Computational Ocean Acoustics

- Ray Tracing
- Wavenumber Integration
- Normal Modes
- Parabolic Equation
- Broadband Modeling
- Ambient Noise Modeling

Ambient Noise Modeling

- Noise in Stratified Ocean
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  - Normal Modes
  - Numerical Examples
- Noise in 3D Ocean
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  - Stochastic Signal and Noise Model
  - Snapshot Synthesis of Signals and Noise

Surface Noise in a Stratified Ocean

Helmholtz Equation - Horizontal Source Distribution

$$(\nabla^2 + k^2)\, \phi_\omega(\mathbf{r}, z) = -S_\omega(\mathbf{r}')\, \delta(z - z')\ ,$$

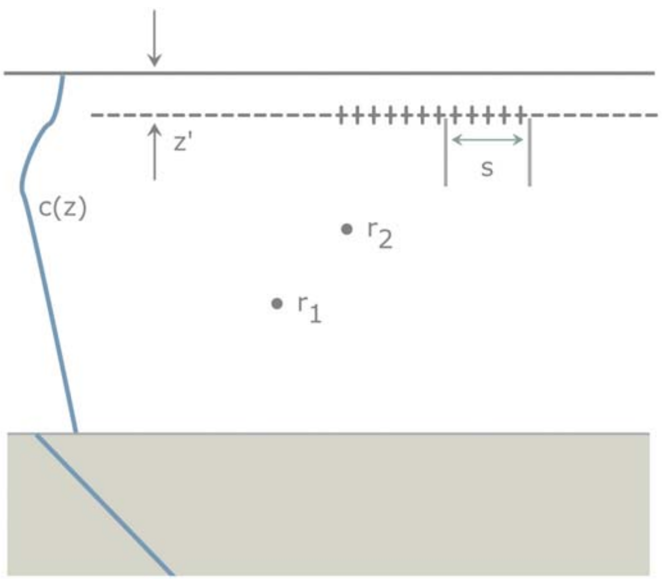
Solution

$$\phi(\mathbf{r}, z) = \int S(\mathbf{r}')\, g(\mathbf{r}, \mathbf{r}'; z, z')\, d^2\mathbf{r}',$$

Green's function

$$(\nabla^2 + k^2)\, g(\mathbf{r}, \mathbf{r}'; z, z') = -\delta^2(\mathbf{r} - \mathbf{r}')\, \delta(z - z')\ ,$$

Surface Noise in a Stratified Ocean



Cross-Spectral Density

$$
\begin{aligned}
C_\omega(\mathbf{r}_1, \mathbf{r}_2, z_1, z_2) &= <\phi(\mathbf{r}_1, z_1)\, \phi^*(\mathbf{r}_2, z_2)> \\
&= \int\!\!\int <S(\mathbf{r}')\, S^*(\mathbf{r}'')> \\
&\quad \times g(\mathbf{r}_1, \mathbf{r}', z_1, z')\, g^*(\mathbf{r}_2, \mathbf{r}'', z_2, z')\, d^2\mathbf{r}'\, d^2\mathbf{r}'',
\end{aligned}
$$

Surface Noise Source Correlation Function

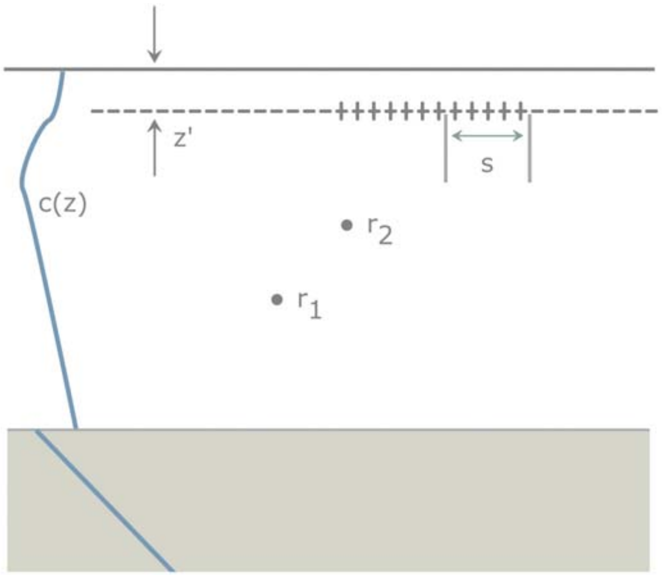
$$q^2 N(\mathbf{s}) \equiv <S(\mathbf{r}')\, S^*(\mathbf{r}'')>$$

$$\mathbf{s} \equiv \mathbf{r}' - \mathbf{r}''$$

Green's function

$$
\begin{aligned}
g(\mathbf{r}_1, \mathbf{r}', z_1, z') &= \frac{1}{2\pi} \int g(k, z_1, z') \exp\left[i\mathbf{k} \cdot (\mathbf{r}_1 - \mathbf{r}')\right]\, d^2\mathbf{k}\,, \\
g^*(\mathbf{r}_2, \mathbf{r}'', z_1, z') &= \frac{1}{2\pi} \int g^*(k', z_2, z') \exp\left[-i\mathbf{k}' \cdot (\mathbf{r}_2 - \mathbf{r}'')\right]\, d^2\mathbf{k}'.
\end{aligned}
$$

Surface Noise in a Stratified Ocean



Noise Correlation

$$\begin{aligned} C_\omega(\mathbf{R}, z_1, z_2) &= q^2 \int\int N(\mathbf{s})\, g(k, z_1, z')\, g^*(k, z_2, z') \\ &\quad \times \exp[i\mathbf{k}\cdot(\mathbf{R}-\mathbf{s})]\, d^2\mathbf{s}\, d^2\mathbf{k} \\ &= 2\pi q^2 \int N(\mathbf{s})\, d^2\mathbf{s} \int_0^\infty g(k_r, z_1, z')\, g^*(k_r, z_2, z') \\ &\quad \times J_0(k_r|\mathbf{R}-\mathbf{s}|)\, k_r\, dk_r\,, \end{aligned}$$

Surface Source Correlation

$$N(\mathbf{s}) = \frac{1}{2\pi} \int P(\mathbf{k}) \exp(i\mathbf{k}\cdot\mathbf{s})\, d^2\mathbf{k}\,.$$

Integration over **s**

$$C_\omega(\mathbf{R}, z_1, z_2) = 2\pi q^2 \int P(\mathbf{k})\, g(k, z_1, z')\, g^*(k, z_2, z') \exp(i\mathbf{k}\cdot\mathbf{R})\, d^2\mathbf{k}\,.$$

Isotropic Noise

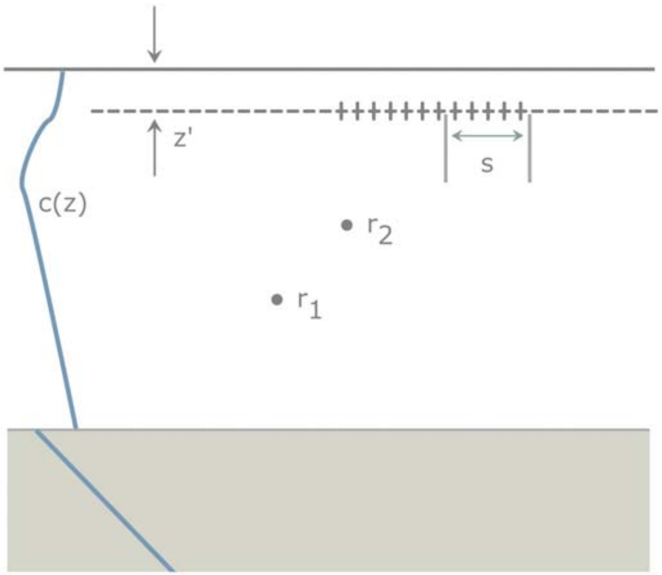
$$N(\mathbf{s}) = N(|\mathbf{s}|)$$

$$C_\omega(R, z_1, z_2) = 4\pi^2 q^2 \int [P(k_r)\, g(k_r, z_1, z')\, g^*(k_r, z_2, z')]\, J_0(k_r R)\, k_r\, dk_r\,.$$

Noise Correlation Function

$$C_\tau(R, z_1, z_2) = \int_{-\infty}^{\infty} C_\omega(R, z_1, z_2) \exp(-i\omega\tau)\, d\omega\,.$$

Surface Noise in a Stratified Ocean



Spatial Distribution of Noise Sources

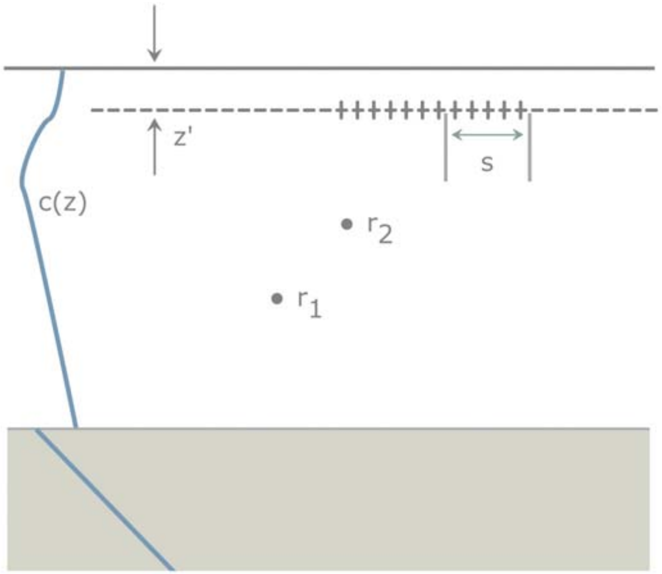
1. Monopole: mass addition, heat addition, volume change—e.g., bubbles, rain droplet impact, *etc.*
2. Dipole: force, translation, acceleration (sloshing)—e.g., vibration of unbaffled rigid bodies.
3. Quadrupole: moment, shear, distortion, rotation, turbulence.

Multipole Source Order p

$$\cos^p \theta$$

$$N(\mathbf{s}) = \begin{cases} \frac{2\delta(k(z')s)}{[k^2(z')s]} & \text{uncorrelated noise sources} \\ 2^p p! \frac{J_p[k(z')s]}{[k(z')s]^p} & \cos^p\theta \text{ radiation pattern.} \end{cases}$$

Surface Noise in a Stratified Ocean
Wavenumber Integral Representation



Uncorrelated Surface Sources

$$C_\omega(R, z_1, z_2) = \frac{8\pi^2 q^2}{k^2(z')} \int_0^\infty g(k_r, z_1, z')\, g^*(k_r, z_2, z')\, J_0(k_r R)$$

Noise Intensity

$$C_\omega(0, z, z) = \frac{8\pi^2 q^2}{k^2(z')} \int_0^\infty |g(k_r, z, z')|^2\, k_r\, dk_r\,.$$

Correlated Noise Sources

$$\int_0^\infty J_p(ax)\, J_0(bx)\, x^{1-p} dx = \begin{cases} 0 & a < b \\ \left[2^{-1}\left(a^2 - b^2\right)\right]^{p-1} a^{-p} \left[\Gamma(p)\right]^{-1} & a \geq b\,, \end{cases}$$

$$P(k_r) = \frac{2p!}{k^{2p}\,\Gamma(p)} \begin{cases} 0 & k_r > k \\ \left(k^2 - k_r^2\right)^{p-1} & k_r \leq k\,, \end{cases}$$

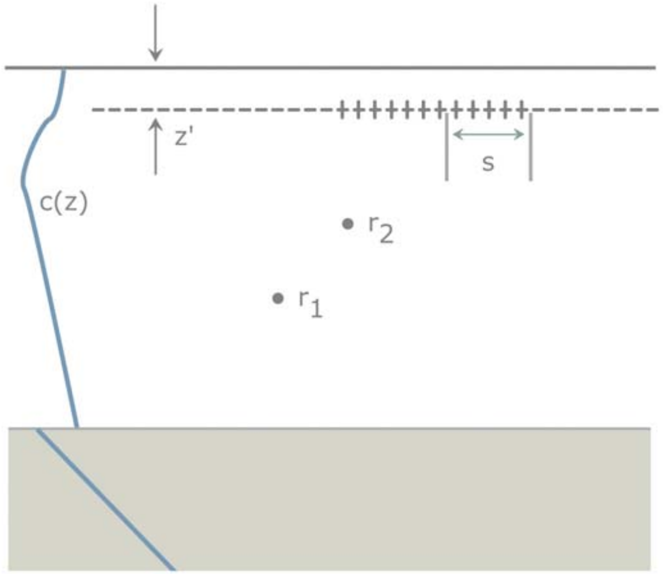
Noise Correlation

$$\begin{aligned} C_\omega(R, z_1, z_2) &= \frac{8\pi^2 p! q^2}{k^{2p}\,\Gamma(p)} \int_0^k \left(k^2 - k_r^2\right)^{p-1} \\ &\quad \times g(k_r, z_1, z')\, g^*(k_r, z_2, z')\, J_0(k_r R)\, k_r\, dk_r \\ &= \frac{8\pi^2 p q^2}{k^{2p}} \int_0^k \left(k^2 - k_r^2\right)^{p-1} \\ &\quad \times g(k_r, z_1, z')\, g^*(k_r, z_2, z')\, J_0(k_r R)\, k_r\, dk_r\,. \end{aligned}$$

Noise Source Normalization

$$q^2(z') = Q^2/16\pi(z')^2\,.$$

Surface Noise in a Stratified Ocean
Normal Mode Representation



Normal Mode Representation

Normal Mode Expansion of Green's Function

$$g(k_r, z, z') = \frac{1}{2\pi\rho} \sum_m \frac{\Psi_m(z')\,\Psi_m(z)}{k_r^2 - k_{rm}^2},$$

Modal Attenuation

$$k_{rm} = \kappa_m + i\alpha_m$$

Noise Correlation

$$C_\omega(\mathbf{R}, z_1, z_2) = \frac{4\pi^2 q^2}{k^2} \int_{-\infty}^{\infty} g(k_r, z_1, z')\, g^*(k_r, z_2, z')\, H_0^{(1)}(k_r R)\, k_r\, dk_r\,.$$

Modal Expansion

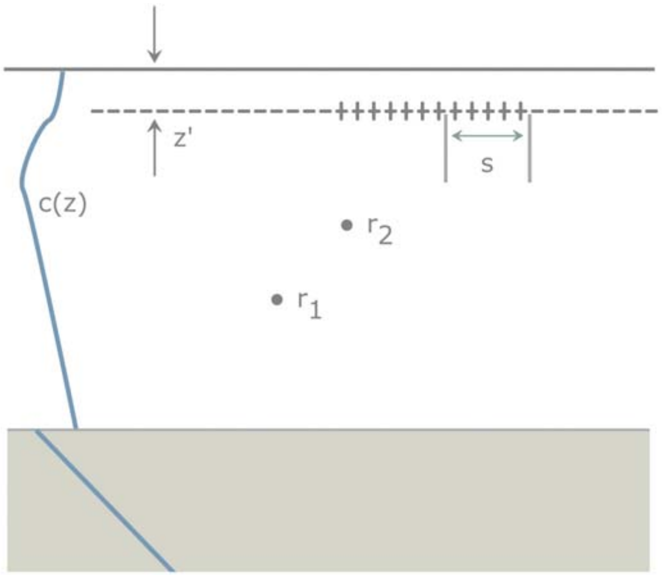
$$\begin{aligned} I_{mn} &\equiv \frac{q^2}{\rho^2 k^2} \int_{-\infty}^{\infty} \frac{k_r\, H_0^{(1)}(k_r R)}{[k_r^2 - k_{rm}^2][k_r^2 - (k_{rn}^*)^2]}\, dk_r \\ &= \frac{i\pi q^2}{\rho^2 k^2} \left[\frac{H_0^{(1)}(k_{rm} R)}{k_{rm}^2 - (k_{rn}^*)^2} + \frac{H_0^{(2)}(k_{rn}^* R)}{k_{rm}^2 - (k_{rn}^*)^2} \right], \end{aligned}$$

Bessel Function Symmetry

$$J_0 = [H_0^{(1)} + H_0^{(2)}]/2$$

$$-H_0^{(1)}(-x) = H_0^{(2)}(x)$$

Surface Noise in a Stratified Ocean
Normal Mode Representation



Noise Correlation

$$C_\omega(\mathbf{R}, z_1, z_2) = \frac{i\pi q^2}{\rho^2 k^2} \sum_{m,n} \Psi_m(z')\,\Psi_m(z_1)\,\Psi_n(z')\,\Psi_n(z_2)\,f_{mn}$$
$$\times \left[H_0^{(1)}(k_{rm}R) + H_0^{(2)}(k_{rn}^* R)\right],$$

Modal Coherence

$$f_{mn} = \frac{1}{k_{rm}^2 - (k_{rn}^*)^2}.$$

$$\kappa_m \gg \alpha_m,\, \kappa_n \gg \alpha_n$$

$$f_{mn} = \begin{cases} \frac{1}{\kappa_m^2 - (\kappa_n^*)^2} & \text{for } m \neq n \\ \frac{1}{4i\alpha_m \kappa_m} & \text{for } m = n\,. \end{cases}$$

Incoherent Modal Summation

$$C_\omega(\mathbf{R}, z_1, z_2) = \frac{\pi q^2}{2\rho^2 k^2} \sum_m \frac{[\Psi_m(z')]^2\,\Psi_m(z_1)\,\Psi_m(z_2)\,J_0(\kappa_m R)}{\alpha_m \kappa_m}\,.$$

Surface Noise in a Stratified Ocean

Noise in a Homogeneous Halfspace

$$C_\omega(R, z_1, z_2) \;=\; \frac{2pq^2}{k^{2p}} \int_0^k \left(k^2 - k_r^2\right)^{p-1} \exp[iZ(k^2 - k_r^2)^{1/2}]$$

$$\times\; \frac{\sin^2[z'(k^2 - k_r^2)^{1/2}]}{k^2 - k_r^2}\, J_0(k_r R)\, k_r\, dk_r\;,$$

$$Z \equiv z_1 - z_2$$

Normalized Noise Correlation

$$\overline{C}_\omega(R, z_1, z_2) \equiv \lim_{z' \to 0} \frac{\mathrm{Re}[C_\omega(R, z_1, z_2)]}{\{\mathrm{Re}[C_\omega(0, z_1, z_1)]\, \mathrm{Re}[C_\omega(0, z_2, z_2)]\}^{1/2}}\;.$$

Cron and Sherman Results

$$I_p(R,Z) = \int_0^k \left(k^2 - k_r^2\right)^{p-1} \cos[Z(k^2-k_r^2)^{1/2}]\, J_0(k_r R)\, k_r\, dk_r \,.$$

$$\begin{aligned} I_p(R,0) &= \int_0^k \left(k^2 - k_r^2\right)^{p-1} J_0(k_r R)\, k_r\, dk_r \\ &= 2^{p-1} k^p R^{-p} (p-1)!\, J_p(kR) \\ &\Rightarrow \end{aligned}$$

$$\overline{C}_\omega(R,z_1,z_1) = 2^p p! \frac{J_p(kR)}{(kR)^p}\,,$$

$$I_p(0,Z) = \int_0^k \left(k^2 - k_r^2\right)^{p-1} \cos[Z(k^2-k_r^2)^{1/2}]\, k_r\, dk_r\,,$$

Transformation of Variable - $\zeta \equiv (k^2 - k_r^2)^{1/2}$

$$I_p(0,Z) = \int_0^k \zeta^{2p-1} \cos(Z\zeta)\, d\zeta \,.$$

Surface Noise in a Stratified Ocean

[See Figs. 9.2, 9.3 in Jensen, Kuperman, Porter and Schmidt. *Computational Ocean Acoustics.* New York: Springer-Verlag, 2000.]

Surface Noise in a Stratified Ocean
Horizontal Correlation

[See Jensen Fig. 9.4]

Surface Noise in a Stratified Ocean
Vertical Correlation

[See Jensen Fig. 9.5]

Seismo-Acoustic Noise in a Stratified Ocean

[See Jensen Figs. 9.6, 9.7, 9.8, 9.9]