

13.002
PS #1 Solution

1.2.4. (a)

$$(1.0110101)_2 = (2^0) + (2^{-2}) + (2^{-3}) + (2^{-5}) + (2^{-7}) = 1.41406250000000$$

1.2.4. (b)

$$(11.0010010001)_2 = (2^1) + (2^0) + (2^{-3}) + (2^{-6}) + (2^{-10}) = 3.14160156250000$$

1.2.5. (a)

$$\begin{aligned} \sqrt{2} - (1.0110101)_2 &= 1.41421356237309 - 1.41406250000000 = \\ &= 1.510623730900385e-004 \text{ (absolute error)} \end{aligned}$$

$$\frac{1.510623730900385e-004}{1.41421356237309} = 1.068172283940988e-004 \text{ (relative error)}$$

1.2.5. (b)

$$\begin{aligned} \pi - (11.0010010001)_2 &= 3.14159265358979 - 3.14160156250000 \\ &= -8.908910209992627e-006 \text{ (absolute error)} \end{aligned}$$

$$\frac{8.908910209992627e-006}{3.14159265358979} = 2.835794194964366e-006 \text{ (relative error)}$$

1.2.13. (b)

$$\left(\frac{1}{10} + \frac{1}{3}\right) + \frac{1}{5} =$$

$$((0.1101)_2 \times 2^{-3} + (0.1011)_2 \times 2^{-1}) = (0.1110)_2 \times 2^{-1}$$

$$(0.1110)_2 \times 2^{-1} + (0.1101)_2 \times 2^{-2} = (0.1010)_2 \times 2^0$$

$$\frac{19}{30} \approx (0.1010)_2 \times 2^0$$

$$0.63333333333333 - 0.62500000000000 = 0.00833333333333 \text{ (absolute error)}$$

$$\frac{0.00833333333333}{0.63333333333333} = 0.01315... \text{ (relative error)}$$

1.3.1.(b)

$$98350-98000 = 350 \text{ (absolute error)}$$

$$\frac{350}{98350} = 0.00355871886121 \text{ (relative error) (2 significant digits)}$$

1.3.1.(c)

$$0.000068-0.00006= 8\text{e-}006 \text{ (absolute error)}$$

$$\frac{8\text{e-}006}{0.000068} = 0.117647058 \text{ (relative error) (no significant digits)}$$

1.3.12

$$x_{1_{new}} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \cdot \frac{b + \sqrt{b^2 - 4ac}}{b + \sqrt{b^2 - 4ac}} = \frac{-2c}{b + \sqrt{b^2 - 4ac}}$$

$$x_{2_{new}} = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \cdot \frac{b - \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}} = \frac{-2c}{b - \sqrt{b^2 - 4ac}}$$

1.3.13. (a)

$$x^2 - 1,000.001x + 1 = 0$$

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = 1000$$

$$x_2 = \frac{-2c}{b - \sqrt{b^2 - 4ac}} = 0.001$$

Programming Exercise 1

```
% script M-file findroots.m

a=input('Enter the value of "a" from ax^2+bx+c=0 :');
b=input('Enter the value of "b" from ax^2+bx+c=0 :');
c=input('Enter the value of "c" from ax^2+bx+c=0 :');
if b>= 0;
    sign=1;
else sign=-1;
end;
q=-0.5*(b+sign*sqrt((b^2)-4*a*c));
x1=q/a;
x2=c/q;
xx1=num2str(x1);
xx2=num2str(x2);
disp(['X1 is equal to ',xx1])
disp(['X2 is equal to ',xx2])
```

Programming Exercise 2

```
x(1)=1/2;r(1)=0.994; p(1)=1; p(2)=0.497; q(1)=1; q(2)=0.497;
for n=2:11
    x(n)=(1/2)*x(n-1);
end
for n=2:11
    r(n)=(1/2)*(r(n-1));
end
for n=3:11
    p(n)=(3/2)*p(n-1)-(1/2)*p(n-2);
end
for n=3:11
    q(n)=(5/2)*q(n-1)-q(n-2);
end
h=1:11;
figure(1)

plot(h, x(h)-r(h), 'bd',h, x(h)-p(h), 'r+',h, x(h)-q(h), 'g')
grid on
legend('r(n)', 'p(n)', 'q(n)')

fprintf('n    x(n)    r(n)    p(n)    q(n)\n')
for i = h
    fprintf('%2d    %+10.8f    %+10.8f    %+10.8f    %+10.8f\n', i, x(i), p(i), r(i), q(i))
end
fprintf(' n    x(n)-r(n)    x(n)-p(n)    x(n)-q(n)\n')
for i = h
    fprintf('%2d    %+10.8f    %+10.8f    %+10.8f\n', i, x(i)-r(i), x(i)-p(i), x(i)-q(i))
end
```

n	x(n)	r(n)	p(n)	q(n)
1	+0.50000000	+1.00000000	+0.99400000	+1.00000000
2	+0.25000000	+0.49700000	+0.49700000	+0.49700000
3	+0.12500000	+0.24550000	+0.24850000	+0.24250000
4	+0.06250000	+0.11975000	+0.12425000	+0.10925000
5	+0.03125000	+0.05687500	+0.06212500	+0.03062500
6	+0.01562500	+0.02543750	+0.03106250	-0.03268750
7	+0.00781250	+0.00971875	+0.01553125	-0.11234375
8	+0.00390625	+0.00185938	+0.00776563	-0.24817188
9	+0.00195313	-0.00207031	+0.00388281	-0.50808594
10	+0.00097656	-0.00403516	+0.00194141	-1.02204297
11	+0.00048828	-0.00501758	+0.00097070	-2.04702148

n	x(n)-r(n)	x(n)-p(n)	x(n)-q(n)
1	-0.49400000	-0.50000000	-0.50000000
2	-0.24700000	-0.24700000	-0.24700000
3	-0.12350000	-0.12050000	-0.11750000
4	-0.06175000	-0.05725000	-0.04675000
5	-0.03087500	-0.02562500	+0.00062500
6	-0.01543750	-0.00981250	+0.04831250
7	-0.00771875	-0.00190625	+0.12015625
8	-0.00385938	+0.00204687	+0.25207813
9	-0.00192969	+0.00402344	+0.51003906
10	-0.00096484	+0.00501172	+1.02301953
11	-0.00048242	+0.00550586	+2.04750977

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