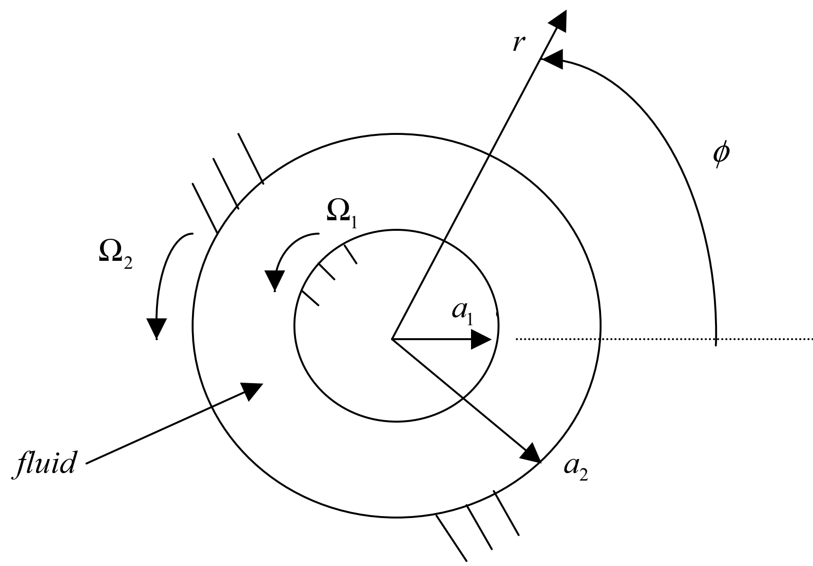
2.20 Problem Set 4A **Name: ____________________**

1. Fluid is contained in the gap between two long concentric cylinders of radii a_1 and a_2. The cylinders rotate about the *z*-axis with constant angular velocities Ω_1 and Ω_2 with respect to a fixed reference frame. Assume the flow is steady and incompressible and that the velocity $\vec{U}$ is only in the ϕ direction, so that in cylindrical coordinates $\vec{U} = (u_r, u_\phi, u_z) = (0, u_\phi, 0)$. Assume initially that the velocity and pressure are independent of ϕ and *z*.



(a) Verify that the continuity equation $\vec{\nabla} \cdot \vec{U} = 0$ is satisfied for the assumed characteristics of the velocity field. You may want to check the text *Fluid Flow* (SAH), Appendix 3.4 for the cylindrical coordinate form of the continuity equation.

(b) First assume that the axis of rotation of the cylinders is horizontal, so that gravity is not important. Write the *full, unsteady* form of the incompressible Navier-Stokes equations for the *r* and ϕ directions for this flow (2 equations). See text SAH, p. 74 for the cylindrical coordinate form of the Navier-Stokes equations.

(c) Simplify the two equations in (b) using the assumptions made in the problem statement (do not try to solve them).

(d) Write the two kinematic boundary conditions for these equations.

Now assume that the axis of rotation (z-axis) of the long cylinders is vertical, so that gravity must be accounted for. Still assume that velocity is independent of both ϕ and z, but that pressure is now only independent of ϕ. Other assumptions are the same.

(e) Write the *full, unsteady* form of the incompressible Navier-Stokes equations for this flow for the z-direction.

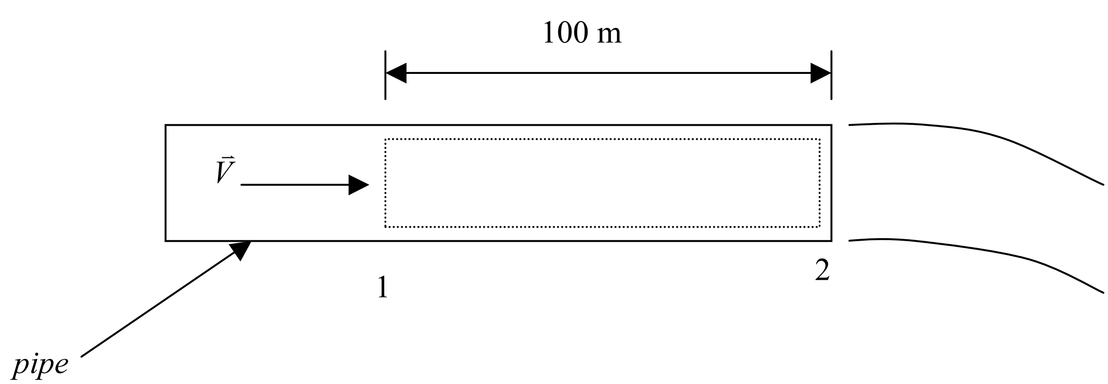
(f) Simplify the equation in (e) using the assumptions in the problem statement.

(g) Integrate the partial differential equation obtained in (f) to solve for the pressure. For the assumptions made, the constant of integration must be a function of the coordinate ___.

(h) Will the solution for the velocity field for the vertical cylinders be different from the solution for the horizontal cylinders? Explain why or why not by referring to the Navier-Stokes equations for the two cases.

(i) If you solved for the pressure $p_1 = p_1(r)$ in the case of the horizontal cylinders, how could you then find the pressure p_2 in the case of the vertical cylinders?

2. A straight, horizontal pipe with a 20-cm internal diameter discharges water into the air. If the rate of flow is 0.01 m^3/s and is increasing at the rate of 0.15 m^3/s^2, what will the pressure be 100 m from the outlet end? Neglect viscous effects (there is no wall shear stress).



Use a fixed control volume analysis as in previous problems, but now include the unsteady terms. For example, recall the unsteady momentum theorem:

$$\sum_{on fluid} \vec{F} = \iiint_{C.V.} \frac{\partial(\rho\vec{V})}{\partial t} dV + \iint_{C.S.} \rho\vec{V}(\vec{V}\cdot\hat{n})dS$$

3. The flow around a model of a turbine blade is investigated. The model is five times larger than the prototype. A maximum pressure of 4 psi is measured at the leading edge, a maximum velocity of 30 ft/s is measured near the top of the blade, and a small device attached to the surface measures a shearing stress of 0.02 psi at a particular location. Determine the associated quantities to be expected on the prototype. Water is the fluid for both model and prototype.

4. An atomic bomb explodes with a known fixed energy release E. In addition to E, assume the expansion of the blast depends on the following physical variables: the blast radius R, the air density ρ, and the time t. The physical units (dimensions) of the problem are mass M, length L, and time T. Using dimensional analysis, obtain an expression for the radius R in terms of a constant and the other physical variables.