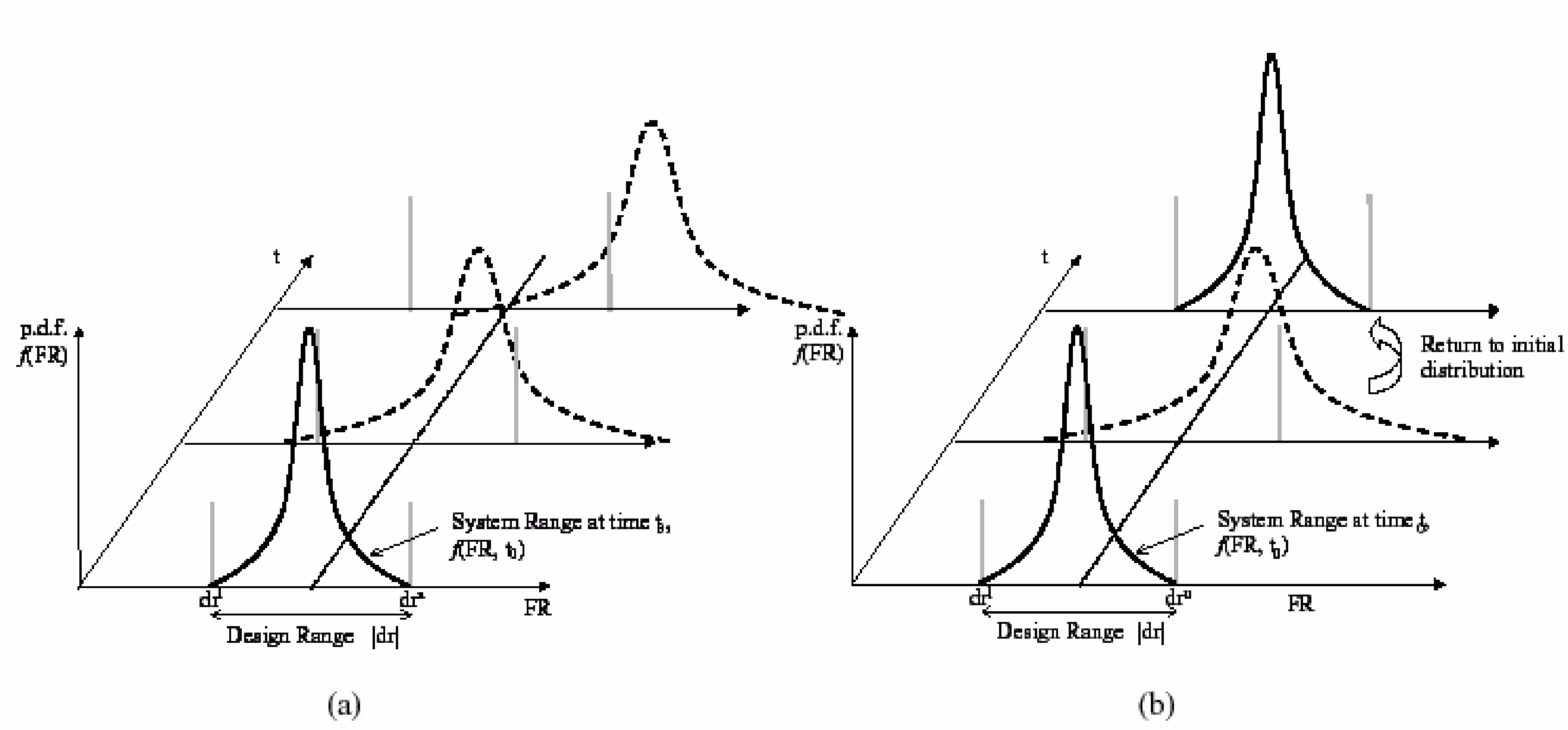
- Minimize Real complexity by
  - Eliminating source of variation
  - Desensitizing w.r.t. variation
  - Compensating error
- Eliminate Imaginary complexity by
  - Achieving uncoupled design
  - Identifying design matrix

Time-dependent complexity

Combinatorial complexity

$\Rightarrow$ Periodic complexity

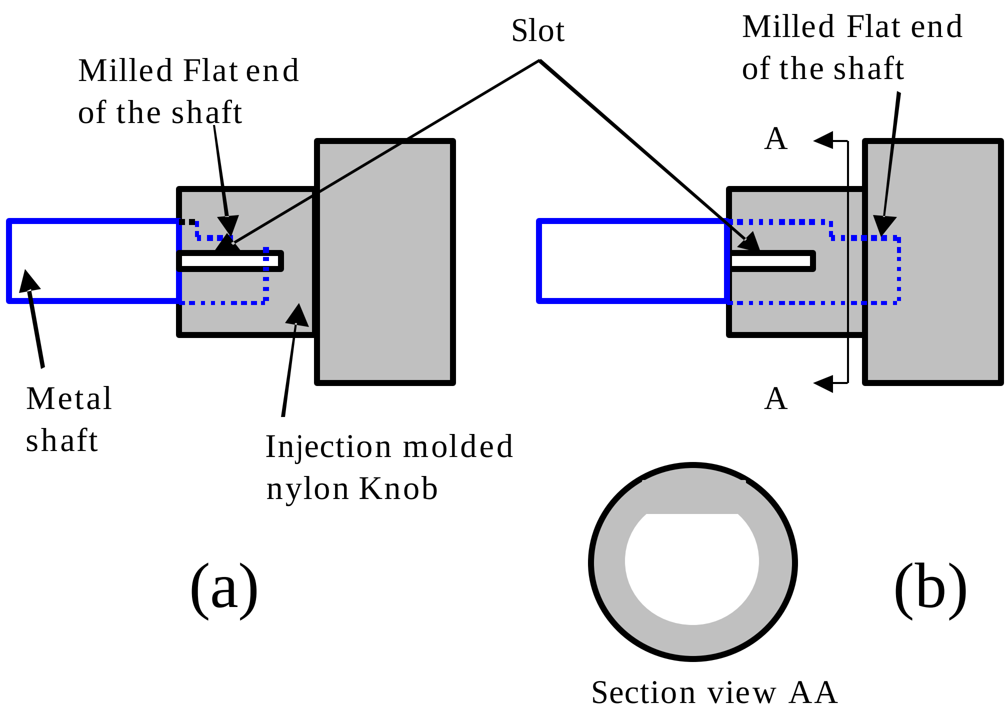
Time-varying system range



- Detect changes in system range
- Prevent system range deterioration by design
- Bring the system range back into design range by re-initialization

Prevent system range deterioration by design

By eliminating coupling between 'turn' and 'grasp', one can effectively delay system range deterioration.



N. P. Suh, Axiomatic Design: Advances and Applications, 2001

Bring the system range back into design range: Re-initialization

Example: Design of Low Friction Surface

- Dominant friction mechanism: Plowing by wear debris

- System range (particle size) moves out of the desired design range
  $\Rightarrow$ Need to re-initialize

Figure removed for copyright reasons.

N. P. Suh and H.-C. Sin, Genesis of Friction, Wear, 1981

Figure removed for copyright reasons.

S. T. Oktay and N. P. Suh, Wear debris formation and Agglomeration, Journal of Tribology, 1992

Design of Low Friction Surface

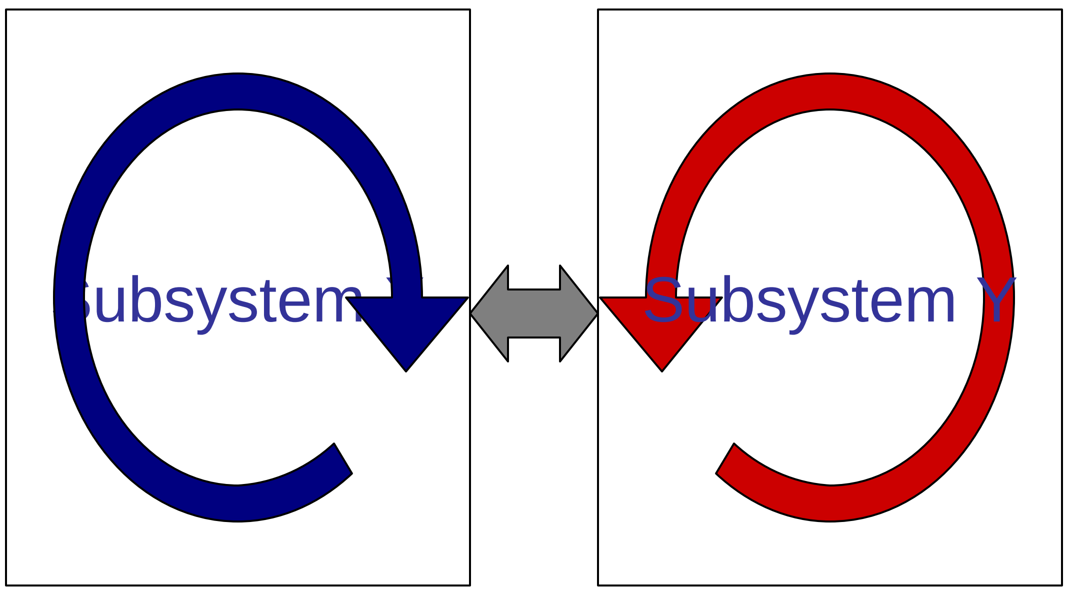
- Periodic undulation re-initializes the system range

Figures removed for copyright reasons.

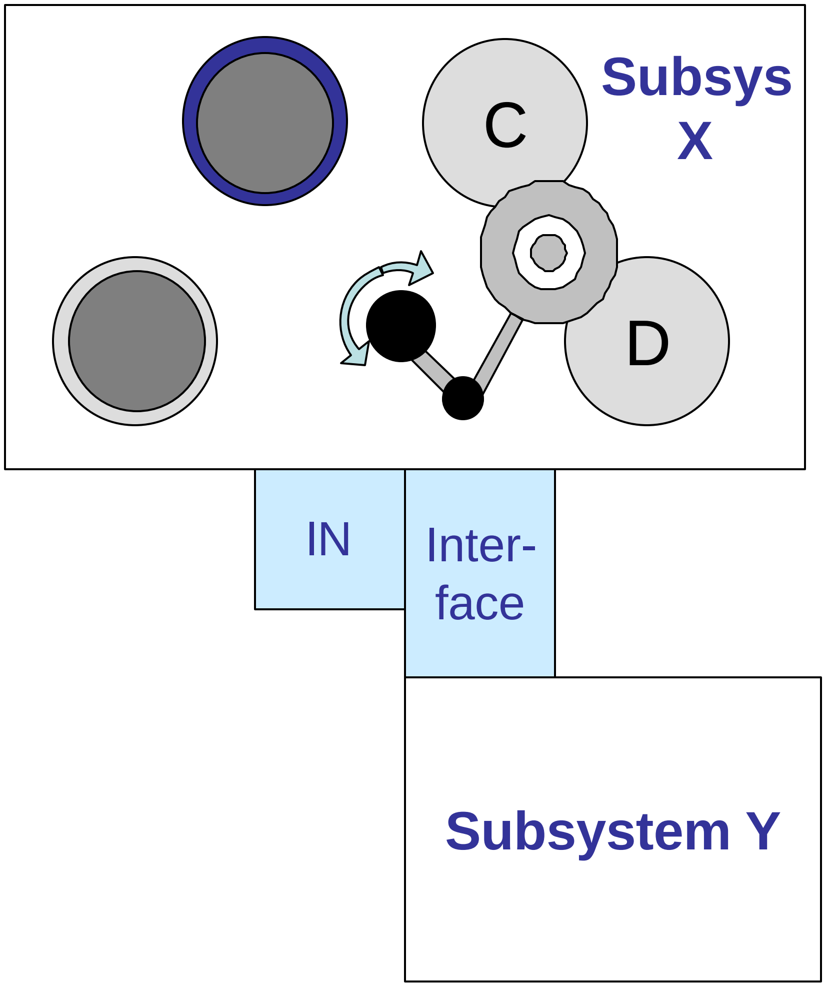
S. T. Oktay and N. P. Suh, Wear debris formation and agglomeration, Journal of Tribology, 1992

Example: Scheduling of mfg system

Periodicity should be introduced & maintained to prevent the system from developing chaotic behavior



Problem description

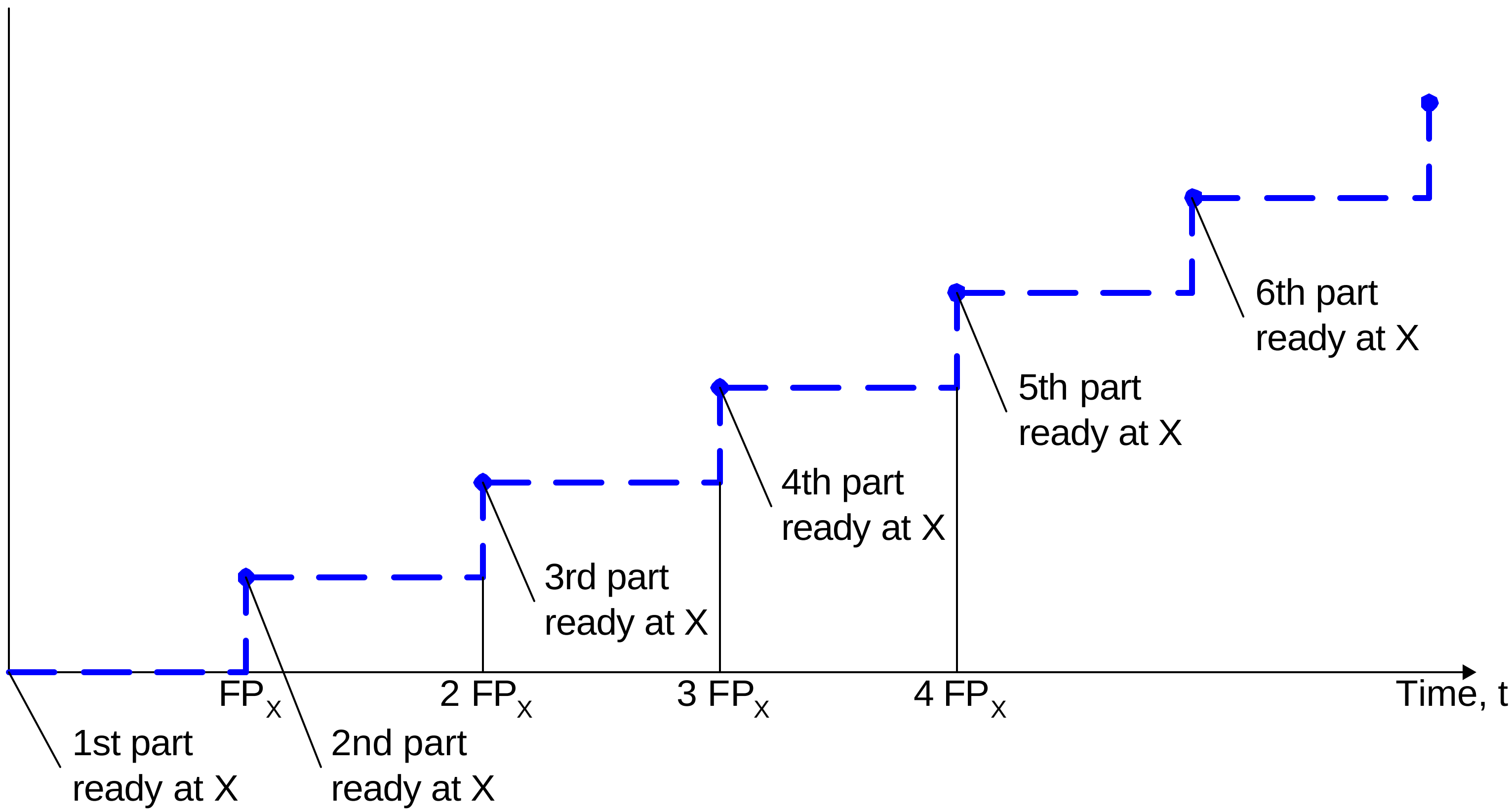


Station		PT_i or CT_Y (sec)	Number of machines	$MvPk_i$ (sec)	$MvPl_i$ (sec)
IN		-	1	5	-
X	a	30	1	5	5
	b	40	1	5	5
	c	50	1	5	5
	d	80	2	5	5
Y		**60±5**	1	-	5

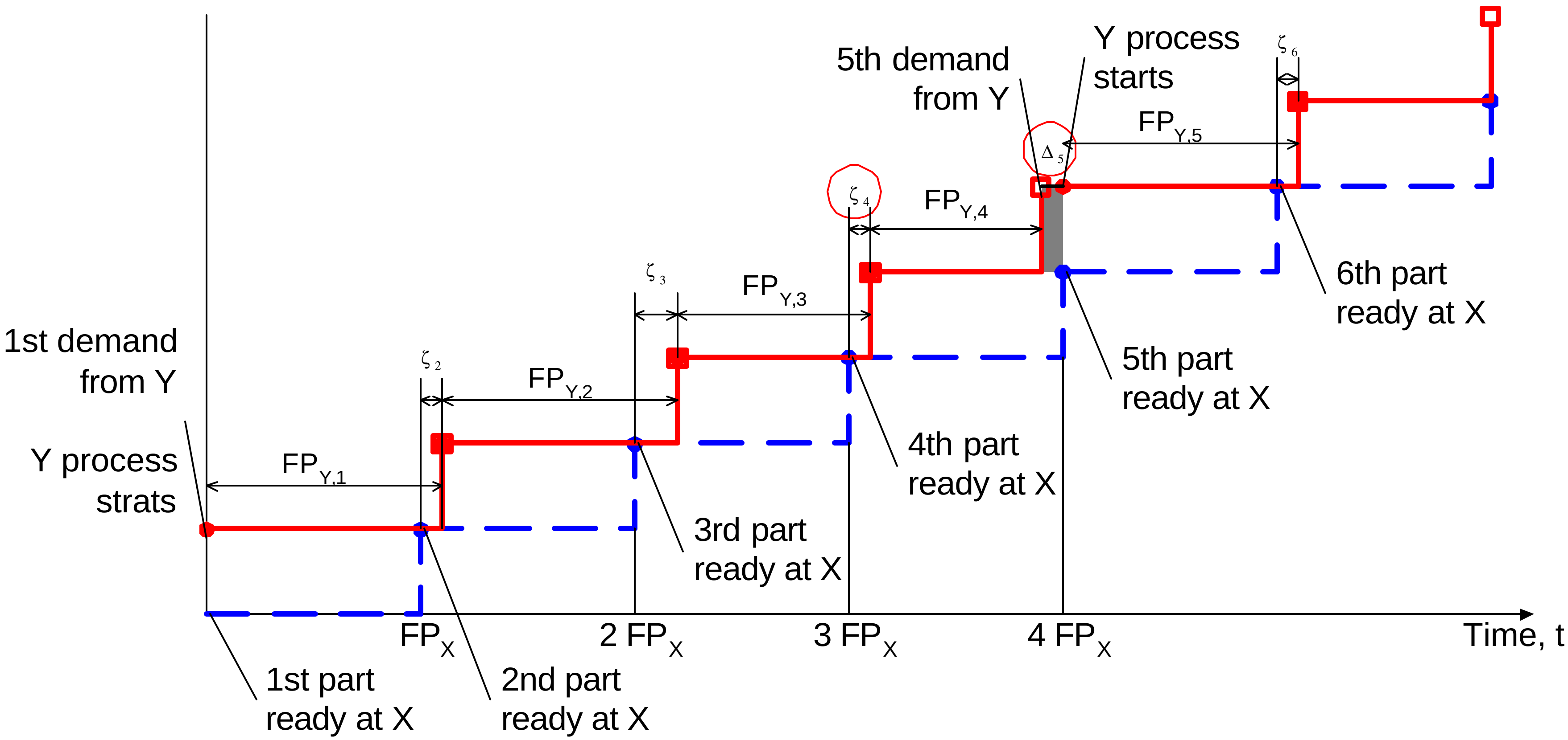
- Fastest speed*
  70 seconds when X determines the system speed
  65-75 seconds when Y determines the system speed

- Objective
  Maximum utilization rate for the machine Y
- Constraint
  Transport from C to D must be immediate

* Speed is measured by throughput time: shorter time means faster speed



6th part
ready at X
5th part
ready at X
4th part
ready at X
3rd part
ready at X
FP X
2 FP X
3 FP X
4 FP X
Time, t
1st part
ready at X
2nd part
ready at X



5th demand
from Y
Y process
starts
ζ 6
FP Y,5
Δ 5
ζ 4
FP Y,4
ζ 3
FP Y,3
6th part
ready at X
5th part
ready at X
1st demand
from Y
ζ 2
FP Y,2
4th part
ready at X
Y process
strats
FP Y,1
3rd part
ready at X
FP X
2 FP X
3 FP X
4 FP X
Time, t
1st part
ready at X
2nd part
ready at X

CT_Y : ... 60sec - 60sec - 60sec - 60sec - ***55sec*** - ...

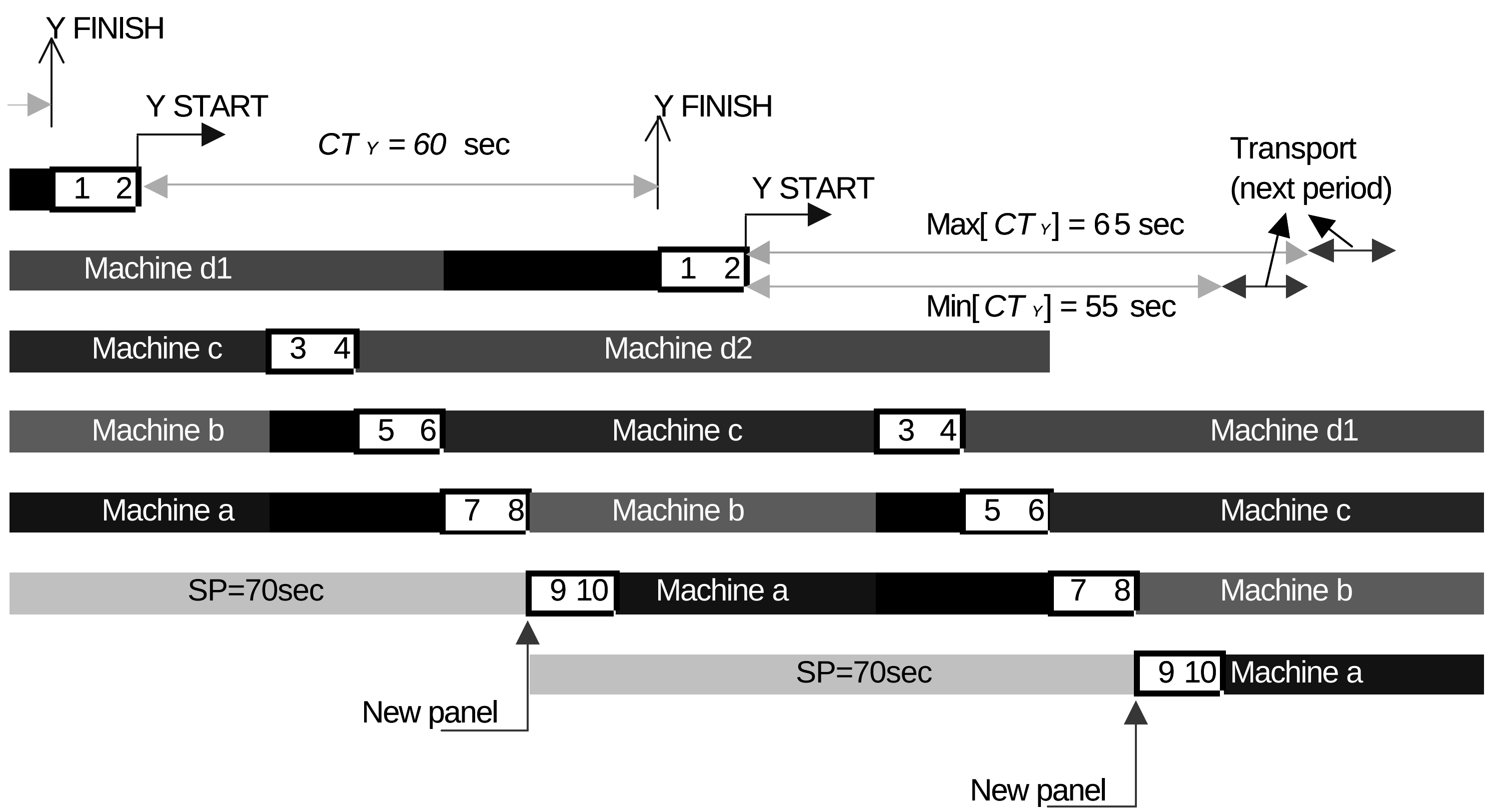


Figure 10. Steady state operation with 70 seconds sending period

From Lee, Taesik. "Complexity Theory in Axiomatic Design." MIT PhD Thesis, 2003.

CT_Y : ... 60sec - 60sec - ***55sec*** - 65sec - 65sec ...

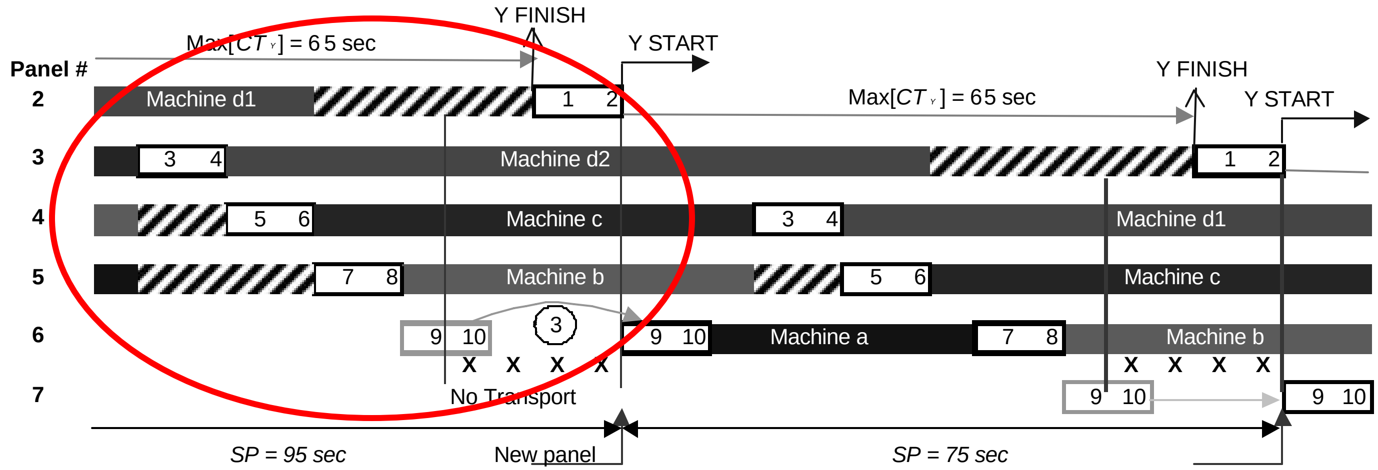


Figure 11 (b)

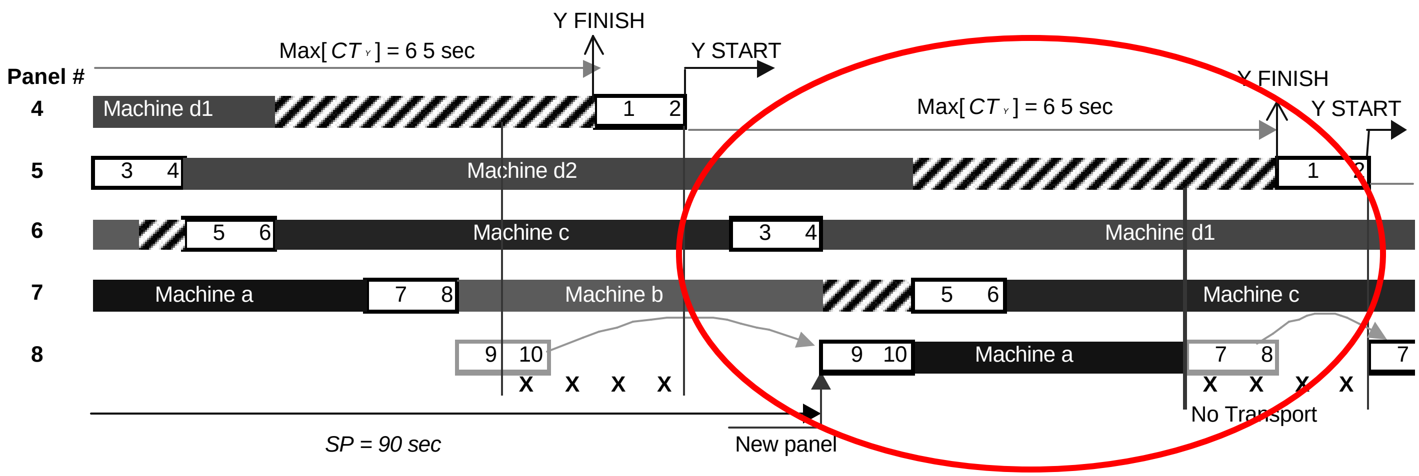
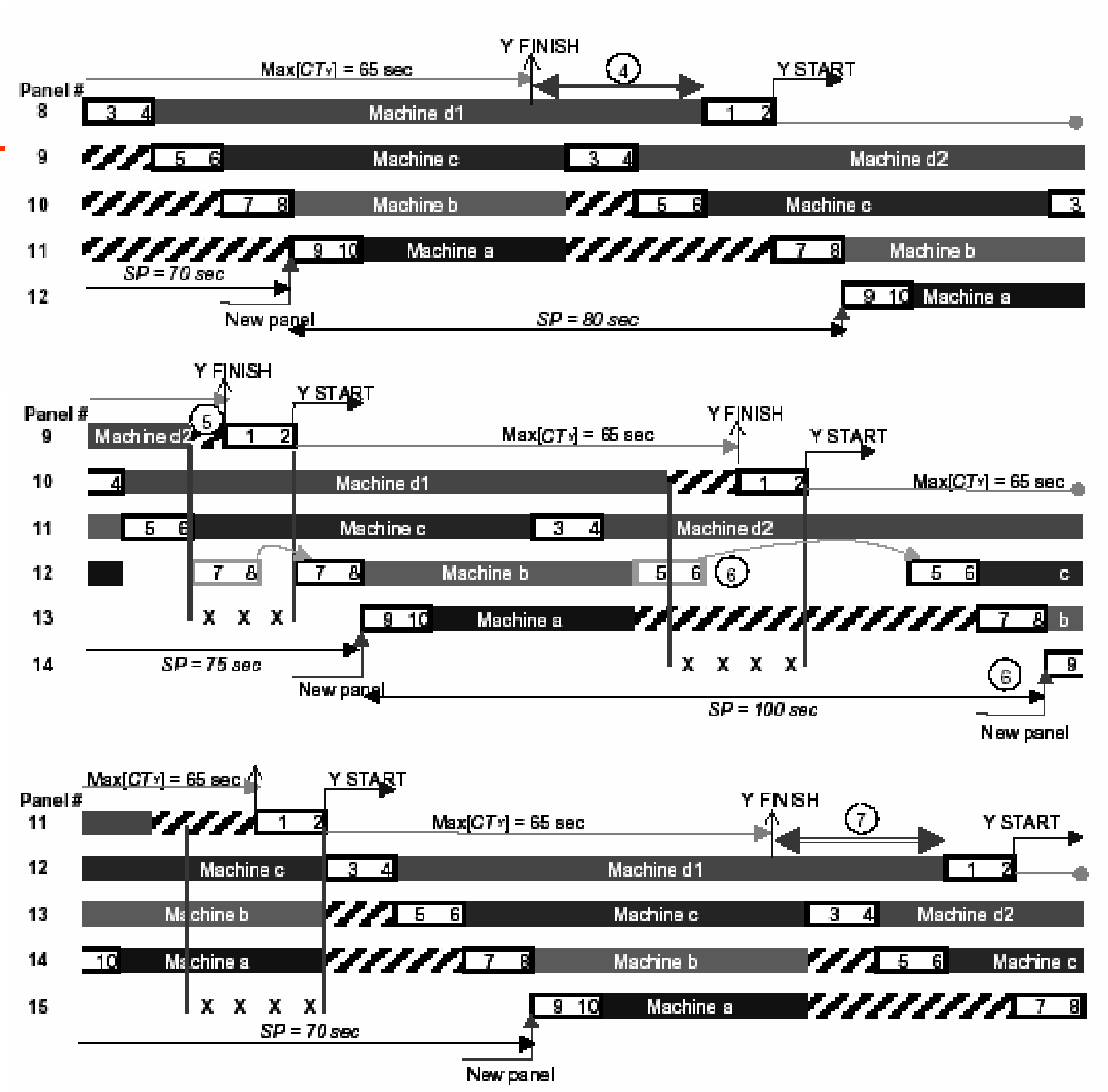


Figure 11 (c)

From Lee, Taesik. "Complexity Theory in Axiomatic Design." MIT PhD Thesis, 2003.



Average throughput time = (75+75+75+100)/4 = 81.25

From Lee, Taesik. "Complexity Theory in Axiomatic Design." MIT PhD Thesis, 2003.

- Single perturbation from subsystem Y causes incomplete period in downstream
- The system regains periodicity after the perturbation is removed but with undesirable performance
- Throughput time is 81.25 seconds in average
  - Slower than the system capability

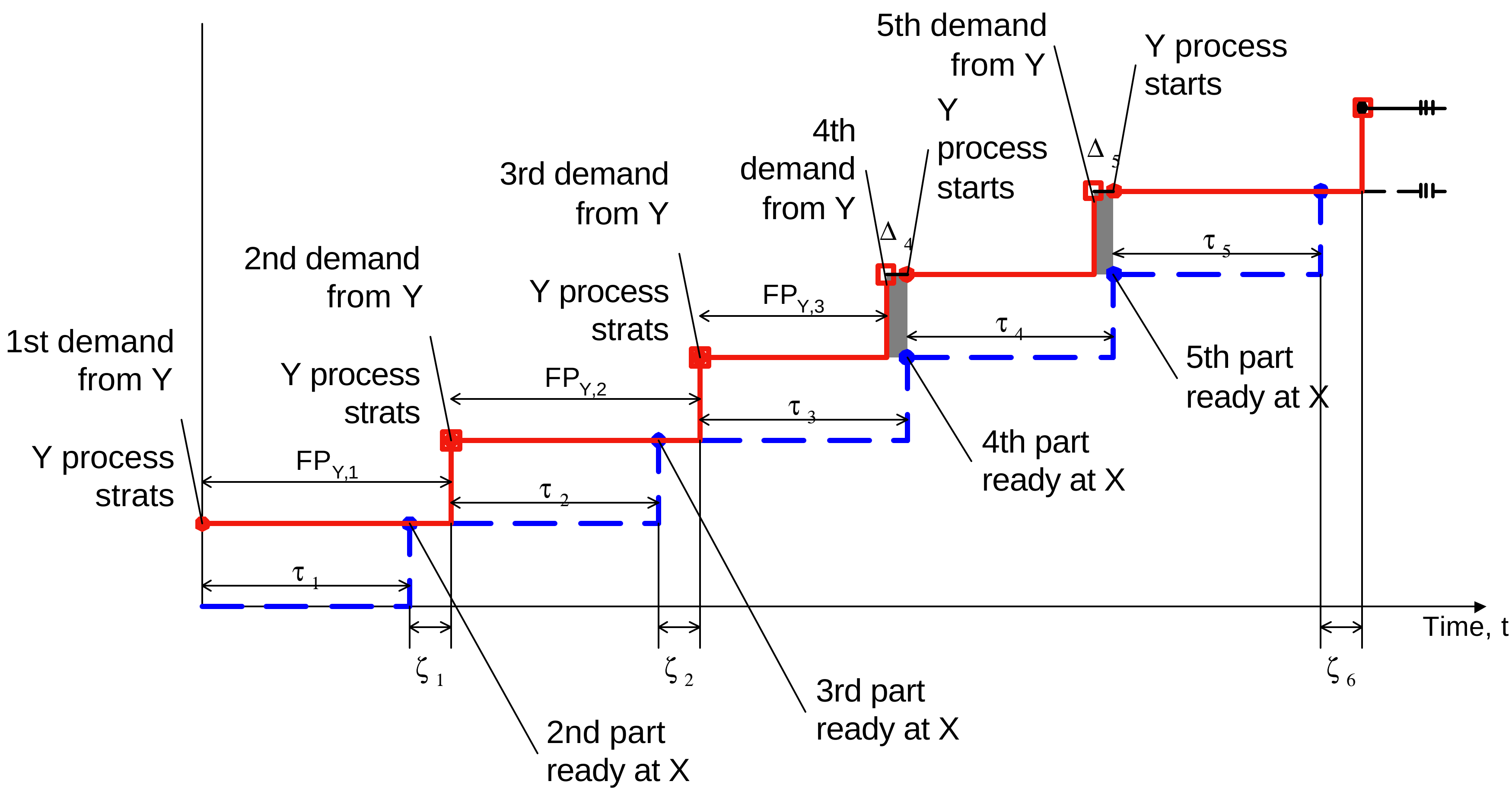
Re-initialization scheme in scheduling

- Define a "renewal" event that imposes period

$$\mathbf{u}(0) = \{ u_0(t), u_1(t), \ldots, u_{k-1}(t), u_k(t), u_{k+1}(t), \ldots, u_N(t)\}$$
$$= \{0, 0, \ldots, 0, 0, 0, \ldots, 0\}$$
$$\vdots$$
$$\mathbf{u}(T-\Delta) = \{1, 1, \ldots, 1, 0, 1, \ldots, 1\}$$
$$\mathbf{u}(T) = \{1, 1, \ldots, 1, 1, 1, \ldots, 1\}$$
$$\mathbf{u}(T+\varepsilon) = \{0, 0, \ldots, 0, 0, 0, \ldots, 0\} = \mathbf{u}(0)$$

- Scheduling activity is confined within such a period with a goal of maintaining "periodicity"
  - Conditional renewal event

$$t_{ini} = t_{request} \quad \text{if } t_{request} \geq 70 \text{ sec } (FP_X)$$
$$t_{ini} = 70\text{sec} \quad \text{if } t_{request} < 70 \text{ sec}$$



- Each period is independent (memoryless)

Conclusion

- Breakdown of functional periodicity results in sub-optimal throughput rate
- Periodicity should be introduced & maintained to prevent the system from developing chaotic behavior

Example: Cell division

- A cell has a mechanism to coordinate cycles of two subsystems such that the overall periodicity is maintained
- Break-down of functional periodicity leads to anomaly of cell division and further chaotic behavior of the system

- Maintaining functional periodicity in the cell cycle is an important functional requirement for cell division

Overview of the Cell Cycle

Figure removed for copyright reasons.

* Figure taken from Molecular Biology of the Cell, Alberts, Garland Science

Chromosome cycle & Centrosome cycle

Figure removed for copyright reasons.

* Figure taken from Molecular Biology of the Cell, Alberts, Garland Science

Importance of the correct number of chromosomes and centrosomes

- Centrosomal abnormalities
  - Chromosome missegregation
  - Aneuploidy

Figure removed for copyright reasons.

Figure 2 in Nigg, E. A. "Centrosome abberation: cause or consequence of cancer progression?" *Nature Reviews Cancer* 2 (2002): 815-825.

Figure removed for copyright reasons.

* Figure taken from http://www.sivf.com.au/chromosomes.htm

Functional periodicity

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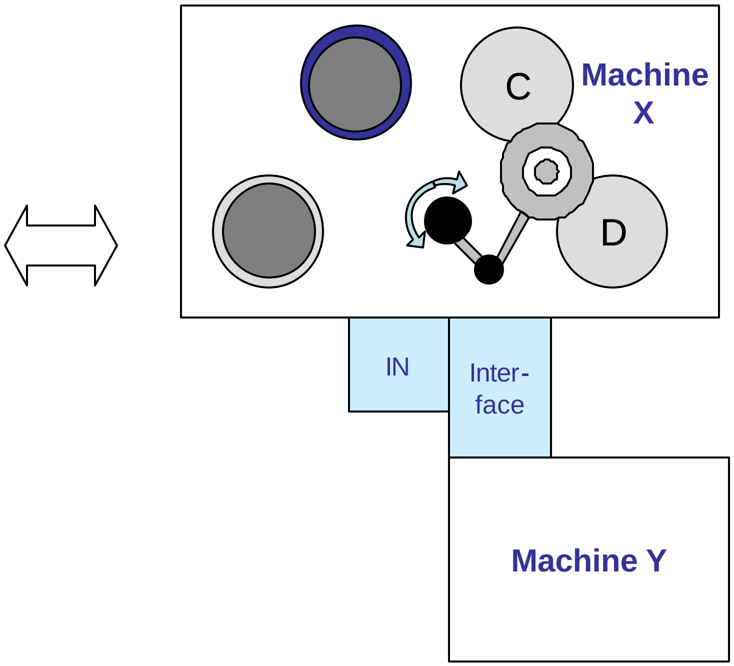
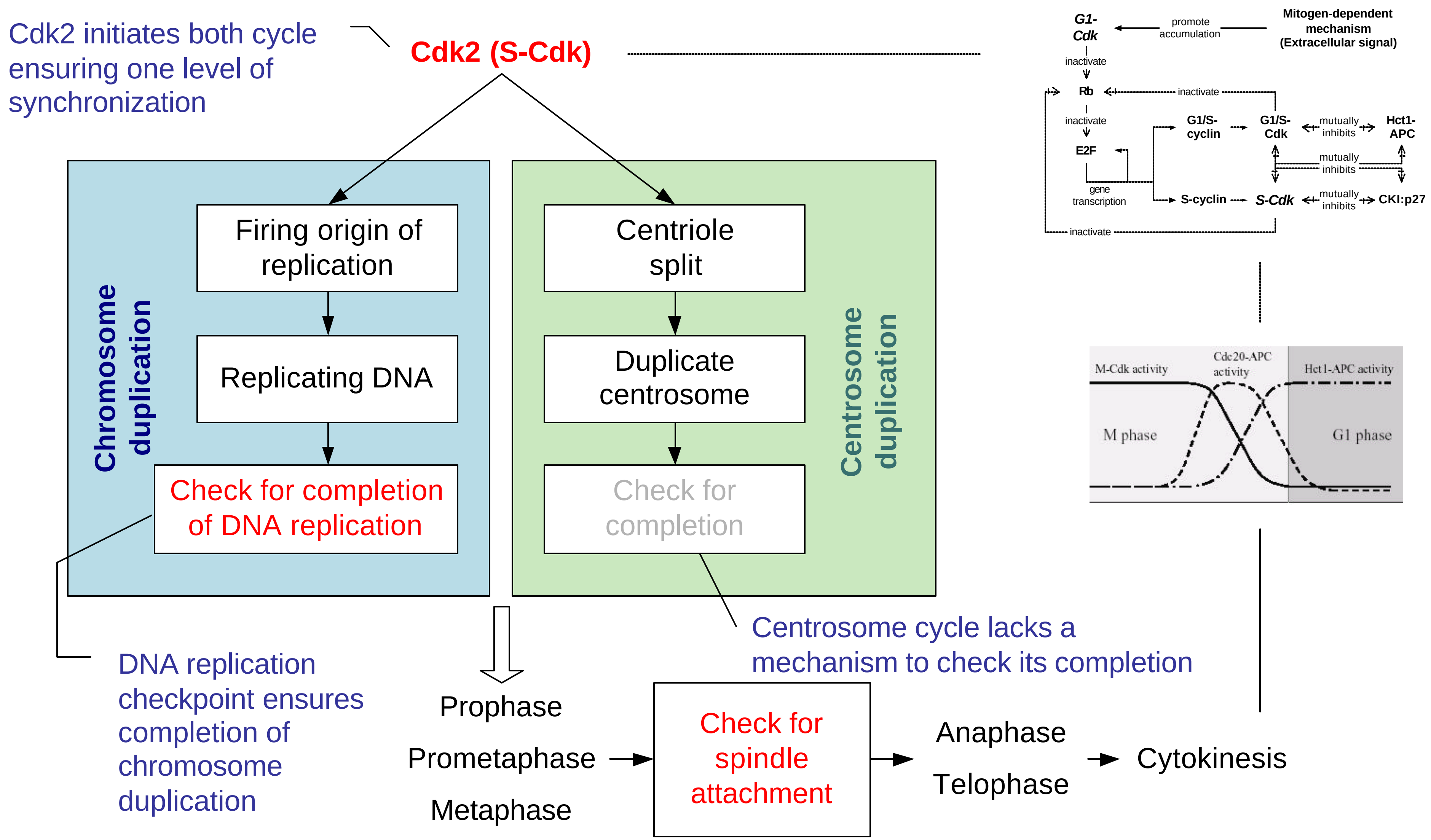
See Figure 1 in Nigg, E. A. "Centrosome abberation: cause or consequence of cancer progression?" *Nature Reviews Cancer* 2 (2002): 815-825.



Mechanism



Cdk2 initiates both cycle ensuring one level of synchronization
Cdk2 (S-Cdk)
Chromosome duplication
Firing origin of replication
Replicating DNA
Check for completion of DNA replication
Centrosome duplication
Centriole split
Duplicate centrosome
Check for completion
DNA replication checkpoint ensures completion of chromosome duplication
Centrosome cycle lacks a mechanism to check its completion
Prophase
Prometaphase
Metaphase
Check for spindle attachment
Anaphase
Telophase
Cytokinesis
G1-Cdk
promote accumulation
Mitogen-dependent mechanism (Extracellular signal)
inactivate
Rb
inactivate
inactivate
E2F
gene transcription
G1/S-cyclin
G1/S-Cdk
mutually inhibits
Hct1-APC
mutually inhibits
S-cyclin
S-Cdk
mutually inhibits
CKI:p27
inactivate
M-Cdk activity
Cdc20-APC activity
Hct1-APC activity
M phase
G1 phase

Conclusion

- A cell has a mechanism to coordinate cycles of two subsystems such that the overall periodicity is maintained

- Maintaining functional periodicity in the cell cycle is an important functional requirement for cell division
  - Can pose questions with new perspective

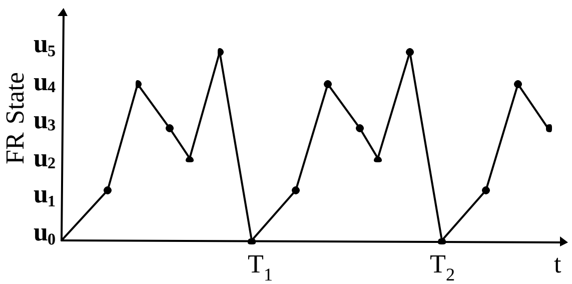
Time-dependent FR

- Functional periodicity
- $\mathbf{u(t)} = \{u_1(t), u_2(t), \ldots , u_N(t)\}$

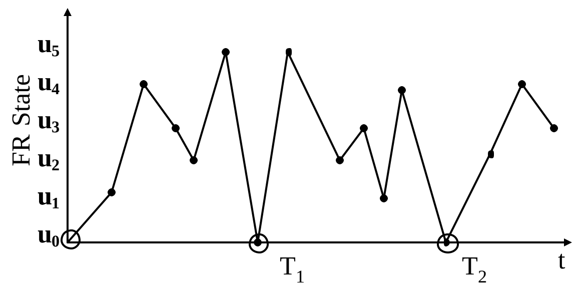
  Periodic – There exist T_i s.t. $\mathbf{u(T_i) = u(T_j)}$ with regular transition pattern

  Semi-periodic – There exist T_i s.t. $\mathbf{u(T_i) = u(T_j)}$ without regular transition pattern

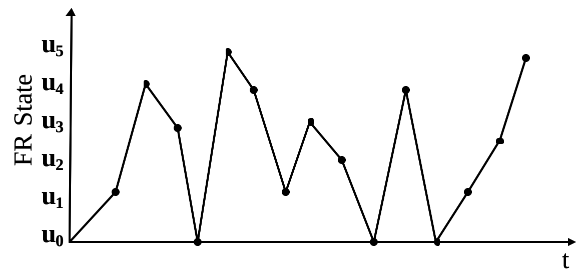
  Aperiodic – None of the above



(a) Periodic



(b) Semi-periodic



(c) Aperiodic

Uncertainty and functional periodicity

- $P_s(t) = P(\mathbf{u}(t) = \mathbf{u}^*(t))$
  - For periodic & semi-periodic ***FR***(t), P_s returns to one at the beginning of a new period
- Predictability of **FR**
  - (Periodicity) $\rightarrow$ (Predictability)
  - (Unpredictability) $\rightarrow$ (Aperiodicity) $\Leftrightarrow$

    ~ (Aperiodicity) $\rightarrow$ ~ (Unpredictability)

- Uncertainty in current period is independent of a prior period only if the initial state is properly established