2.882 System Design and Analysis

February 16

What we'll do today

- Project discussion
- Information content, Robustness

Term Project Overview

- Key dates
  - Today: Project topic discussion, kick-off
    [~ 6 wks]
  - April 4: Interim progress report
    [~ 5 wks]
  - May 11: Project presentation
  - May 16: Written project report due

- Deliverables
  - Conceptual design solution
  - AD/Complexity analysis
  - Presentation, report

Project Topic Presentation

Project Examples from the previous year

- Engine project
- CEV architecture project

Information content

- Design range
- System range
- Probability of success
- (Allowable) Tolerance

Review

Design Range

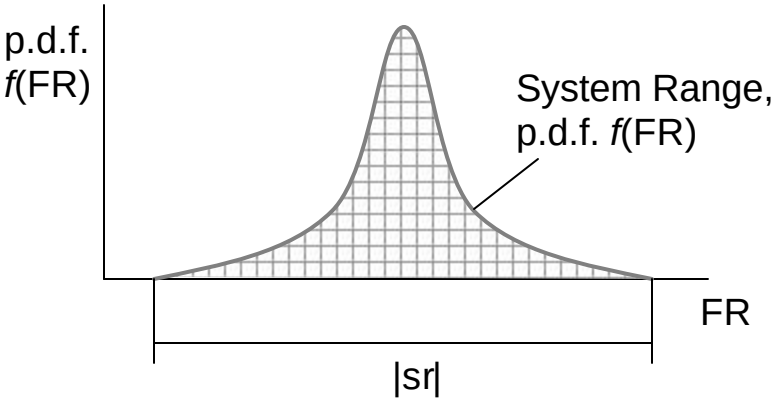
- Examples of “range” in FR statements
  - Maintain the speed of a vehicle *at a x mph +/- 5mph*
  - Ensure *no* leakage under pressure *up to 100 bar*

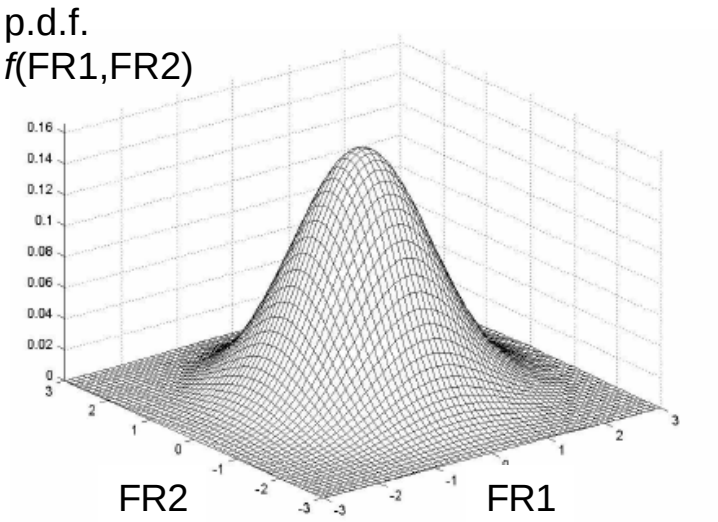
- Specification for FR
- Acceptable range of values of a chosen FR metric; Goal-post
- Different from “tolerance”
- Different from “operating range”
- Target value (nominal), Upper bound, Lower bound

Review

System Range

- Response/performance in FR domain, resulting from the chosen 'design'
  - Here, 'design' includes both a chosen set of DPs and the way they deliver/affect FRs
- Due to various factors such as the input (DP) variation, internal/external noise, etc., FR takes a range of values, forming a range

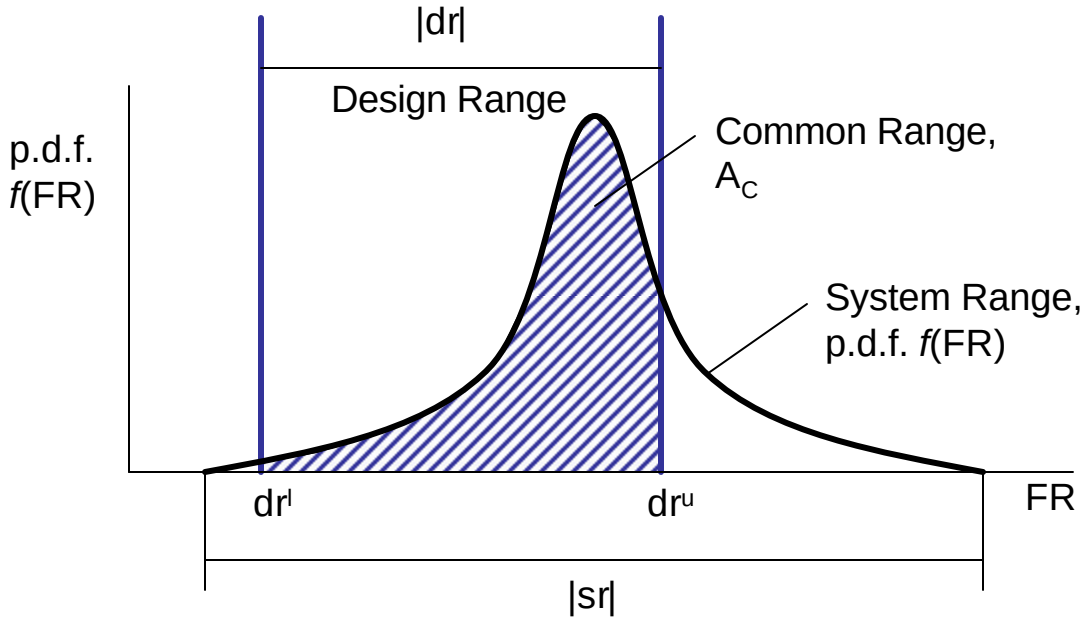




Information content

$$P(FR) = \int_{dr^l}^{dr^u} f(FR)\,dFR$$

$$I = -\log_2 P = -\log_2 P(FR) = -\log_2 \int_{dr^l}^{dr^u} f(FR)\,dFR$$



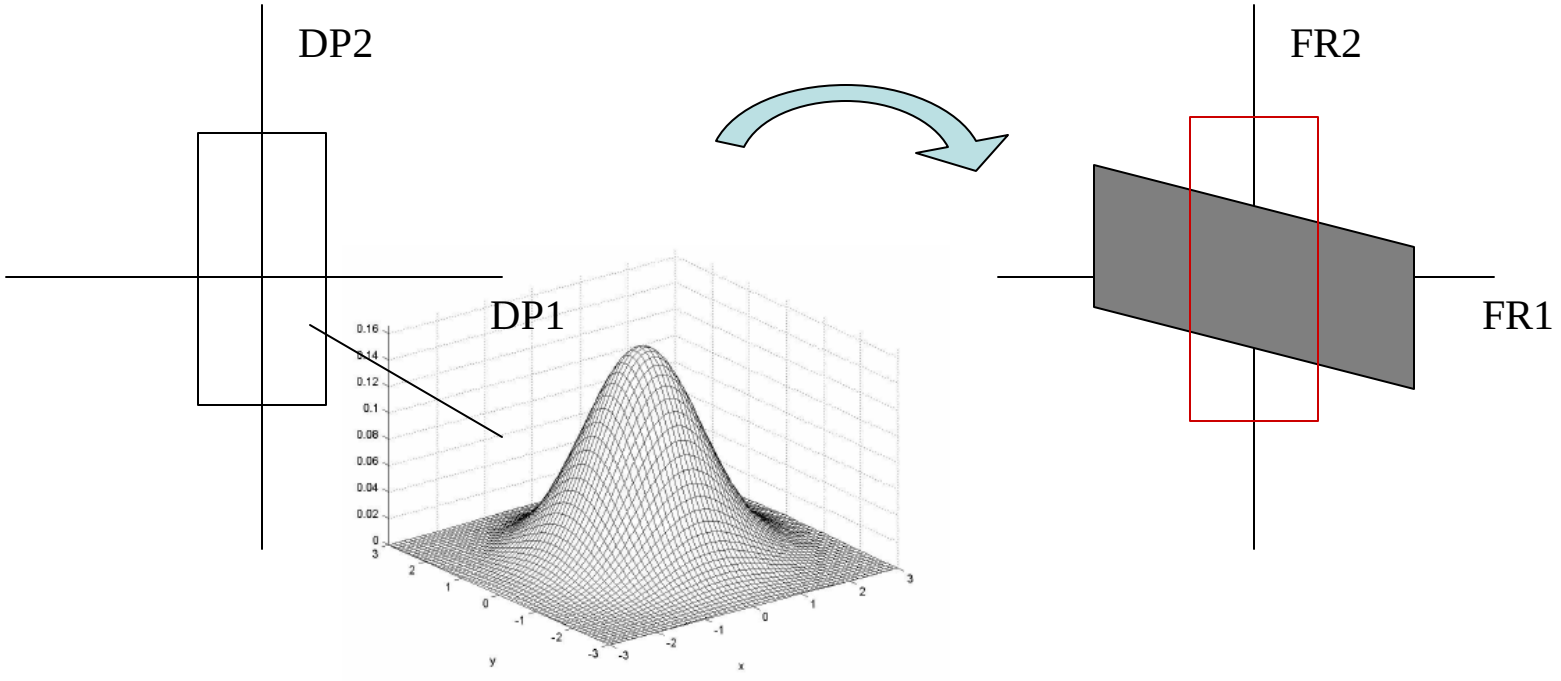
Multiple FR system range

Example

$$\begin{Bmatrix} FR1 \\ FR2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \end{Bmatrix}$$

Design range

FR1: [-0.5 , 0.5]
FR2: [-2.0 , 2.0]



Detecting change in system range

"Monitoring marginal probability of each FR is not only inaccurate but potentially misleading"

Example

$$\begin{Bmatrix} FR1 \\ FR2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \end{Bmatrix}$$

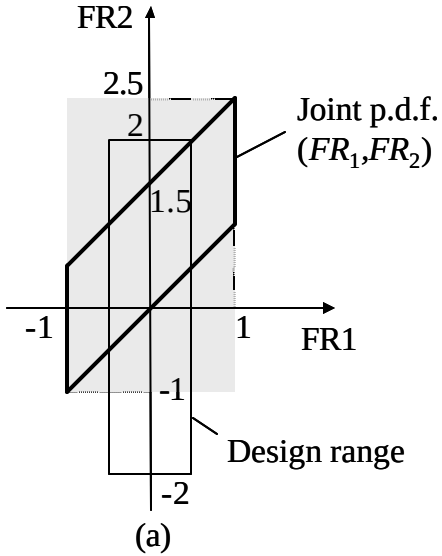
Design range

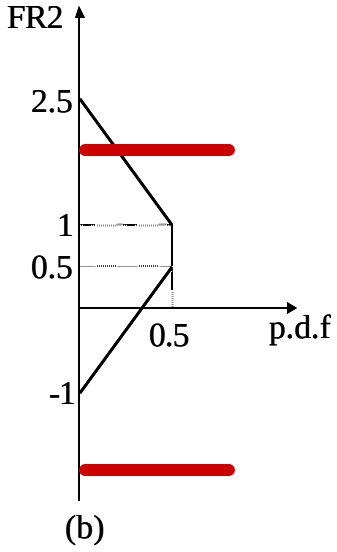
FR1: [-0.5,0.5]
FR2: [-2,2]

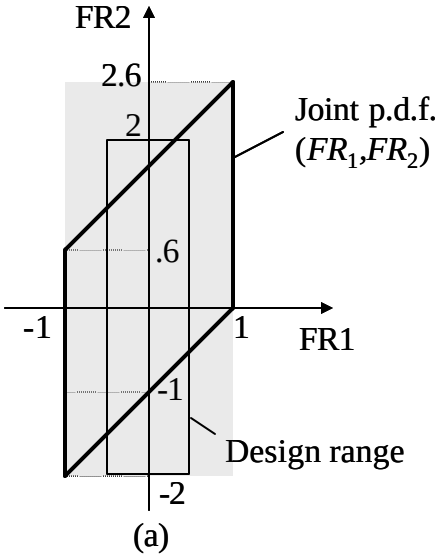
Design parameter variation

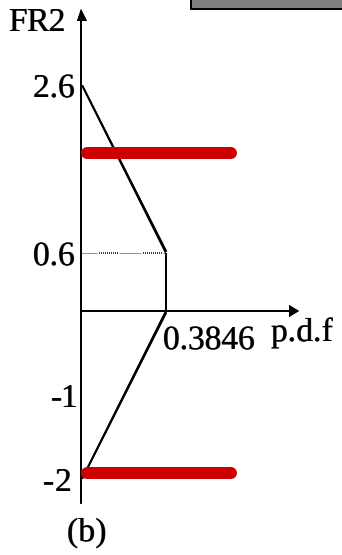
Initial
DP1: U[-1,1]
DP2: U[0,1.5]

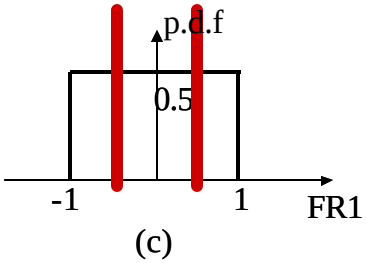
After change
DP1: U[-1,1]
DP2: U[-1,1.6]



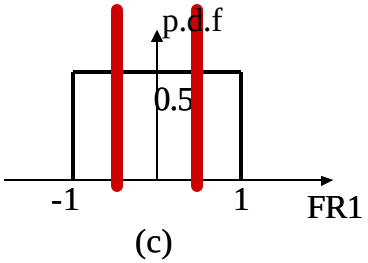








DP1: U[-1,1]
DP2: U[0,1.5]



DP1: U[-1,1]
DP2: U[-1,1.6]

Before DP2 change

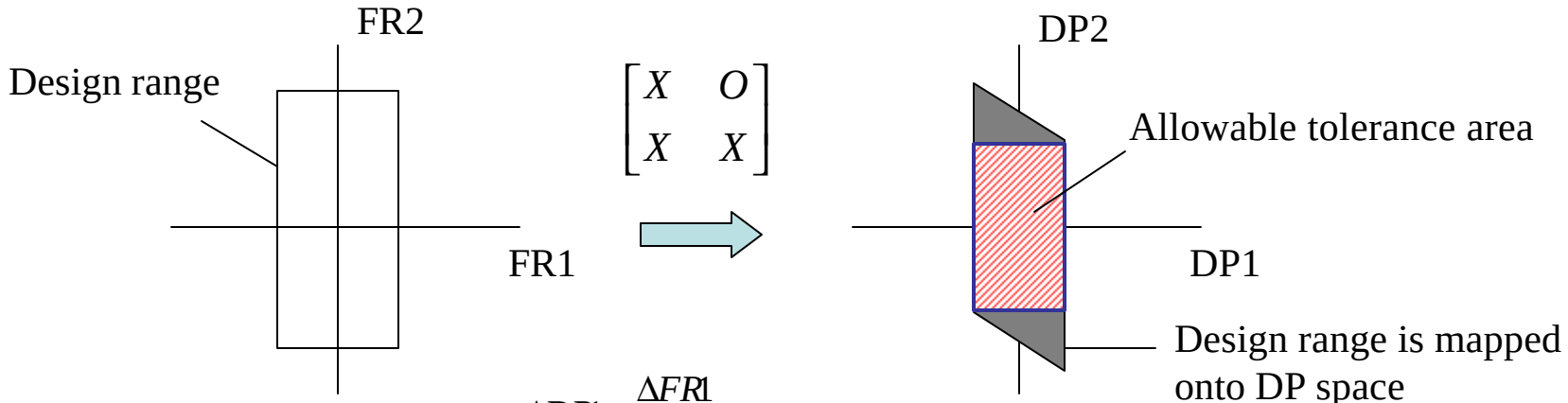
After DP2 change

	p_{FR1}	p_{FR2}	$p_{FR1} \times p_{FR2}$	$p_{FR1,FR2}$
Before	0.5	0.9583	**0.4792**	**0.5**
After	0.5	0.9654	**0.4827**	**0.499**

Review

Allowable tolerance

- Defined for DP
- Tolerances that DPs can take while FRs still remaining completely inside design ranges
- Unconditional tolerance
- Conservative tolerancing



$$\Delta DP1 = \frac{\Delta FR1}{A11}$$

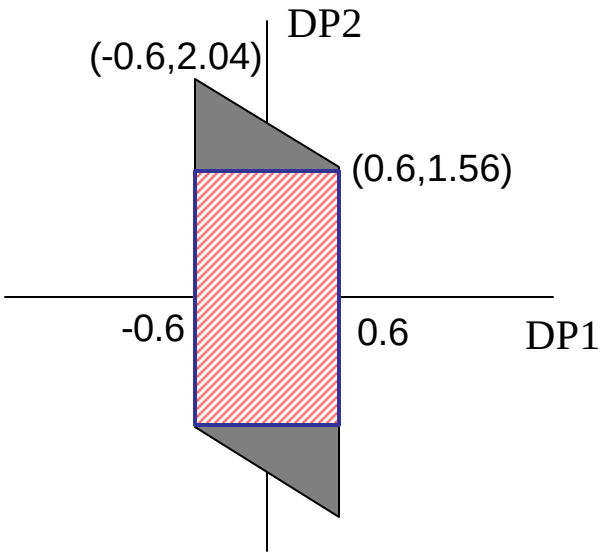
$$\Delta DP2 = \frac{\Delta FR2 - |A21 \cdot \Delta DP1|}{A22}$$

Linear tolerancing vs. Statistical tolerancing

$$\begin{Bmatrix} FR1 \\ FR2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.4 & 1 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \end{Bmatrix}$$

Design range FR1: [-0.6,0.6]
FR2: [-1.8,1.8]

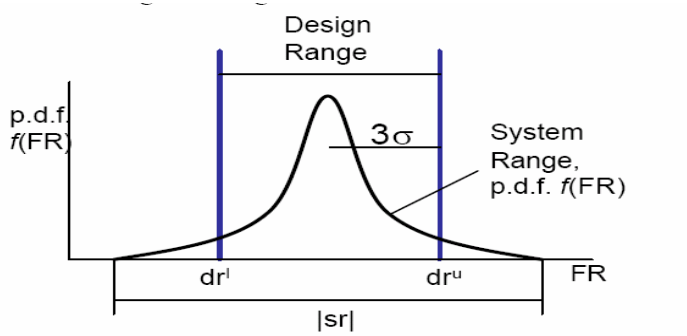
Linear tolerancing



Allowable tolerance

DP1: [-0.6,0.6]
DP2: [-1.56,1.56]

Statistical tolerancing



$3\sigma_{FR1} = 0.6 \rightarrow \sigma_{FR1} = 0.2$
Therefore, $\sigma_{DP1} = 0.2$

$Var(FR2) = 0.4^2 Var(DP1) + Var(DP2)$
Thus, $\sigma_{DP2} = 0.5946$

$3\ \sigma_{DP1} = 0.6$
$3\ \sigma_{DP2} = 1.784$

Review

Robustness

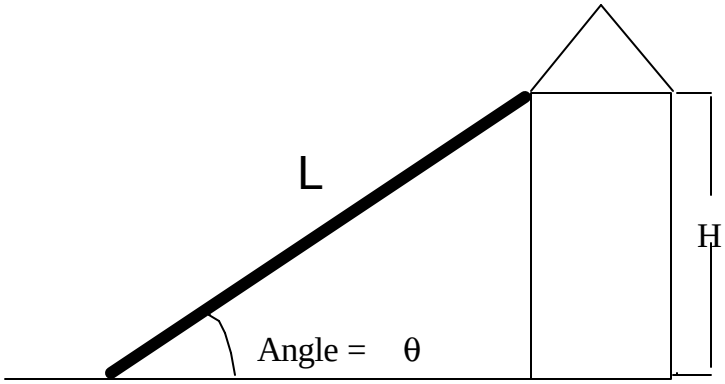
- In axiomatic design, robust design is defined as a design that always satisfies the functional requirements,

  $\Delta FRi > \delta FRi$

  even when there is a large random variation in the design parameter δDPi.

- Two different concepts in robustness
  - Insensitive to 'noise'
    - Information Axiom
    - Traditional robust design
  - Adaptive to change
    - Independence Axiom
    - Hod Lipson, Jordan Pollack, and Nam P. Suh, "On the Origin of Modular Variation", *Evolution*, *Evolution*, 56(8) pp. 1549-1556, 2002

Example: Measuring the Height of a House with a Ladder



$$H + dH = \sin q\, L + L\cos q\, dq$$

$$dH = L\cos q\, dq$$

What if L also has uncertainty?

Review

$$\overrightarrow{FR} - \overrightarrow{FR}^{*} = \left.\frac{\partial \overrightarrow{FR}}{\partial \vec{n}}\right|_{\vec{n}=0} d\vec{n} + \left.\frac{\partial \overrightarrow{FR}}{\partial \overrightarrow{DP}}\right|_{\overrightarrow{DP}=\overrightarrow{DP}^{*}} (\overrightarrow{DP} - \overrightarrow{DP}^{*}) + \left.\frac{\partial \overrightarrow{FR}}{\partial \vec{C}}\right|_{\vec{C}=\vec{C}^{*}} (\vec{C} - \vec{C}^{*})$$

2 1 3

0. Assign the largest possible tolerance
0. Eliminate the bias (E[FR] = FR*)

1. Eliminate the variation: SPC, Poka-Yoke, etc.
2. De-sensitize: Taguchi robust design
3. Compensate

$$\left.\frac{\partial \overrightarrow{FR}}{\partial \vec{C}}\right|_{\dot{C}=\dot{C}^{*}} (\vec{C} - \vec{C}^{*}) = -\left(\left.\frac{\partial \overrightarrow{FR}}{\partial \vec{n}}\right|_{\vec{n}=0} d\vec{n} + \left.\frac{\partial \overrightarrow{FR}}{\partial \overrightarrow{DP}}\right|_{\dot{DP}=\dot{DP}^{*}} (\overrightarrow{DP} - \overrightarrow{DP}^{*}) \right)$$

Robustness built into a system by design

Example: Design of Low Friction Surface

- Dominant friction mechanism: Plowing by wear debris

- System range (particle size) moves out of the desired design range
  $\Rightarrow$ Need to re-initialize

Graph and diagram removed for copyright reasons.

N. P. Suh and H.-C. Sin, Genesis of Friction, Wear, 1981

Two diagrams removed for copyright reasons.

S. T. Oktay and N. P. Suh, Wear debris formation and Agglomeration, Journal of Tribology, 1992

Design of Low Friction Surface

- Periodic undulation re-initializes the system range

Two figures (6-part diagram and pair of graphs) removed for copyright reasons.

S. T. Oktay and N. P. Suh, Wear debris formation and agglomeration, Journal of Tribology, 1992