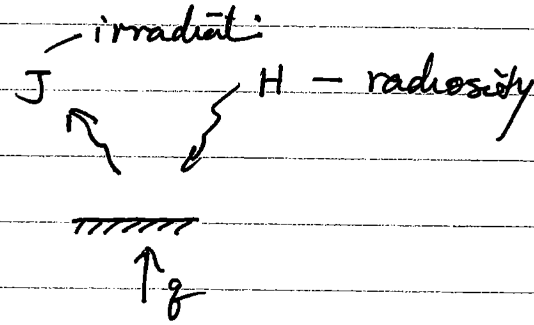
Summary of last lecture

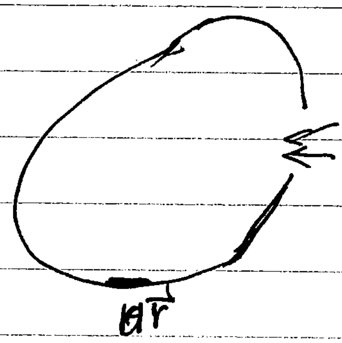
1.



$$q = J - H$$
$$= \varepsilon E_b - \alpha H$$

$$q = \frac{E_b - J}{(1-\varepsilon)/\varepsilon}$$

2.



$$H(\vec{r}) = \int_A J(\vec{r}\,')\, dF_{dA-dA'} + H_0(\vec{r})$$

$$q(\vec{r}) = \varepsilon(\vec{r}) E_b(\vec{r}) - \alpha(\vec{r}) \times \left[\int_A J(\vec{r}\,')\, dF_{dA-dA'} + H_0(\vec{r}) \right]$$

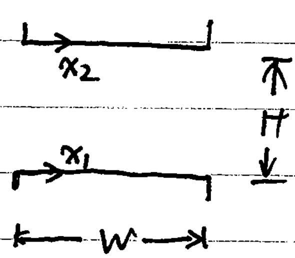
— Integral equation

0. View facter.

$$dF_{dA_i-dA_j} = \frac{\cos\theta_i \cos\theta_j}{\pi S^2}\, dA_j$$

$$A_i F_{ij} = A_j F_{ij}$$
$$F_{i(j+k)} = F_{ij} + F_{ik}$$
$$\sum_{j=1}^{N} F_{ij} = 1$$

4.



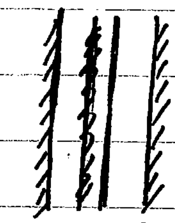
$$J_2(x_2) = \varepsilon \sigma T^4 + \frac{(1-\varepsilon)}{2} h^2 \int_0^W \frac{J_1(x_1)\, dx_1}{[h^2 + (x_2 - x_1)^2]^{3/2}}$$

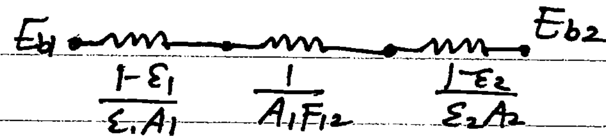
$$\varphi(x) = f(x) + \int_0^W K(x, x')\, \varphi(x')\, dx'$$

↑Kernel

Fredholm Integral eq. of 2nd kind.

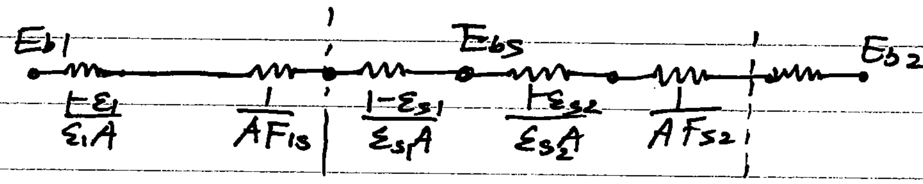
Count all resistance in the system.



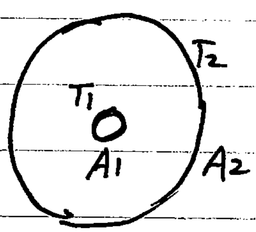


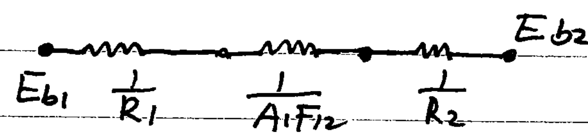
$$Q_{12} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A} + \frac{1}{A} + \frac{1-\varepsilon_2}{\varepsilon_2 A}} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

Radiation shield



$$Q_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \underbrace{\left(\frac{1}{\varepsilon_{s1}} + \frac{1}{\varepsilon_{s2}} - 1\right)}_{\text{2 surfaces of shield}}}$$

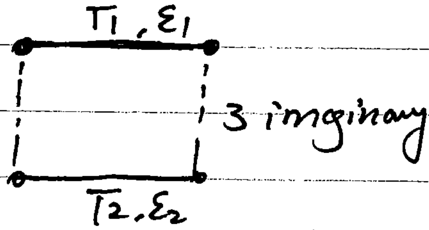




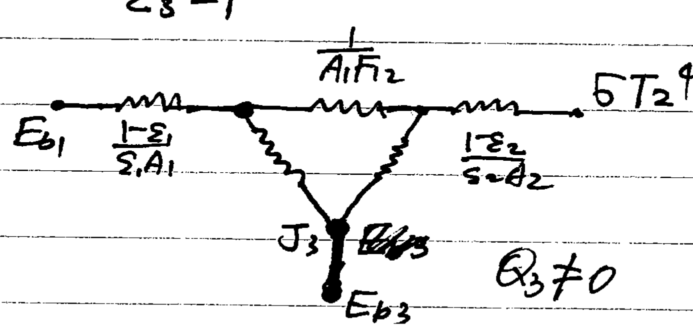
$$Q_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

$$= \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \underbrace{\frac{A_1}{A_2}}_{0}\left(\frac{1}{\varepsilon_2} - 1\right)} \approx \varepsilon_1 A_1 \sigma (T_1^4 - T_2^4)$$

T_∞

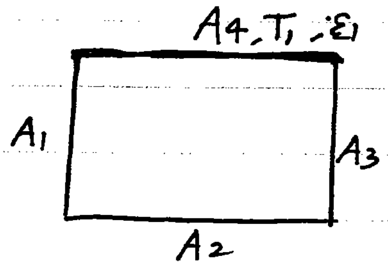


$\varepsilon_3 = 1$



$Q_3 \neq 0$

- If we assume temperature or heat flux are uniform across certain surfaces



$$J_i = \varepsilon_i E_{bi} + (1-\varepsilon)\left[\sum_{j=1}^{N} J_i F_{i-j} + H_{oi}\right]$$

or $$J_i = q_i + \sum_{j=1}^{N} J_j F_{i-j} + H_{oi}$$

or $$\frac{q_i}{\varepsilon_i} - \sum_{j=1}^{N}\left(\frac{1}{\varepsilon_i} - 1\right) F_{i-j}\, q_j + H_{oi}$$
$$= \sum_{j=1}^{N} F_{i-j}\left(E_{bi} - E_{bj}\right)$$

4 surfaces 4 equations for the above example.

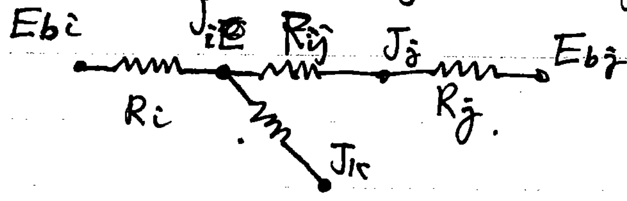
- Radiation Network Method < 3 surfaces

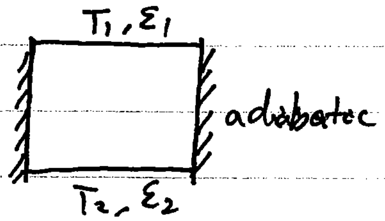
we already have $$q_i = \frac{\varepsilon_i}{1-\varepsilon_i}\left(E_{bi} - J_i\right)$$

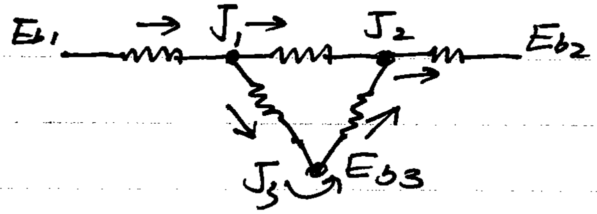
$$Q_i = \frac{A_i \varepsilon_i}{1-\varepsilon_i}\left(E_{bi} - J_i\right)$$
$$= \frac{E_{bi} - J_i}{1-\varepsilon_i / \varepsilon_i A_i} = \frac{E_{bi} - J_i}{R_i}$$

surface resistance

$$Q_{i\to j} = J_i A_i F_{i-j}$$
$$Q_{ij} = J_i A_i F_{ij} - J_j A_j F_{j\to i} = A_i F_{i-j}\left(J_i - J_j\right)$$
$$= \frac{J_i - J_j}{1/A_i F_{ij}} = \frac{J_i - J_j}{R_{ij}}$$ — spatial resistance

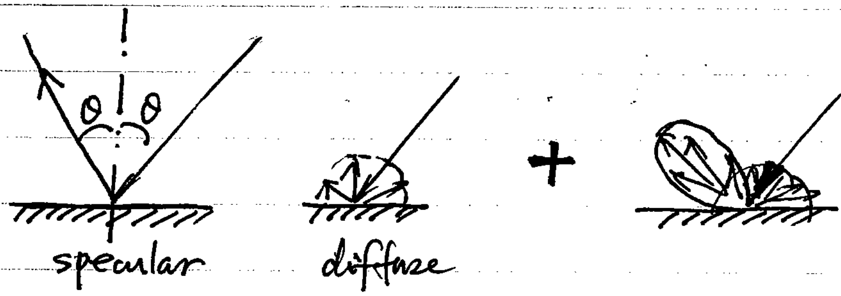






$J_3 = E_{b3}$.
$Q_3 = 0$.

* partially specular & partially diffuse surfaces.



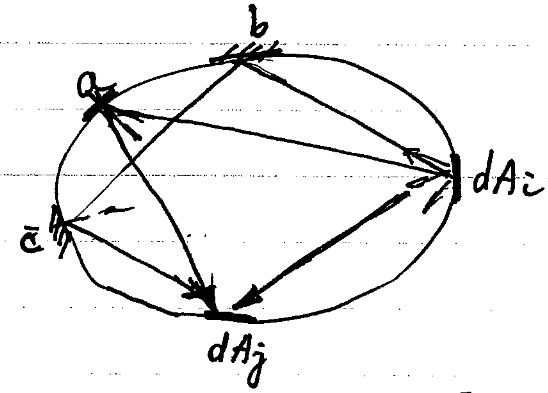
ρ''_λ gray $\rho'' \overset{\Delta\Delta}{\Rightarrow} \rho^s$

— total, hemispherical-hemispherical reflectivity.

$\rho = \rho^d + \rho^s = 1-\alpha = 1-\varepsilon$

ρ^d: diffuse; ρ^s: specular.

$1-\varepsilon$: gray — diffuse emitter



specular view factor

$$dF^s_{dA_i-dA_j} = \frac{\text{diffuse power leaving } dA_i \text{ intercepted by } dA_j \text{, by direct or specular refl.}}{\text{total diffuse power leaving } dA_i}$$

$$dF^s_{dA_i-dA_j} = dF_{dA_i-dA_j} + \rho^s_a\, dF_{dA_i(a)-dA_j} + \rho^s_b \rho^s_c\, dF_{dA_i(bc)-dA_j} + \cdots$$

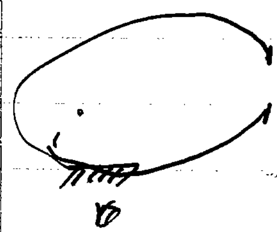
flat } mirrors dA_i – image into { same
curved   different area

Skip proof: $$dA_i F^s_{di-dj} = dA_j \, dF^s_{dj-di}$$

⇑ independent of flat or curved.

$$A_i F^s_{i-j} = A_j F^s_{j-i}$$

energy balance



$$q(\vec{r}) = \varepsilon E_b(\vec{r}) - \alpha H(\vec{r})$$
$$= \varepsilon(r)\,[E_b(\vec{r}) - H(\vec{r})]$$

$$q(\vec{r}) = J^{tot} - H(\vec{r})$$

$$J^{tot}(\vec{r}) = \underline{\rho^d(r) H(\vec{r})} + \rho^s(\vec{r}) H(\vec{r}) + \underline{\varepsilon(r) E_b(\vec{r})}$$

$$= J(\vec{r}) + \rho^s(\vec{r}) H(\vec{r})$$

↑ still call radiosity

$$q(\vec{r}) = J(\vec{r}) - [1 - \rho^s(\vec{r})]\,H(\vec{r})$$

↳ $$q(\vec{r}) = \frac{\varepsilon(\vec{r})}{\rho^d(\vec{r})}\,[(1-\rho^s)E_b - J]$$

$$H(\vec{r}) = \int_A J(\vec{r}')\, dF^s_{dA-dA'} + H_o^s(\vec{r})$$

↑ diffuse part only   after considering specular reflection

from here ⇒ Integral equation, or if $J(r')$ uniform,
set of algebraic equations.