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2.002 MECHANICS AND MATERIALS II
HOMEWORK NO. 4

Distributed: Friday, April 2, 2004
Due: **Friday, April 9, 2004**

Problem 1 (20 points)

Note: for reference material, consult the laboratory write-up on elastic-plastic beam bending

Consider the square cross-section beam shown, of dimensions h by h, subject to "diamond-orientation" bending in the plane shown (neutral axis: plane $y = 0$). The beam can be considered to be composed of an elastic/perfectly-plastic material having Young's modulus E, and tensile yield strength σ_y.

1. Using the standard assumptions of engineering beam theory, evaluate the magnitude of applied moment, M_y, just sufficient to bring the most highly-stressed region to the verge of yielding. Express your answer in terms of h and material properties, as appropriate. (Aside: are you "surprised" by the value you got for $I = \int y^2 \, dA$ in this orientation?)

2. If the applied curvature is increased to very large values, the elastic/plastic boundaries (tension and compression sides) in this geometry, like those in the bending of rectangular cross-sections studied earlier, will move inward, toward the neutral axis. At "infinite" curvature, the boundaries will reach opposite sides of the $y = 0$ surface, resulting in tensile yielding stress values of magnitude σ_y in one "triangle" half of the cross-section, and compressive yielding stress values of magnitude $-\sigma_y$ in the other triangular half of the cross-section. At this point, the bending moment carried by the cross-section reaches a limiting value, M_L. Evaluate M_L for this section.

3. Using your answers to the two previous questions, evaluate the ratio M_L/M_y for bending of this section. How does this value compare with the ratio for bending of this same cross-section, but on rotated axes, so that the cross-section appears as a square? (Our usual orientation for bending.)

4. Compare M_y for the "diamond" cross-section with the corresponding M_y for the square orientation. What is the ratio of these first-yield bending moments? Explain why they differ in the way that they do. Evaluate the same ratio for the corresponding limit moments, and M_L, and comment on reasons why they differ. Which axes should be used for applying bending moments to a square section, and why?

5. Discuss the residual stress state when the diamond-orientation is unloaded to $M = 0$ immediately after being deformed to large curvature at $M = M_L$. How does this residual stress state compare or contrast with the state for unloading of the square orientation from its limit value of M? Can any negative moment be applied to the diamond cross-section after unloading from limit load, without causing further plasticity? Discuss

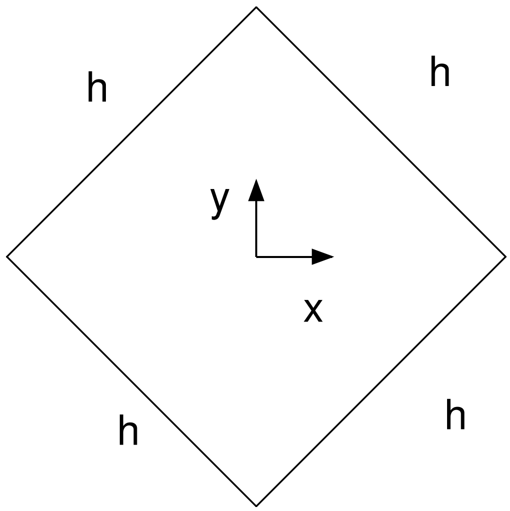

Figure 1: Square cross-section of beam, oriented for bending along "diamond" orientation.

Problem 2 (30 points)

A great deal of the mechanisms and phenomenology of the strengthening of metallic crystals can be summarized in the following phrase:

"Smaller is stronger . . ."

Discuss three specific examples of strengthening mechanisms, and explain how and why the aphorism "smaller is stronger" applies to each strengthening mechanism.

Problem 3 (30 points)

Standard cylindrical compression specimens have an initial height to diameter ratio of $H_0/D_0 = 2$. It is desired to conduct a compression test in a demonstration lab, and to compress the specimen to a final height of $H = H_0/2$.

From prior testing, it is known that the material has Young's modulus $E = 200\,GPa$, Poisson ratio $\nu = 0.3$, and its plasticity can be well characterized by an initial value of tensile/compressive yield strength as $s_0 = 500\,MPa$, along with a constant hardening modulus, $h = 2\,GPa$, governing the evolution of uniaxial flow strength, s, with equivalent plastic strain, $\bar{\epsilon}^p$, according to

$$\frac{ds}{d\bar{\epsilon}^p} = h = \text{constant}.$$

In turn, this expression can be integrated to express the current value of strength, for any given value of $\bar{\epsilon}^p \geq 0$, as

$$s(\bar{\epsilon}^p) = s_0 + h\,\bar{\epsilon}^p.$$

The load cell on the testing machine to be used for the compression test has a maximum load capacity of $100\,kN$.

You are asked to provide an answer to the following question:

"What is the largest allowed value of initial diameter in a compression specimen of this material ($D_{0(\text{max})}$) that can be safely compressed to half its initial height in the testing machine?"

In particular:

- **(10 points) Explain why the elastic strain is <u>not</u> an important feature in answering this problem. That is, explain why, for this application, you may assume that the material is rigid/plastic, so that the total strains and strain rates are essentially equal to the plastic strains and strain rates, respectively.**

- **(20 Points) What is the largest diameter that can safely be used for the compression specimen, under the imposed conditions?**

HINTS:

- Remember, for active yielding in uniaxial compression, the axial [true]stress, σ, is negative, so the yield criterion becomes $s = \bar{\sigma} = -\sigma$.

- For monotonic loading in compression, the plastic portion of the [true] axial strain, $\epsilon \doteq \epsilon^{(p)}$, is negative, and is thus related to the equivalent plastic strain by $-\epsilon^{(p)} \doteq -\epsilon = \bar{\epsilon}^p$.

Problem # 4 (20 points)

Long bars of an alloy steel are available in stock of rectangular cross-section, with [initial] thickness $t_0 = 25\,mm$ and width $w_0 = 100\,mm$. It is desired to use these bars as tensile-loaded truss members, and to be able to apply tensile loads up to $P_{\max} = 1.1MN$ without causing plastic yielding in the bars. The initial tensile yield strength of the steel is $\sigma_y = 350\,MPa$.

- **Can the as-received bars support a load of magnitude $P_{\max} = 1.1MN$ without yielding? How much tensile load can it support without yielding?**
- It is known that the tensile flow strength, s, of this steel increases with equivalent tensile plastic strain, $\bar{\epsilon}^p$, according to

  $$s(\bar{\epsilon}^p) = \sigma_y \left(1 + \frac{\bar{\epsilon}^p}{c}\right)^N ,$$

  where the strain hardening exponent is $N = 0.14$, and the constant $c = 0.01$. Someone suggests that it may be possible to cold-roll the bar stock to a new cross-sectional shape, of reduced thickness t, but essentially the same width, $w = w_0$, and in the process generate enough equivalent plastic strain and associated strain-hardening so that the rolled bar stock can be used as truss members that <u>can</u> support tensile loads up to $P_{\max} = 1.1MN$ without [further] plastic yielding, even though the rolling reduces the thickness and cross-sectional area of the bar. We will explore this possibility.

  First note that the equivalent plastic strain increment, $d\bar{\epsilon}^p$, can be expressed in terms of the cartesian components of the plastic strain increment tensor, $d\epsilon_{ij}^{(p)}$, by

  $$d\bar{\epsilon}^p = \sqrt{\frac{2}{3}\sum_{i=1}^{3}\sum_{j=1}^{3} d\epsilon_{ij}^{(p)} d\epsilon_{ij}^{(p)}}.$$

  Let the rolling direction (along the length of the bar) be cartesian direction number 1, let the through-thickness direction be 2, and let the breadth direction be 3. In the process of rolling, there is an incremental reduction in thickness, $dt < 0$, so that

  $$d\epsilon_{22}^{(p)} = \frac{dt}{t} < 0.$$

  As noted above, there is negligible transverse plastic straining in rolling, so $d\epsilon_{33}^{(p)} \doteq 0$. Assume further that rolling introduces no change in plastic shear strains (i.e., $d\epsilon_{12}^{(p)} = d\epsilon_{13}^{(p)} = d\epsilon_{23}^{(p)} = 0$).

  Obtain an expression for $d\bar{\epsilon}^p$ in terms of t and $|dt|$, and show how this expression can be integrated to give

  $$\bar{\epsilon}^{(p)} = \frac{2}{\sqrt{3}} \ln\left(\frac{t_0}{t}\right).$$

  HINT: something needs to be done about evaluating $d\epsilon_{11}^{(p)}$...

- **What is the maximum rolling-reduced bar thickness, $t = t_{\max}$, which gives a strain-hardened strength s and rolling-reduced thickness $t = t_{\max}$ combination such that the cold-rolled bar stock does, indeed, support tensile load $P_{\mathrm{mx}} = 1.1MN$ without further yielding?**

  Note: this part of the problem may best be solved by performing a set of numerical evaluations, for different values of thickness, and finding out which t-value answers the question.