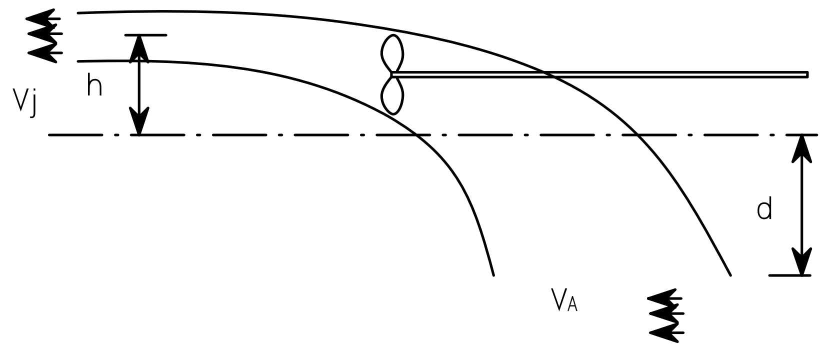
Waterjet

first draft 9/23/04 from Prof. Carmichael notes.
9/17/06: modified to reflect w (V=>V_A) and separate inlet and outlet pressure loss (in addition to drag) to reflect paper

V_A velocity inlet
w wake fraction
Vs ship velocity
Vj nozzle (outlet) velocity



$$V_A := V_s \cdot (1 - w)$$

$$T = m\_dot \cdot (V_J - V_A)$$

$$m\_dot = mass\_flow\_rate$$

at inlet centerline ... $p_{local} = p_{atmos} + \rho \cdot g \cdot d$

at this point ... total pressure (pitot tube)

$$p_{oin} = p_{local} + \frac{1}{2} \cdot \rho \cdot V_A^{\,2} = p_{atmos} + \rho \cdot g \cdot d + \frac{1}{2} \cdot \rho \cdot V_A^{\,2}$$

pressure at inlet to pump ...
total pressure (pitot tube) ...

$$p_{op} = p_{oin} - \rho \cdot g \cdot (d + h) = p_{atmos} - \rho \cdot g \cdot h + \frac{1}{2} \cdot \rho \cdot V_A^{\,2}$$

pressure at pump exit... total pressure (pitot tube) ..

$$p_{oj} = p_{atmos} + \frac{1}{2} \cdot \rho \cdot V_j^{\,2}$$

total pressure increase across pump ...

$$p_{oj} - p_{op} = p_{atmos} + \frac{1}{2} \cdot \rho \cdot V_j^{\,2} - p_{atmos} + \rho \cdot g \cdot h - \frac{1}{2} \cdot \rho \cdot V_A^{\,2} = \frac{1}{2} \cdot \rho \cdot \left(V_j^{\,2} - V_A^{\,2}\right) + \rho \cdot g \cdot h$$

energy rise across the pump per unit mass flow is ...

$$\frac{p_{oj} - p_{op}}{\rho} = \frac{1}{2} \cdot \left(V_j^{\,2} - V_A^{\,2}\right) + g\,h$$

power absorbed by ideal pump is therefore ...

$$P_{pi} = m\_dot \cdot \frac{p_{oj} - p_{op}}{\rho} = m\_dot \cdot \left[\frac{1}{2} \cdot \left(V_j^{\,2} - V_A^{\,2}\right) + g\,h\right]$$

ideal efficiency is then ... and quasi propulsive coefficient is ...

$$\eta_i = \frac{P_{Ti}}{P_{pi}} \qquad \eta_D = \frac{effective\_power}{power\_delivered} = \frac{P_E}{P_{pi}} = \frac{R \cdot V_s}{P_{Pi}} = \frac{(1 - t) \cdot T \cdot V_s}{P_{Pi}} = \frac{1 - t}{1 - w} \cdot \frac{T \cdot V_A}{P_{Pi}} = \frac{1 - t}{1 - w} \cdot \frac{P_{Ti}}{P_{pi}} = \frac{1 - t}{1 - w} \cdot \eta_i$$

$$\eta_i = \frac{P_{Ti}}{P_{pi}} = \frac{T \cdot V_A}{P_{pi}} = \frac{m\_dot \cdot V_A \cdot (V_J - V_A)}{m\_dot \cdot \left[\frac{1}{2} \cdot \left(V_j^{\,2} - V_A^{\,2}\right) + g\,h\right]} = \frac{2 \cdot V_A \cdot (V_J - V_A)}{V_j^{\,2} - V_A^{\,2} + 2g\,h} = \frac{2 \cdot V_A \cdot (V_J - V_A)}{V_j^{\,2} - V_A^{\,2} + 2g\,h} = \frac{2 \cdot \left(\frac{V_J}{V_A} - 1\right)}{\left(\frac{V_j}{V_A}\right)^2 - 1 + 2\frac{g\,h}{V^2}}$$

if h = 0

$$\eta_i = \frac{2\cdot\left(\frac{V_J}{V_A} - 1\right)}{\left(\frac{V_j}{V_A}\right)^2 - 1} = \frac{2}{\frac{V_j}{V_A} + 1}$$

same as propeller (we developed following in actuator disk

from actuator_disk.mcd using new variables to avoid duplication

$$\Delta v := VV_j - VV_A$$

$$VV := VV_A + \frac{\Delta v}{2}$$

$$\eta_I := \frac{T\cdot VV_A}{T\cdot V} \text{ simplify } \rightarrow 2\cdot\frac{VV_A}{VV_A + VV_j}$$

$$\eta_I = \frac{2}{1 + \frac{VV_j}{VV_A}}$$

what are implications of $VV_j = VV_A$?

h cannot be negative (would be ducted prop, h limits efficiency

Real waterjet with losses

net thrust of waterjet $T_{net} = T - Drag_{inlet}$ $T = m\_dot\cdot\left(V_j - V_A\right)$

conventional drag coefficient $C_d = \frac{Drag}{\frac{1}{2}\cdot\rho\cdot v^2\cdot A}$

drag coefficient of inlet $C_D = \frac{Drag}{\frac{1}{2}\cdot\rho\cdot v^2\cdot A} = \frac{Drag}{\frac{1}{2}\cdot\rho\cdot V_A\cdot A\cdot V_A} = \frac{Drag}{\frac{1}{2}\cdot m\_dot\cdot V_A}$

net thrust

$$T_{net} = T - Drag_{inlet} = m\_dot\cdot\left(V_j - V_A\right) - C_D\cdot\frac{1}{2}\cdot m\_dot\cdot V_A = m\_dot\cdot V_A\cdot\left[\left(\frac{V_j}{V_A} - 1\right) - C_D\cdot\frac{1}{2}\right]$$

net thrust power

$$P_{T\_net} = T_{net}\cdot V_A = m\_dot\cdot V_A^2\cdot\left[\left(\frac{V_j}{V_A} - 1\right) - C_D\cdot\frac{1}{2}\right]$$

delta p across pump must be increased to account for losses ...
we'll assume separate inlet and outlet losses
assume internal losses are ... ~ 1/2*ρ*v^2
and the pump pressure rise is ...

$$\Delta p_{loss} = \Delta p_{in\_loss} + \Delta p_{out\_loss}$$

non-dimensional pressure loss coefficient is ....
and the real pump pressure rise is ...

$$K_{in} = \frac{\Delta p_{in\_loss}}{\frac{1}{2}\cdot\rho\cdot V_A^2} \qquad K_{out} = \frac{\Delta p_{out\_loss}}{\frac{1}{2}\cdot\rho\cdot V_j^2}$$

$$p_{oj} - p_{op} = \frac{1}{2}\cdot\rho\cdot\left(V_j^2 - V_A^2\right) + \rho\cdot g\cdot h + \Delta p_{loss} = \frac{1}{2}\cdot\rho\cdot\left(V_j^2 - V_A^2\right) + \rho\cdot g\cdot h + K_{in}\cdot\frac{1}{2}\cdot\rho\cdot V_A^2 + K_{out}\cdot\frac{1}{2}\cdot\rho\cdot V_j^2$$

$$p_{oj} - p_{op} = \frac{1}{2}\cdot\rho\cdot V_A^2\cdot\left[\left(\frac{V_j}{V_A}\right)^2 - 1 + 2\cdot\frac{g\cdot h}{V_A^2} + K_{in} + K_{out}\cdot\left(\frac{V_j}{V_A}\right)^2\right]$$

ideal pump power is ... $$P_{pi} = m\_dot \cdot \frac{p_{oj} - p_{op}}{\rho} = m\_dot \cdot \frac{1}{2} \cdot V_A^{\,2} \cdot \left[\left(\frac{V_j}{V_A} \right)^2 - 1 + 2 \cdot \frac{g \cdot h}{V^2} + K_{in} + K_{out} \cdot \left(\frac{V_j}{V_A} \right)^2 \right]$$

define η_p such that $$\eta_p = \frac{P_{pi}}{P_p} \qquad P_p = actual\_pump\_power$$

$$P_p = \frac{P_{Pi}}{\eta_p} = \frac{m\_dot}{\eta_p} \cdot \frac{p_{oj} - p_{op}}{\rho} = \frac{1}{2} \cdot \frac{m\_dot \cdot V_A^{\,2}}{\eta_p} \cdot \left[\left(\frac{V_j}{V_A} \right)^2 - 1 + 2 \cdot \frac{g \cdot h}{V^2} + K_{in} + K_{out} \cdot \left(\frac{V_j}{V_A} \right)^2 \right]$$

$$\eta_{real} = \frac{P_{T\_net}}{P_p} = \frac{m\_dot \cdot V_A^{\,2} \cdot \left[\left(\frac{V_j}{V_A} - 1 \right) - C_D \cdot \frac{1}{2} \right]}{\frac{1}{2} \cdot \frac{m\_dot \cdot V_A^{\,2}}{\eta_p} \cdot \left[\left(\frac{V_j}{V_A} \right)^2 - 1 + 2 \cdot \frac{g \cdot h}{V_A^{\,2}} + \left[K_{in} + K_{out} \cdot \left(\frac{V_j}{V_A} \right)^2 \right] \right]}$$

for a different form ... multiply numerator and denominator by (V_A/V_j)^2 ...

$$\eta_{real} = \frac{2 \cdot \eta_p \cdot \left[\left(\frac{V_j}{V_A} - 1 \right) - C_D \cdot \frac{1}{2} \right]}{\left(\frac{V_j}{V_A} \right)^2 - 1 + 2 \cdot \frac{g \cdot h}{V_A^{\,2}} + K_{in} + K_{out} \cdot \left(\frac{V_j}{V_A} \right)^2} = \frac{2 \cdot \eta_p \cdot \frac{V_A}{V_j} \cdot \left[\left(1 - \frac{V_A}{V_j} \right) - C_D \cdot \frac{1}{2} \cdot \frac{V_A}{V_j} \right]}{1 - \left(\frac{V_A}{V_j} \right)^2 + 2 \cdot \frac{g \cdot h}{V_j^{\,2}} + K_{in} \cdot \left(\frac{V_A}{V_j} \right)^2 + K_{out}}$$

and substitute μ for V_A/V_j ... $$\eta_{real} = \frac{2 \cdot \eta_p \cdot \mu \cdot \left[(1 - \mu) - C_D \cdot \frac{1}{2} \cdot \mu \right]}{\mu^2 \cdot (K_{in} - 1) + 1 + K_{out} + 2 \cdot \frac{g \cdot h}{V_j^{\,2}}} = \frac{2 \cdot \eta_p \cdot \mu \cdot \left[(1 - \mu) - C_D \cdot \frac{1}{2} \cdot \mu \right]}{1 + K_{out} - \mu^2 \cdot (1 - K_{in}) + 2 \cdot \frac{g \cdot h}{V_j^{\,2}}}$$

and the quasi propulsive coefficient is then ..

$$\eta_D = \frac{1 - t}{1 - w} \cdot \eta_p \cdot \frac{2 \cdot \eta_p \cdot \left[\left(\frac{V_j}{V_A} - 1 \right) - C_D \cdot \frac{1}{2} \right]}{\left(\frac{V_j}{V_A} \right)^2 - 1 + 2 \cdot \frac{g \cdot h}{V_A^{\,2}} + K_{in} + K_{out} \cdot \left(\frac{V_j}{V_A} \right)^2} = \frac{1 - t}{1 - w} \cdot \eta_p \cdot \frac{2 \cdot \mu \cdot \left[(1 - \mu) - C_D \cdot \frac{1}{2} \cdot \mu \right]}{1 + K_{out} - \mu^2 \cdot (1 - K_{in}) + 2 \cdot \frac{g \cdot h}{V_j^{\,2}}}$$

as from above ...

net thrust power $$P_{T\_net} = T_{net} \cdot V = m\_dot \cdot V_A^{\,2} \cdot \left[\left(\frac{V_j}{V_A} - 1 \right) - C_D \cdot \frac{1}{2} \right]$$

first some comments to relate to previous lecture/notes version and Wärtsilä paper

with K_{out} = 0 (N.B. this just means lumping all the pressure losses into a factor of 1/2* ρ*V_A^2 and accounting for a drag increase due to the inlet ...

$$\eta_D = \frac{1-t}{1-w}\cdot\eta_p\cdot\frac{2\cdot\eta_p\cdot\left[\left(\frac{V_j}{V_A}-1\right)-C_D\cdot\frac{1}{2}\right]}{\left(\frac{V_j}{V_A}\right)^2-1+2\cdot\frac{g\cdot h}{V_A{}^2}+K}$$

this is the form previously

and ... with C_D = 0 and assuming h = 0 (i.e. head loss is small compared to other terms ...

this is the form in the paper with

$$\eta_D = \frac{1-t}{1-w}\cdot\eta_p\cdot\frac{2\cdot\mu\cdot(1-\mu)}{1+K_{out}-\mu^2\cdot(1-K_{in})}$$

$$K_{out} = \phi = \text{nozzle\_loss\_coefficient} \qquad K_{out} = \frac{\Delta p_{out\_loss}}{\frac{1}{2}\cdot\rho\cdot V_j{}^2}$$

$$K_{in} = \varepsilon = \text{inlet\_loss\_coefficient} \qquad K_{in} = \frac{\Delta p_{in\_loss}}{\frac{1}{2}\cdot\rho\cdot V_A{}^2}$$

at this point, assuming K, C_D, and η_p are constant, could differentiate wrt V_j/V_A (or μ) and determine V_j/V_A for max propulsive coefficient, but minimum weight usually determines parameters.

▶ pump background (Wislicenus)

▶ example