

MIT OpenCourseWare  
<http://ocw.mit.edu>

2.004 Dynamics and Control II  
Spring 2008

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
DEPARTMENT OF MECHANICAL ENGINEERING

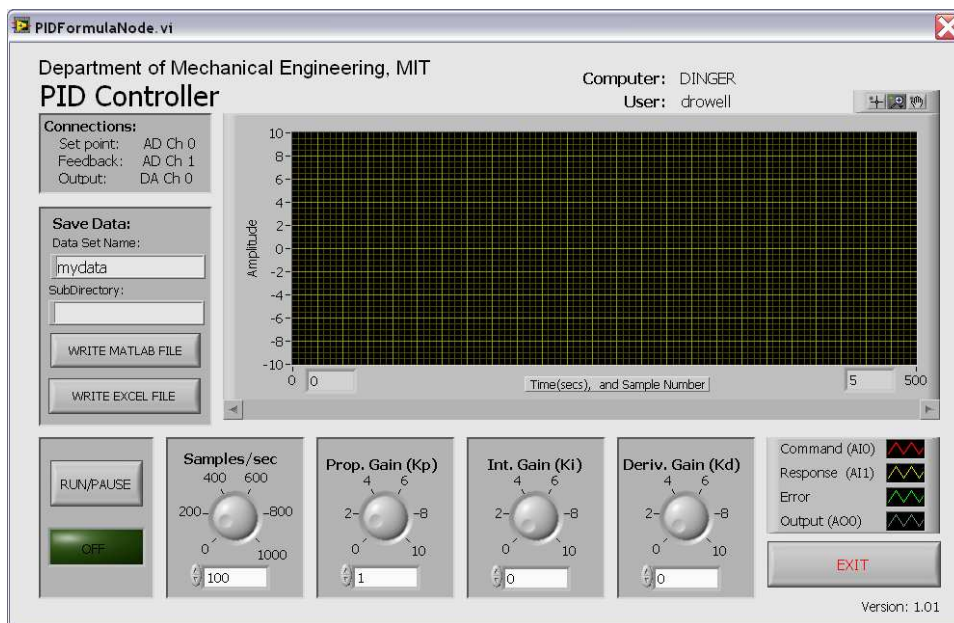
2.004 *Dynamics and Control II*

**Laboratory Session 4:**  
**Closed-Loop Performance of a Proportional Velocity Controller**<sup>1</sup>

**Laboratory Objectives:**

- (i) Introduction to the 2.004 digital PID Controller.
- (ii) Detailed study of the effect of controller gain on transient and steady-state behavior
- (iii) Study of the ability of the closed-loop system to “follow” time varying commands.

**The 2.004 PID Controller** In this session you will set up the closed-loop proportional velocity control, just as you did last week, but this time you will use the 2.004 computer-based controller. This is a digital, “discrete-time” controller, written in the LabVIEW language, that allows you to adjust the controller parameters by a set of virtual knobs on the front panel. This controller is a more general form than we have met so far, it is known as a *PID* (proportional + integral + derivative) controller, but in this lab we will only use the proportional function. In later labs we will examine the effects of the integral and derivative terms.



The controller software runs in an *infinite loop*, reading the values of the command and feedback signals on two of the input channels, computing the value of the output to the

---

<sup>1</sup>March 6, 2008

amplifier, and writing that value as a voltage to a digital-analog converter (DAC). In the case of proportional control the computation is trivial:

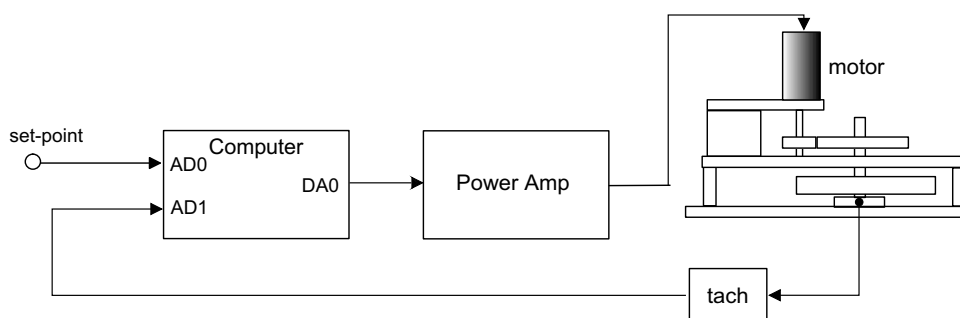
$$v_{out} = K_p(v_{com} - v_{resp})$$

where  $v_{com}$  is the command signal (set-point), and  $v_{resp}$  is the system response (in this case the tachometer output). The loop is synchronized to an internal “clock”, and runs at a speed controlled by a knob on the front panel.

The controller includes a chart-recorder that generates four traces: the input command (red), the system response (yellow), the error signal (green), and the voltage output (blue). As in the Chart Recorder software, the recorded data may be saved to a MATLAB or Excel file - with one difference: *the data saved is the data displayed on the screen*. You must use the pan and zoom tools to select the data you want before saving it. In both MATLAB and Excel the data is saved in arrays with four columns, where the column order is command, response, error, and output.

To use the controller, you simply need to choose a sample/update rate - we suggest you use 50 samples/sec, and the Proportional Gain  $K_p$ . Make sure that the two other knobs  $K_I$  and  $K_D$  are set to zero. These invoke more advanced control algorithms that we will introduce in later labs. You start/stop the controller with the button at the lower left of the panel.

The set-up is shown below:



**Experiment #1** In the first experiment you will repeat the measurements that you made with the op-amp controller in Lab 3. The goal is to measure and plot 1) the closed-loop time-constant, 2) the fractional steady-state error, and 3) the peak motor current in response to a step input. Proceed as follows:

- (a) Connect the computer-based controller.
- (b) Start the controller, and select a sampling rate of 50 samples/sec.
- (c) Set the function generator to output a *square wave* with 1.0v amplitude, 0.5v offset and 0.2 Hz frequency.
- (d) Install one magnet to act as a disturbance torque.
- (e) Set  $K_p = 2$  on the controller. Start the system and record the waveforms.

- (f) Select a relevant section of the trace, save it, and then estimate (1) the time constant, (2) the fractional steady-state error, and (3) the peak motor current. Enter your data on the results sheet (attached).
- (g) Repeat the above procedure with  $K_p = 3, 4, 5$  (for  $K_p = 5$ , reduce the amplitude to 0.5v and the offset to 0.25v).

Use the closed-loop transfer function (see the attached summary in the Appendix) to predict the variation of the time-constant and steady-state error as a function of  $K_p$ . Generate a pair of plots comparing the theoretical values and your measured values. Also make a plot of the peak motor current as a function of controller gain.

**Experiment #2** Imagine that our rotational plant was part of a numerically controlled machine. The accuracy of the parts produced by the machine depends on how well the plant can follow a command signal. In other words, an important specification for many controlled systems is how they should respond to a time-varying input. Ideally, of course, the response should follow the command exactly - but that rarely happens. Our second experiment today is to investigate just how well the angular velocity of the plant follows a sinusoidal input command at several frequencies.

Let the input command to the closed-loop system from the function generator be a sinusoid

$$v_{in}(t) = A_{in} \sin \omega t$$

where  $A$  is the amplitude, and  $\omega$  is the angular frequency. We will show in class soon that the response of a linear system to a sinusoidal input is another sinusoid *with the same frequency*  $\omega$ , but differing in amplitude and with a shift in phase, that is

$$v_{out}(t) = A_{out} \sin(\omega t + \phi)$$

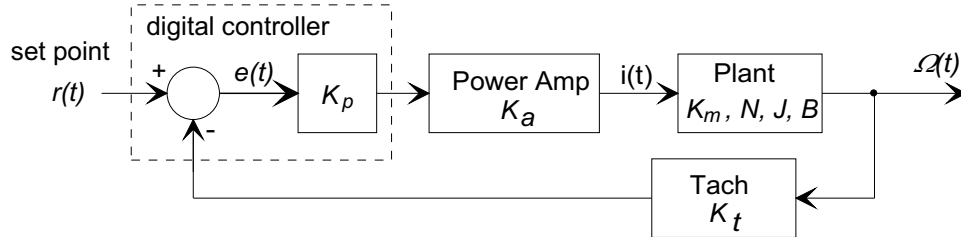
The ratio of the amplitudes  $A_{out}/A_{in}$  at any frequency  $\omega$  is known as the *gain* at that frequency. Your task is to measure the gain at several frequencies, and to comment on the ability of this system to follow high and low frequency commands. With the closed-loop system set up as in Experiment # 1, proceed as follows:

- (a) Set the function generator to generate a sinusoidal output at a frequency of 0.2 Hz, with a peak-to-peak amplitude of 2 volts.
- (b) Set  $K_p = 2$ , start the controller, and record the response.
- (c) Either record the traces, or directly from the screen (adjust the y-scale), estimate the gain  $A_{out}/A_{in}$  at this frequency. Record your result on the results sheet.
- (d) Repeat for frequencies of 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0 and 2.0 Hz.

Make a plot of gain as a function of frequency.

Now we will let you in on a little secret. For a first-order plant such as this, at an angular frequency of  $\omega = 1/\tau$  the gain should have fallen to a fraction  $0.707 = 1/\sqrt{2}$  of the value at a very low frequency. Check your plot to see if this is the case. Record your estimate of this frequency on the results sheet.

## Appendix A: The Closed-Loop Transfer Function



In Lab 3 we found the closed-loop transfer function to be

$$G_{cl}(s) = \frac{\Omega_d(s)}{\Omega_r(s)} = \frac{K_p K_a K_m K_t / N}{J_{eq} s + B_{eq} + K_p K_a K_m K_t / N}$$

The time constant is

$$\tau = \frac{J_{eq}}{B_{eq} + K_p K_a K_m K_t / N}$$

The steady-state response to a step input of magnitude A, that is  $r(t) = A u_s(t)$  where  $u_s(t)$  is the unit step function, is

$$\Omega_{ss} = \lim_{t \rightarrow \infty} \Omega(t) = \lim_{s \rightarrow 0} s \left( \frac{A}{s} G_{cl}(s) \right) = A \frac{K_p K_a K_m K_t / N}{B_{eq} + K_p K_a K_m K_t / N}$$

In the labs to date, we have measured or calculated the following numbers:

- $J_{eq} = 0.03 \text{ N-m}^2$ .
- $B_{eq} = 0.014 \text{ N-m-s/rad}$  (lab average).
- $K_a = 2.0 \text{ A/v}$ .
- $K_m = 0.0292 \text{ N-m/A}$  (lab average).
- $K_t = (0.016 \frac{\text{V}}{\text{rev/min}}) (60 \frac{\text{s}}{\text{min}}) (\frac{1 \text{ rev}}{2\pi \text{ rad}}) = 0.153 \text{ v/(rad/s)}$ .
- $N = \frac{44}{180} = 0.244$

## Lab 4 Summary Sheet

### Experiment # 1

Let  $\tau$  be the closed-loop time-constant (sec), and  $\Delta$  be the fractional steady-state velocity error, that is

$$\Delta = \frac{\Omega_{desired} - \Omega_{ss}}{\Omega_{desired}},$$

and fill in your results below:

$K_p$	$\tau_{predicted}$ (sec.)	$\tau_{measured}$ (sec.)	$\Delta_{predicted}$	$\Delta_{measured}$	$I_{peak}$
2					
3					
4					
5					

Attach your plots and make comments below:

## Lab 4 Summary Sheet

### Experiment # 2

Fill in your results of the measured gain factor below:

Input frequency (Hz)	$A_{in}$	$A_{out}$	Gain ( $A_{out}/A_{in}$ )
0.2			
0.3			
0.4			
0.5			
0.6			
0.7			
0.8			
0.9			
1.0			
2.0			

Attach your plots and make comments below: