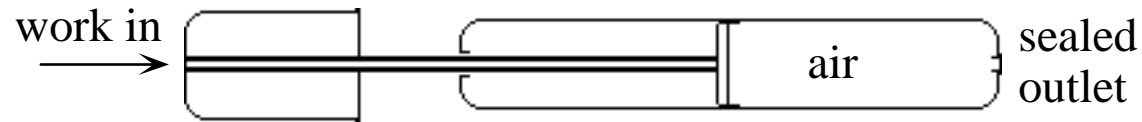


EXAMPLE: THERMAL DAMPING



A BICYCLE PUMP WITH THE OUTLET SEALED.

When the piston is depressed, a fixed mass of air is compressed.

- mechanical work is done.

The mechanical work done on the air is converted to heat.

- the air temperature rises

A temperature difference between the air and its surroundings induces heat flow.

- entropy is produced

The original work done is not recovered when the piston is withdrawn to the original piston.

- available energy is lost

MODEL THIS SYSTEM

GOAL:

the simplest model that can describe thermal damping
(the loss of available energy)

ELEMENTS:

TWO KEY PHENOMENA

work-to-heat transduction

a two port capacitor represents thermo-mechanical transduction

entropy production

a two port resistor represents heat transfer and entropy production

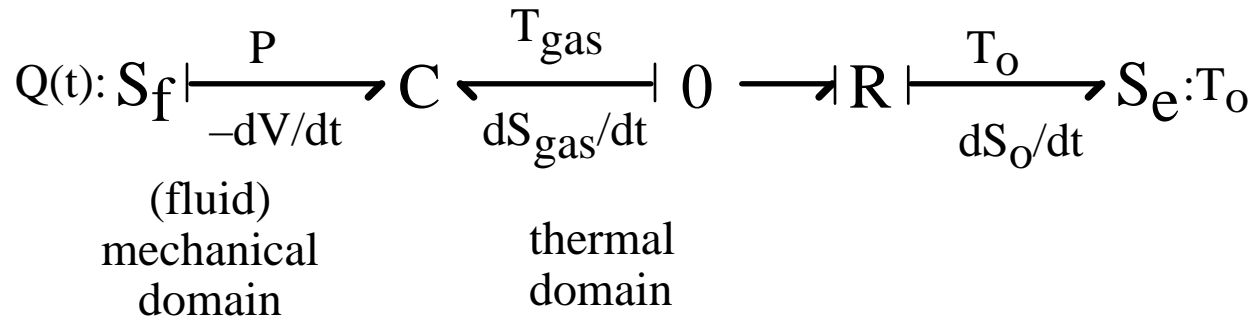
BOUNDARY CONDITIONS:

For simplicity assume

a flow source on the (fluid-)mechanical side

a constant temperature heat sink on the thermal side

A BOND GRAPH IS AS SHOWN.



CAUSAL ANALYSIS:

The integral causal form for the two-port capacitor (pressure and temperature outputs)

is consistent with the boundary conditions

and with the preferred causal form for the resistor

CONSTITUTIVE EQUATIONS:

Assume air is an ideal gas and use the constitutive equations derived above.

$$\left(\frac{T}{T_o}\right) = \left(\frac{V}{V_o}\right)^{-\frac{R}{c_v}} \exp\left(\frac{S - S_o}{mc_v}\right)$$

$$\left(\frac{P}{P_o}\right) = \left(\frac{V}{V_o}\right)^{-\left(\frac{R}{c_v} + 1\right)} \exp\left(\frac{S - S_o}{mc_v}\right)$$

Assume Fourier's law describes the heat transfer process.

$$\dot{Q} = \frac{kA}{l} (T_1 - T_2)$$

ANALYSIS:

For simplicity, linearize the capacitor equations about a nominal operating point defined by S_o and V_o

Define

$$\delta S = S - S_o$$

$$\delta V = V - V_o$$

$$\delta T = T - T_o(S_o, V_o)$$

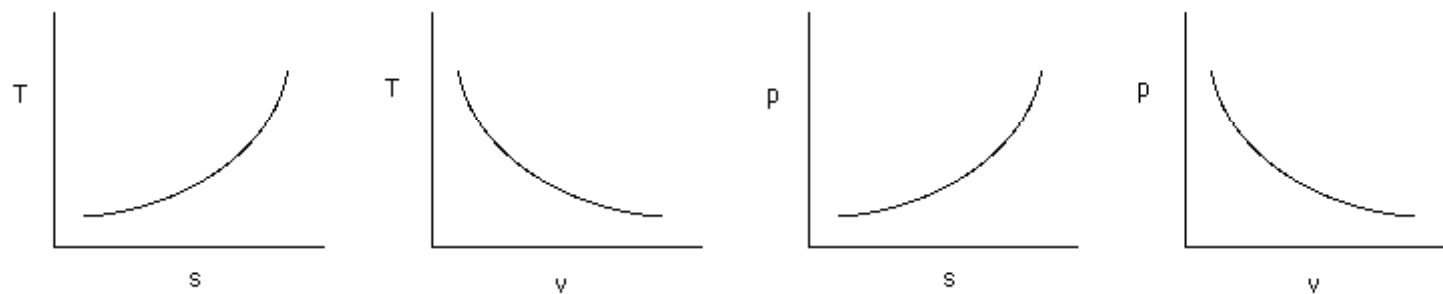
$$\delta P = P - P_o(S_o, V_o)$$

Taylor series expansion:

$$\delta T = \left. \frac{\partial T}{\partial S} \right|_{S_o, V_o} \delta S + \left. \frac{\partial T}{\partial V} \right|_{S_o, V_o} \delta V + \text{higher order terms}$$

$$\delta P = \left. \frac{\partial P}{\partial S} \right|_{S_o, V_o} \delta S + \left. \frac{\partial P}{\partial V} \right|_{S_o, V_o} \delta V + \text{higher order terms}$$

Rough sketches of the general form of the temperature and pressure equations are shown below.



From them we can see that

$$\partial T / \partial S > 0$$

$$\partial T / \partial V < 0$$

$$\partial P / \partial S > 0$$

$$\partial P / \partial V < 0$$

Neglecting the higher-order terms

(and dropping the δ prefix for convenience)

– a linearized model of the two-port capacitor is

$$T \approx AS - BV$$

$$P \approx BS - CV$$

where A, B and C are positive constants defined by

$$A = \partial T / \partial S$$

$$B = \partial P / \partial S = - \partial T / \partial V$$

$$C = - \partial P / \partial V$$

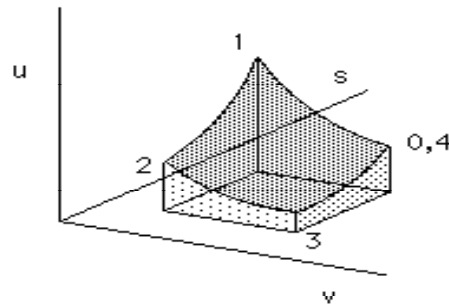
Internal energy corresponding to this linearized approximation is a quadratic form in S and V

$$U = \frac{1}{2} A S^2 - B S V + \frac{1}{2} C V^2$$

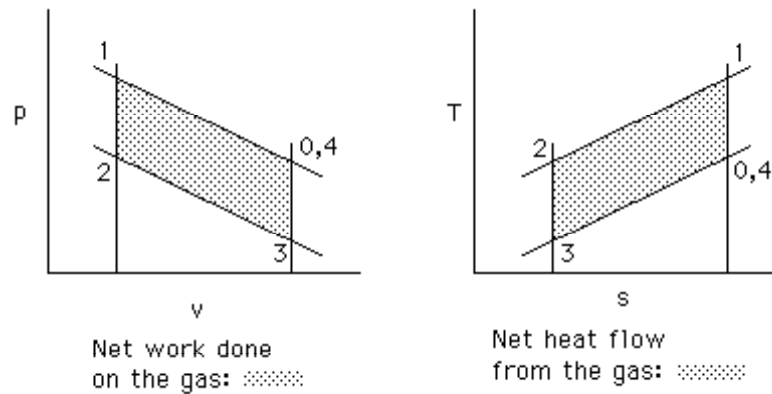
This define a quadratic “surface” with S and V as coordinates.

A WORK-TO-HEAT CYCLE

Consider a sequence of volume and entropy changes that traverse a closed cycle on the energy surface.



The same sequence of changes may also be depicted on the (linearized) pressure-volume and temperature-entropy diagrams



STATE 0:

everything at thermal equilibrium (S_0, V_0)

$$T_0 = AS_0 - BV_0$$

$$P_0 = BS_0 - CV_0$$

$$U_0 = 1/2AS_0^2 - BS_0V_0 + 1/2CV_0^2$$

TRANSITION FROM STATE 0 TO STATE 1:

Rapidly compress the gas

i.e., adiabatically – meaning no heat transfer

entropy remains constant

volume change is $-\Delta V$.

STATE 1:

immediately after compression ($S_0, V_0 - \Delta V$)

$$T_1 = AS_0 - B(V_0 - \Delta V) = T_0 + B\Delta V$$

temperature increases

$$P_1 = BS_0 - C(V_0 - \Delta V) = P_0 + C\Delta V$$

pressure increases

$$U_1 = 1/2AS_0^2 - BS_0(V_0 - \Delta V) + 1/2 C(V_0 - \Delta V)^2$$

internal energy increases by an amount equal to the work done on the gas.

$$U_1 - U_0 = P_0\Delta V + 1/2C\Delta V^2$$

TRANSITION FROM STATE 1 TO STATE 2:

Wait at constant volume while the elevated gas temperature causes heat to flow from the gas to the environment.

This heat (and therefore entropy) flow changes the entropy of the gas by $-\Delta S$.

STATE 2:

at thermal equilibrium again ($S_o - \Delta S$, $V_o - \Delta V$)

$$T_2 = T_o = A(S_o - \Delta S) - B(V_o - \Delta V) = T_o - A\Delta S + B\Delta V$$

Thus

$$A\Delta S = B\Delta V$$

The entropy change also reduces the gas pressure.

$$P_2 = B(S_o - \Delta S) - C(V_o - \Delta V) = P_1 - B\Delta S$$

$$U_2 = 1/2A(S_o - \Delta S)^2 - B(S_o - \Delta S)(V_o - \Delta V) + 1/2C(V_o - \Delta V)^2$$

The internal energy decreases by an amount equal to the heat flow out of the gas.

$$U_2 - U_1 = -T_1\Delta S + 1/2A\Delta S^2$$

TRANSITION FROM STATE 2 TO STATE 3:

Rapidly (adiabatically) expand the gas back to the original volume
entropy remains constant
volume change is ΔV

KEY POINT:

The reduced pressure means that the work done to compress the gas will *not be completely recovered on expansion.*

STATE 3:

immediately after expansion ($S_0 - \Delta S, V_0$)

$$T_3 = A(S_0 - \Delta S) - BV_0 = T_0 - A\Delta S$$

temperature decreases

$$P_3 = B(S_0 - \Delta S) - CV_0 = P_0 - B\Delta S$$

pressure decreases

$$U_3 = 1/2A(S_0 - \Delta S)^2 - B(S_0 - \Delta S)V_0 + 1/2CV_0^2$$

The internal energy decreases by an amount equal to the work done by the gas.

$$U_3 - U_2 = -P_3\Delta V - 1/2C\Delta V^2$$

THE NET WORK DONE ON THE GAS IS

$$(U_1 - U_0) + (U_3 - U_2) = B\Delta V\Delta S = A\Delta S^2$$

This is a positive quantity

– the shaded area on the pressure volume diagram

TRANSITION FROM STATE 3 TO STATE 4:

Wait at constant volume

The depressed gas temperature causes heat (and therefore entropy) to flow into the gas.

STATE 4:

at equilibrium again (S_0, V_0)
(the original state of the gas)

$$T_4 = T_0 = A S_0 - B V_0$$

$$P_4 = P_0 = B S_0 - C V_0$$

$$U_4 = U_0 = 1/2 A S_0^2 - B S_0 V_0 + 1/2 C V_0^2$$

The internal energy of the gas increases by an amount equal to the heat flow into the gas.

$$U_4 - U_3 = T_0 \Delta S - 1/2 \Delta S^2$$

THE NET HEAT FLOW OUT OF THE GAS IS

$$-(U_2 - U_1) - (U_4 - U_3) = B\Delta V\Delta S = A\Delta S^2$$

This exactly balances the net work done on the gas

– the shaded area on the temperature entropy diagram

BECAUSE THE GAS IS AT EQUILIBRIUM

(its original state)

there is no pressure gradient to induce volume change

there is no temperature gradient to induce heat flow

THE NET WORK DONE OVER THE CYCLE HAS BEEN “DISSIPATED”.

SUMMARIZING

THE GAS STORES ENERGY.

It also acts as a transducer because there are two ways to store or retrieve this energy

- two interaction ports

energy can be added or removed as work or heat.

The “energy-storing transducer” behavior is modeled as a two port capacitor.

- just like the energy-storing transducers we examined earlier.

IF POWER FLOWS VIA THE THERMAL PORT, AVAILABLE ENERGY IS REDUCED

– the system also behaves as a dissipator.

The dissipative behavior is due to heat transfer.

Gas temperature change due to compression and expansion does not dissipate available energy.

If the walls were perfectly insulated,
no available energy would be lost,
but then, no heat would flow either.

Without perfect insulation temperature gradients induce heat flow

Heat flow results in entropy generation.

Entropy generation means a loss of available energy.

THAT'S CONSISTENT WITH THE SECOND LAW.

DISCUSSION

ALL MODELS ARE FALSE.

It is essential to understand what errors our models make, and when the errors should not be ignored.

It is commonly assumed that modeling errors become significant at higher frequencies.

– not so!

Compression and expansion of gases is common in mechanical systems.

Hydraulic systems typically include accumulators (to prevent over-pressure during flow transients).

The most common design uses a compressible gas.

Compression and expansion of the gas can dissipate (available) energy.

This dissipation requires heat flow, but heat flow takes time.

For sufficiently rapid compression and expansion, little or no heat will flow, and little or no dissipation will occur.

The simplest model of a gas-charged accumulator may justifiably ignore “thermal damping”.

That is an eminently reasonable modeling decision but that model will be in error

at low frequencies

not high frequencies.

THIS IS A GENERAL CHARACTERISTIC OF PHENOMENA DUE TO THERMODYNAMIC IRREVERSIBILITIES.