Massachusetts Institute of Technology
Biological Engineering Division
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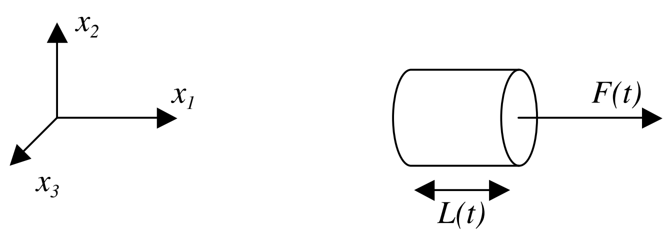
MOLECULAR, CELLULAR, & TISSUE BIOMECHANICS

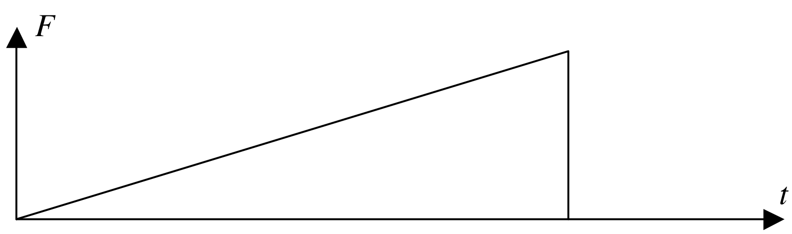
Problem Set # 5 Issued: 10/26/06
Due: 11/2/06

Problem 1
A cylindrical specimen of a particular biological sample with cross-sectional area *A* elastic modulus *E* and Poisson ration ν is being tested under a time-varying axial load, *F(t)*.



a) Give values of all non-zero components of the stress tensor.

b) If the time varying axial load is shown in the graph below, plot the deflection [*L(t)*] vs. time on the assumption that the material is linear and elastic.



c) Obtain algebraic expressions for the lateral strain (ε_{22} or ε_{33}) vs. time on the assumption that the material is isotropic and incompressible. Express your answer in terms of the applied force, cross-sectional area, and material properties.

d) How would your answer in (c) change if the material were anisotropic? (You only need to provide a qualitative answer to this.)

Problem 2
We are all aware of the fact that small organisms can support their weight with extremities that have a higher aspect ratio (length-to-radius ratio) than larger ones.

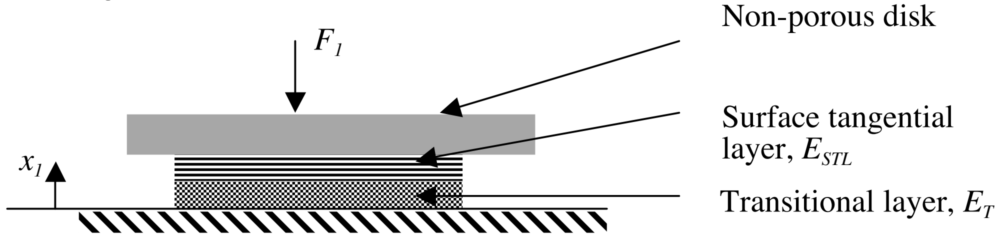
a) Given that, at least for mammals, the supporting structure is bone, and than bone has essentially the same structural properties (Young's modulus, yield stress) regardless of animal size, how would you expect bone cross-sectional area to scale as the mass of the organism changes?

b) What are the consequences if the aspect ratio of the organism were to remain constant (e.g., if a human was scaled up to have a mass comparable to that of an elephant)?

Problem 3
Cartilage is often described as being comprised of several layers: the calcified, radial, transitional and superficial tangential, each with its unique composition and structural organization. Consider specimens that are obtained from the cartilage of a horse that comprise just two of these, the transitional (T) layer and the superficial tangential layer (STL), as shown in the sketch.

a) Treating both of these layers as homogeneous, isotropic and elastic, with elastic moduli E_{STL} and E_T (with $E_{STL} > E_T$) as indicated in the sketch, consider the case of an unconfined compression experiment. Given a vertical load F_1 applied by means of a non-porous disk, and assuming that the top and bottom surfaces are free to slip in the horizontal directions, plot (qualitatively) the following variables:

σ_{11}
ε_{11}
u_1
ε_{22}



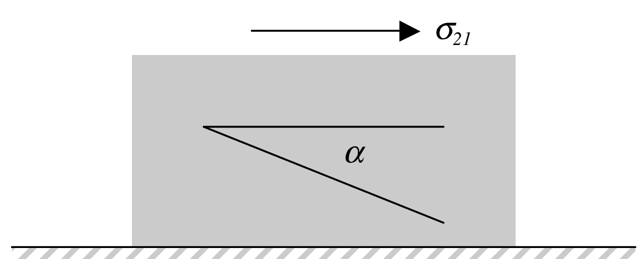
each as a function of x_1.

Are each of these variables uniform in the radial direction at each x_1 position?

Problem 4
Bone, and similar materials, often fracture due to the tensile stress exceeding some critical limit. When such a specimen is subjected to a shear stress as shown in the diagram below, it tends to fraction along a plane at a non-zero angle to the horizontal.

a) Calculate the normal and tangential (shear) stresses on this surface oriented at an arbitrary angle α to the horizontal (see diagram).

b) At what angle α does the maximum normal stress occur?



Problem 5

A small cubic tissue specimen of dimension $h = 2$ cm is tested to determine its elastic properties. If we assume that the material can be modeled as linear, isotropic and homogeneous, answer each of the following:

a) The sample is placed between two rigid surfaces and compressed by 5% of its initial height (Fig. 1a). If the material is incompressible and has a Young's modulus of 10^6 Pa, how much force would need to be applied?

b) Next, a shearing force is applied as in Fig. 1b. What is the shear modulus of the specimen, based on your measurements in (a) and the stated assumptions? How great a shear force would need to be applied to displace the top surface a distance of $0.1h$?

c) If, in the process of conducting the experiment in (a), it is observed that the lateral dimensions of the specimen increase from h to $1.02h$, what is the final volume of the specimen? Is the specimen incompressible as was assumed above? What is the Poisson ratio?

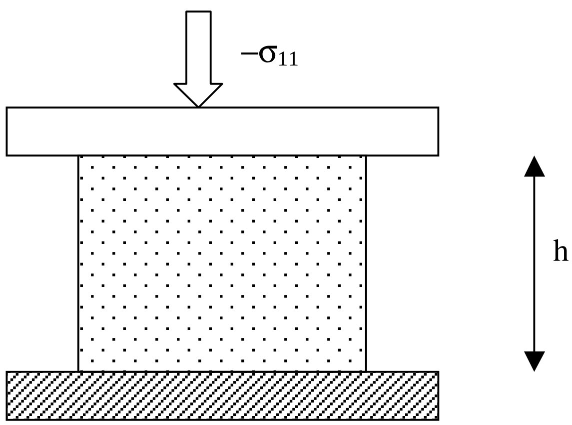


Fig. 1a

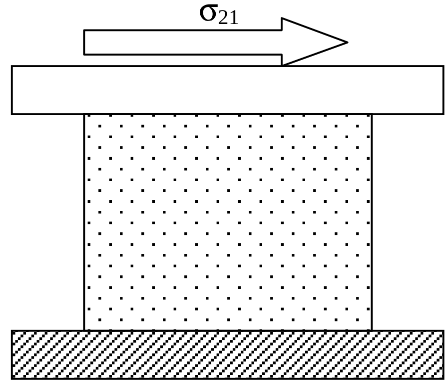


Fig. 1b

(c) Find an expression for the Young's modulus E in terms of the shear modulus G and Poisson's ratio ν by applying both forms of the constitutive laws we discussed in class to the shear configuration shown in Fig. 1b.