2.092/2.093

FINITE ELEMENT ANALYSIS OF SOLIDS AND FLUIDS I

FALL 2009

Quiz #2

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Problem 1 (10 points):

A planar (two-dimensional) analysis of a fluid-structure system is to be performed. The simple model shown below is considered.

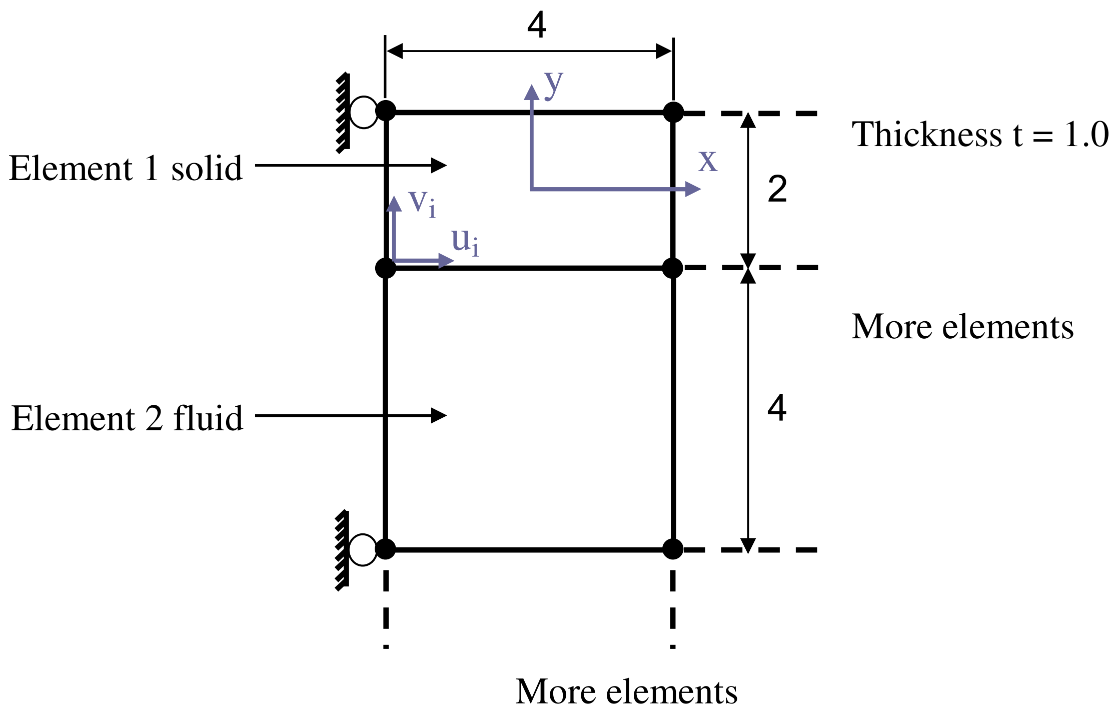


Fig. 1

Infinitesimally small motions are assumed to take place. The solid is in plane strain conditions with $\underline{C}$ known,

$$\begin{bmatrix} \tau_{xx} \\ \tau_{yy} \\ \tau_{xy} \end{bmatrix} = \underline{C} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix}$$

and mass density ρ_s.

The fluid is inviscid with bulk modulus β,

$$p = -\beta \varepsilon_v ; \quad \varepsilon_v = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

and mass density ρ_f. Here u and v are displacements for the fluid and acoustic motions are considered.

(a) Establish the expressions for the "stiffness elements" for the degrees of freedom u_i and v_i.

(b) Establish the expressions for the "consistent mass elements" for these degrees of freedom.

In other words: Establish for

$$\begin{bmatrix} x & x & \cdots \\ x & x & \cdots \\ \vdots & \vdots & \cdots \end{bmatrix} \begin{bmatrix} \ddot{u}_i \\ \ddot{v}_i \\ \vdots \end{bmatrix} + \begin{bmatrix} x & x & \cdots \\ x & x & \cdots \\ \vdots & \vdots & \cdots \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ \vdots \end{bmatrix} = \underline{R}$$

<u>ONLY</u> the entries shown as "x" above in $\underline{M}$ and $\underline{K}$ for the two degrees of freedom shown in Fig. 1.

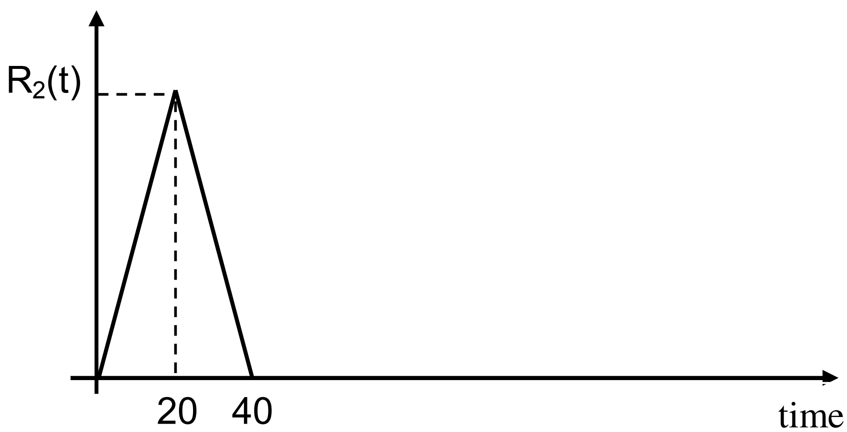
Note:

<u>Give all answers but write as little as possible, and do not perform any integration.</u>

Problem 2 (10 points):

A system is governed by the dynamic equilibrium equations

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{U}_1 \\ \ddot{U}_2 \\ \ddot{U}_3 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0 \\ R_2(t) \\ 0 \end{bmatrix}; \quad {}^0\underline{U}=\underline{0}; \quad {}^0\underline{\dot{U}}=\underline{0}$$



You are required to use the central difference direct time integration method to calculate the response for U_1, U_2, and U_3.

(a) Give the solution steps you would use to calculate the response, but do not perform any actual step by step solution.

(b) Give the time step Δt you would employ for an efficient solution, and give your reasons for the choice.

Note:

You can get the solution, although there are zero masses at the degrees of freedom U_1 and U_3!