2.20 - Marine Hydrodynamics, Spring 2005

Lecture 19

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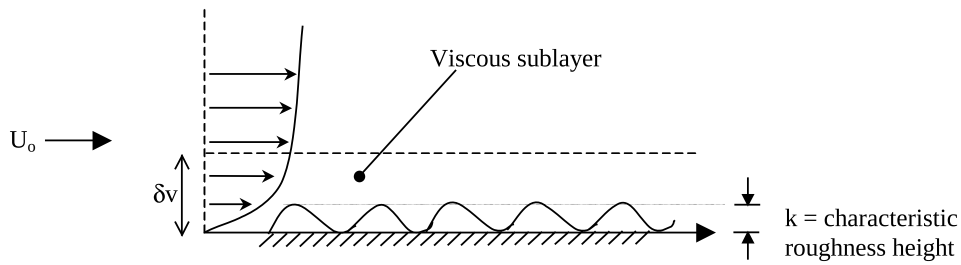
Turbulent Boundary Layers: Roughness Effects

So far, we have assumed a 'hydraulically smooth' surface. In practice, it is rarely so, due to fouling, rust, rivets, etc....



To account for roughness we first *define* an 'equivalent sand roughness' coefficient k (units: $[L]$), a measure of the characteristic roughness height.

The parameter that determines the significance of the roughness k is the ratio

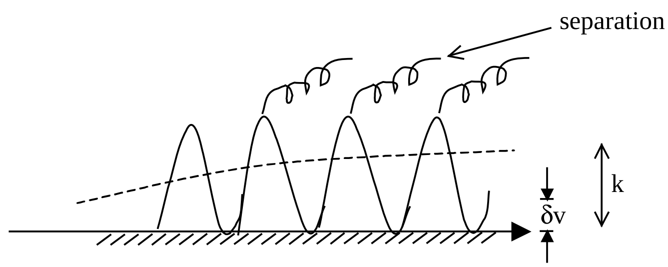
$$\frac{k}{\delta}$$

We thus distinguish the following two cases, depending of the value of the ratio $\frac{k}{\delta}$ on the actual surface - e.g., ship hull.

1. **Hydraulically smooth surface** For $k < \delta_v << \delta$, where δ_v is the viscous sub-layer thickness, k does **not** affect the turbulent boundary layer significantly.

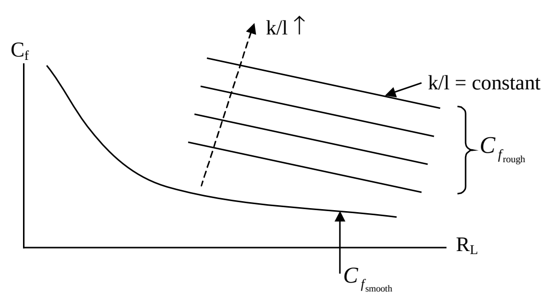
$$\frac{k}{\delta} << 1 \Rightarrow C_f \simeq C_{f,\text{ smooth}} \Rightarrow C_f = C_f(R_{e_L})$$

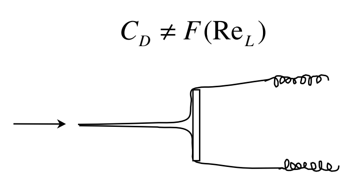
2. **Hydraulically rough surface** For $k >> \delta >> \delta_v$, the flow will resemble what is sketched in the following figure.



In terms of sand grains: each sand grain can be thought of as a bluff body. The flow, thus separates downstream of each sand grain. Recalling that drag due to 'separation' = form drag >> viscous drag we can *approximate* the friction drag as the resultant drag due to the separation behind each sand grain.

$$\frac{k}{\delta} >> 1 \Rightarrow C_f \equiv C_{f,\text{ rough}} \Rightarrow C_f = C_f(\frac{k}{L}, \underbrace{R_{e_L}}_{\text{weak dependence}})$$





$C_{f,\text{ rough}}$ has only a weak dependence on R_{e_L}, since for bluff bodies $C_D \neq F(R_{e_L})$

In summary The important parameter is k/δ:

$$\frac{k}{\delta(x)} << 1: \text{ hydraulically smooth}$$

$$\frac{k}{\delta(x)} >> 1: \text{ rough}$$

Therefore, for the same k, the smaller the δ, the more important the roughness k.

4.11.1 **Corollaries**

1. **Exactly scaled models** (e.g. hydraulic models of rivers, harbors, etc...)

   Same relative roughness: $\dfrac{k}{L} \sim$ const for model and prototype

   $$\frac{k}{\delta} = \frac{k}{L}\frac{L}{\delta} \sim \left(\frac{k}{L}\right) {R_{e_L}}^{1/5}$$

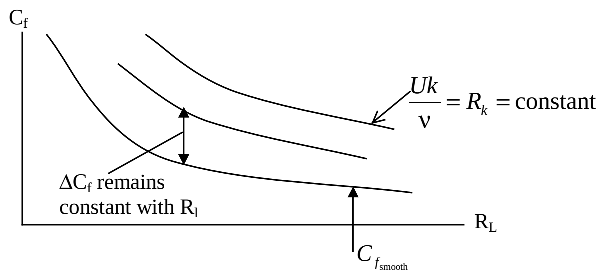
   $$\frac{k}{\delta} \uparrow \text{ for } R_{e_L} \uparrow$$

   For $R_{e\ \text{model}} << R_{e\ \text{prototype}}$:

   $$\left(\frac{k}{\delta}\right)_m < \left(\frac{k}{\delta}\right)_p$$

   $$(C_f)_m < (C_f)_p$$

2. **Roughness Allowance**. Often, the model is hydraulically smooth while the prototype is rough. In practice, the roughness of the prototype surface is accounted for 'indirectly'.



- For the same ship (R_e same), different k gives different $R_{e_k} = \dfrac{Uk}{\nu}$.
- For a given R_{e_k}, the friction coefficient C_f is increased by **almost** a constant for $\dfrac{Uk}{\nu} = R_{e_k} = const$ over a wide range of R_{e_L}.
- If the model is hydraulically smooth, can we account for the roughness of the prototype?

  Notice that $\Delta C_f = \Delta C_f\left(R_{e_k}\right)$ has only a weak dependence on R_{e_L}. We can therefore, run an experiment using hydraulically smooth model, and add ΔC_f to the final friction coefficient for the prototype

$$C_f(R_{e_L}) = C_{f\ \text{smooth}} + \Delta C_f \underbrace{(R_{e_k})}_{\text{not } (R_{e_L})}$$

  Gross estimate: For ships, we typically use $\Delta C_f = 0.0004$.

$$\textit{Reality:} \quad \frac{k}{\delta} = \frac{R_{e_k}}{(\underbrace{\delta/L}_{\sim R_{e_L}{}^{-1/5}})R_{e_L}} \cong \frac{R_{e_k}}{R_{e_L}{}^{4/5}} \Longrightarrow$$

$$\frac{k}{\delta} \downarrow \text{ as } R_{e_L} \uparrow \text{, i.e., } \Delta C_f \text{ smaller for larger } R_{e_L}.$$

- **Hughes' Method** Adjust for R_{e_L} dependence of $C_{f_{\text{rough}}}$.

$$C_{f_{\text{rough}}} = C_{f_{\text{smooth}}}(1+\gamma) \Longrightarrow \Delta C_f = \gamma C_{f_{\text{smooth}}}(R_{e_L})$$

  i.e., As $R_{e_L} \uparrow$, $\Delta C_f \downarrow$.

Chapter 5 - Model Testing.

5.1 Steady Flow Past General Bodies

- In general, $C_D = C_D(R_e)$.

- For bluff bodies

$$\boxed{\text{Form drag} >> \text{Friction drag} \Rightarrow C_D \approx const \equiv C_P (\text{within a regime})}$$

Recall that the form drag (C_P) has only regime dependence on Reynold's number, i.e, its NOT a function of Reynold's number within a regime.

- For streamlined bodies

$$\boxed{C_D(R_e) = C_f(R_e) + C_P}$$

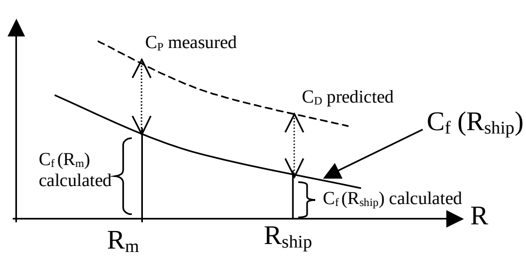
5.1.1 **Steps followed in model testing**:

(a) Perform an experiment with a smooth model at $R_{e_M}(R_{e_M} << R_{e_S})$ and obtain the model drag C_{DM}.

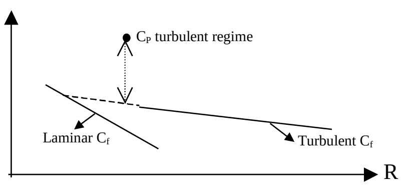
(b) Calculate $C_{PM} = C_{DM} - C_{fM}(R_{e_M}) = C_{PS} = C_P; C_{DM}$ measured, $C_{fM}(R_{e_M})$ calculated.

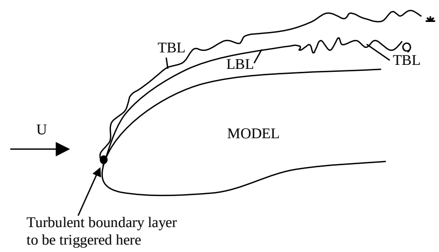
(c) Calculate $C_{DS} = C_P + C_{fS}(R_{e_S})$

(d) Add ΔC_f for roughness if needed.



Caution: In an experiment, the boundary layer must be in the same regime (i.e., turbulent) as the prototype. Therefore turbulence stimulator(s) must be added.





5.1.2 **Drag on a ship hull** For bodies near the free surface, the Froude number F_r is important, due to wave effects. Therefore $C_D = C_D(R_e, F_r)$. In general the ratio $\frac{R_e}{F_r} = \frac{\sqrt{gL^3}}{\nu}$. It is impossible to easily scale both R_e and F_r. For example $\frac{R_e}{F_r} = \text{constant}$ and $\frac{L_m}{L_p} = \frac{1}{10} \Rightarrow \frac{\nu_m}{\nu_p} = 0.032$ or $\frac{g_m}{g_p} = 1000!$

This makes ship model testing seem unfeasible. **Froude's Hypothesis** proves to be invaluable for model testing

$$C_D(R_e, F_r) \approx \underbrace{\overbrace{C_f(R_e)}^{\textit{calculate}}}_{\substack{C_f \text{ for flat plate} \\ \text{of equivalent wetted area}}} + \underbrace{\overbrace{C_R(F_r)}^{\textit{measure indirectly}}}_{\text{residual drag}}$$

In words, Froude's Hypothesis assumes that the drag coefficient consists of two parts, C_f that is a *known* function of R_e, and C_R, a *residual drag* that depends on F_r number *only* and *not* on R_e. Since $C_f(R_e) \sim C_f(R_e)_{\text{flat plate}}$, we need to run experiments to (indirectly) get $C_R(F_r)$.

Thus, for ship model testing we require *Froude* similitude to measure $C_R(F_r)$, while $C_f(R_e)$ is estimated theoretically.

5.1.3 OUTLINE OF PROCEDURE FOR FROUDE MODEL TESTING (S $\equiv$ 'SHIP' M $\equiv$'MODEL'; in general $\nu_S \neq \nu_M$, and $\rho_S \neq \rho_M$)

1. Given U_S, calculate: $F_{r_S} = U_S/\sqrt{gL_S} = F_{r_M}$

2. For Froude similitude, tow model at: $U_M = F_{r_S}\sqrt{gL_M}$

3. Measure total resistance (drag) of model: Measure D_M

4. Calculate total drag coefficient for model: $$C_{DM} = \frac{D_M}{0.5\rho_M U_M^2 \underbrace{S_M}_{\textbf{wetted}\text{ area}}}$$

5. Use ITTC line to calculate $C_f(R_{e_M})$: $$C_f(R_{e_M}) = \frac{0.075}{(\log_{10} R_{e_M} - 2)^2}$$

6. Calculate residual drag of model: $C_{RM} = C_{DM} - C_f(R_{e_M})$

7. Froude's Hypothesis: $C_{RM}(R_{e_M}, F_r) = C_{RM}(F_r) = C_{RS}(F_r) = C_R(F_r)$

8. Use ITTC line to calculate $C_f(R_{e_S})$: $$C_f(R_{e_S}) = \frac{0.075}{(\log_{10} R_{e_S} - 2)^2}$$

9. Calculate total drag coefficient for ship: $$C_{DS} = C_R(F_r) + C_f(R_{e_S}) + \underbrace{\Delta C_f}_{\substack{\cong 0.0004 \\ \text{typical value}}}$$

10. Calculate the total drag of ship: $$D_S = C_{DS} \cdot \left(0.5\rho_S U_S^2 \underbrace{S_S}_{\textbf{wetted}\text{ area}}\right)$$

11. Calculate the power for the ship: $P_S = D_S U_S$

12. Repeat for a series of U_S