Challenge Problem 2 (OPTIONAL) **Name: ____________________**

Consider steady flow in a circular tube whose radius $a(x)$ varies slowly with x along the centerline. A constant pressure difference is maintained between the two ends of the tube, and the axial pressure gradient $\frac{dp}{dx} = -G(x)$ also varies slowly with x. The flow is axisymmetric so that $\vec{V} = (v_x, v_r)$. Because $a(x)$ and $G(x)$ vary slowly, the local flow near some point x in the tube is approximately described by Poiseuille flow and the axial velocity component is given by

$$v_x(x,r) = \frac{G(x)}{4\mu}(a(x)^2 - r^2)$$

where r is the radial distance from the tube centerline.

1. What terms must be neglected in the full Navier-Stokes equation for v_x in order to obtain this approximate solution?

2. What is the constant volume flux Q along the tube in terms of $G(x)$ and $a(x)$?

3. Using your answer for Q, show that

$$P_1 - P_2 = 8\mu \frac{Q}{\pi} \int_{x_1}^{x_2} a^{-4} dx$$

where P_1 and P_2 are the pressures at x_1 and x_2.

4. Assume the tube radius is given by $a(x) = 1 + \alpha x$, where α is a constant. Obtain an equation for the pressure gradient $-G(x)$ assuming the pressures P_1 and P_2 are known at points x_1 and x_2. Express your answer in terms of x_1, x_2, P_1, P_2, α and x.

5. If $\alpha = 0$, what is the pressure gradient?