2.25 ADVANCED FLUID MECHANICS Fall 2004

QUIZ 2

THURSDAY, November 18, 2004, 7:00-9:00 P.M.

OPEN QUIZ WHEN TOLD AT 7:00 PM

THERE ARE TWO PROBLEMS
OF EQUAL WEIGHT

Please answer both questions in the same book
Clearly showing where one question ends and another begins!

Question 1 The transition from dripping to jetting

Everyone is familiar with the phenomenon shown in Figure 1. which shows what happens when the flow rate of water through a vertically aligned faucet (or 'tap') is slowly increased. Initially a 'pendant drop' forms at the exit which grows until it is large enough that it pinches off (Fig1a). Then for slightly higher flow rates, the faucet periodically drips (Fig 1b); finally at higher flow rates a uniform jet forms (Fig1c). In fact, the transition from dripping to jetting is more complex than shown in this figure. In this question we wish to consider this transition from dripping to jetting.

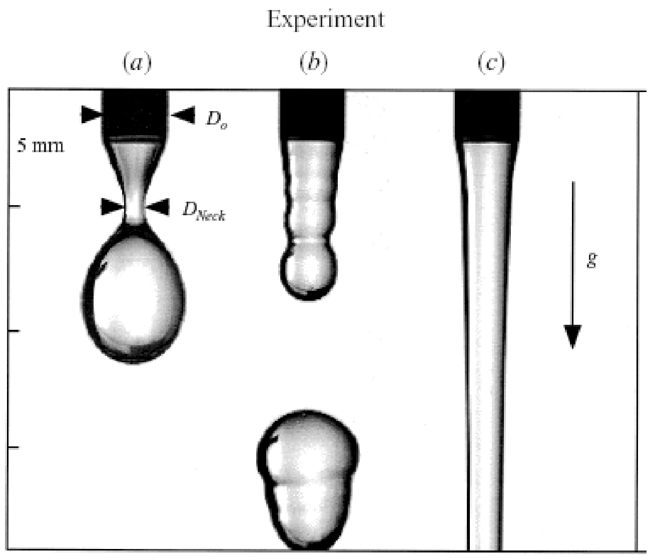


Figure 1

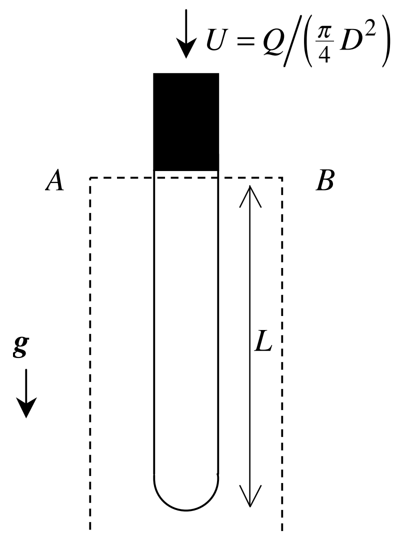


Figure 2: (snapshot at a single instant in time *t*)

For fluids exiting from typical size orifices, viscous stresses are negligible and the flow may be considered inviscid. The surface tension coefficient is σ. The flow rate discharging from the orifice is given by $Q = \frac{1}{4}\pi D^2 U$; where both Q and U are defined in the laboratory reference frame. The exit orifice from the faucet may be considered to be circular with diameter D.

(a) We shall initially analyze the flow using the stationary control volume shown in figure 2. First we need to determine the appropriate force distribution on the plane AB shown in the sketch. Use an appropriate boundary condition normal to the jet surface to show that the pressure in the jet at the plane AB is

$$p_i = p_a + \frac{2\sigma}{D} \tag{1}$$

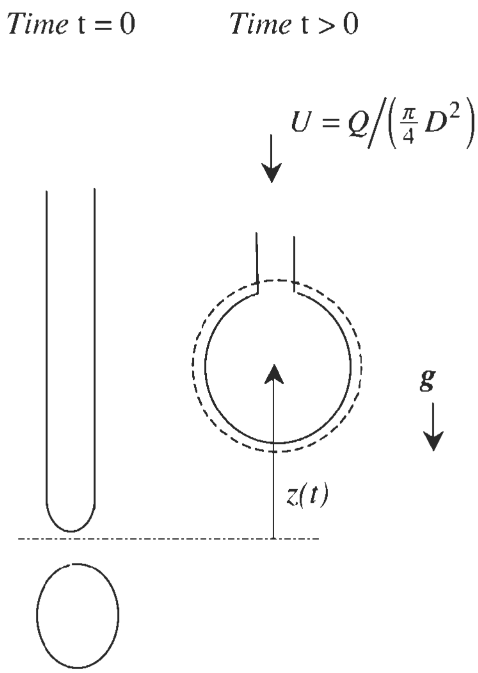
Also give an expression for the axial force arising from surface tension acting along the axial direction of the jet. Check that your expression is dimensionally correct.

(b) One necessary condition for the flow to undergo a transition from dripping to jetting is proposed to be the following:

The flux of momentum into the control volume shown must always be greater than zero (otherwise a stationary pendant drop will have formed, which is attached to the orifice).

Mathematically this can be written $\frac{dP_{cv}}{dt} > 0$ where P_{cv} is the linear momentum of the fluid in the control volume (CV) in the z *direction*. Use an appropriate conservation law to find an expression for the critical jet velocity for transition from dripping to jetting.

(c) Now consider an experiment in which the flow rate supplying a steady cylindrical jet (such as shown in figure 1(c)) is slowly (i.e. "quasi-statically") reduced. At a critical flow rate, dripping begins. At a specific location, denoted $z = 0$, the jet breaks (due to the action of surface tension). We are then left with the unsteady situation shown in the figure below.:



A large terminal drop grows and consumes the incoming jet. (*This is also the situation shown in that figure on the MIT server homepage that you have been staring at all semester!*)
To analyze this, consider a coordinate system oriented vertically upwards (so that $\mathbf{g} = -g\boldsymbol{\delta}_z$). To simplify the analysis, ignore the small contribution of the mass in the long thin jet and consider only the simple spherical control volume shown which consists of a large terminal drop that has a mass $M_{cv}(t)$ and a velocity $\dot{z}(t)$. Draw a large and very clear control volume in your book and carefully label all the forces and appropriate momentum fluxes acting on the CV. Copy the table below into your exam book and fill in the remaining contributions. Use this table to then write down expressions for the conservation of mass and linear momentum for the control volume shown in the figure.

Term	Inflow	Outflow
Outward facing normal $\boldsymbol{n}$	$+\boldsymbol{\delta}_z$	$-\boldsymbol{\delta}_z$
Velocity vector $\boldsymbol{v}$		0
Velocity of the control surface, $\boldsymbol{v_c}$		0
Normal component relative to CS velocity		0

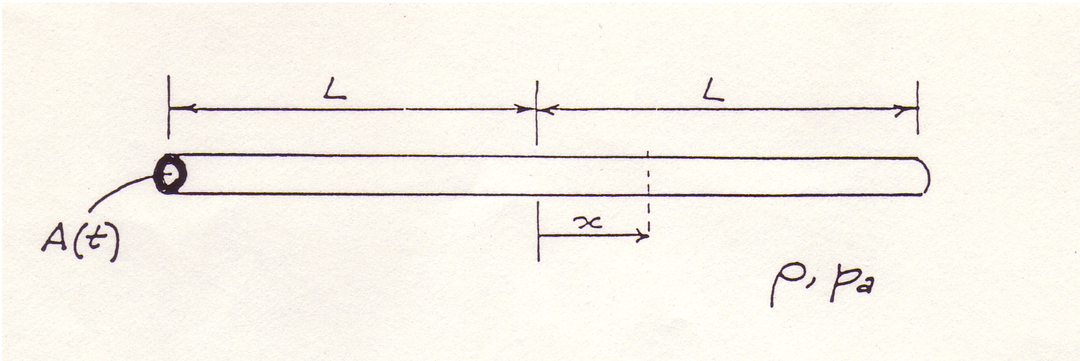
(d) Show that the mass $M_{cv}(t)$ in the terminal drop grows linearly in time. Eliminate the mass $M_{cv}(t)$ from the two equations you derived in (c) in order to obtain a nonlinear second order differential equation for the position $z(t)$. Show that this horrendous looking equation is actually satisfied by a simple quadratic form:

$$z(t) = \tfrac{1}{2}at^2 + (b - U)t \qquad (2)$$

and find expressions for the constants a, b in terms of the variables in the problem.

⇒*When you leave this room, go to the MIT server home page and look at the picture again. Does the trajectory of the pendant blob look parabolic?*

PROBLEM 2 (Déja vue all over again?)



A small cylindrical tube has an inside radius *R(t),* length 2*L,* and two open ends. It is submerged in—and filled with—a liquid that has density ρ and kinematic viscosity ν. The pressure at both open ends is atmospheric, p_a. The tube is made of piezoelectric material, and its inside radius *R(t),* which is uniform over the tube's length, can be programmed by the application of an electric voltage.

Suppose the tube's inner radius is programmed so that $R(t)$ decreases from R_1 at *t*=0 to R_2 at $t = \Delta t$ at a constant rate

$$dR/dt = -\kappa \qquad (\kappa > 0)$$

Assuming *incompressible, inertia-free flow*, derive, in terms of the given quantities, expressions for:

(a) the mean flow speed distribution *u*(x, t) inside the tube, and

(b) the pressure at the tube's center point $p_0(t)$ at $0 \le t \le \Delta t$.

(c) Write down, or derive, a complete set of inequalities that together guarantee that the flow is to a good approximation inertia-free, as assumed. Express these in terms of given quantities: $R_1, R_2, L, \kappa, \Delta t, \rho, \nu$. NOTE: *You will be penalized for including superfluous (i.e. unnecessary, or redundant) criteria.*