Issued: February 3rd, 2005

13.002 Introduction to Numerical Methods for Engineers

In-class programming exercises

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1. Let a be a positive real number, and let the sequence of real numbers x_i be given by

$$x_0 = 1, \quad x_{i+1} = \frac{1}{2} * (x_i + \frac{a}{x_i}),$$

for $i = 0, 1, 2, 3, \ldots$.

The value x_i will converge to $\sqrt{a}$ as $i \longrightarrow \infty$ Write a program that reads in the value of a interactively and uses this algorithm to compute the square root of a.

Test your program as you vary the maximum number of iterations of the algorithm is increased from $1, 2, 3, \ldots$ and determine how many significant digits of precision that you obtain for each. How many iterations are necessary to reach the machine precision of matlab?

2. Write a program to evaluate e by the series:

$$e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \ldots$$

Test your program as you increase the number of terms in the series. Determine how many significant digits of precision that you obtain in your answer as a function of the number of terms in the series. How many terms are necessary to reach machine precision?

3. Consider the function $x\sin(x) - 1$.

(a.) How many roots does this function have in the interval $[0, \pi]$?

(b.) Write a matlab program to find the root(s) using Newton-Raphson iteration with appropriate starting values.

(c.) Make a graph of relative error vs. iteration step for all roots.

(d.) How many iterations are needed to reach an error of less than 10^{-8}?