2.25 Advanced Fluid Mechanics
QUIZ 2

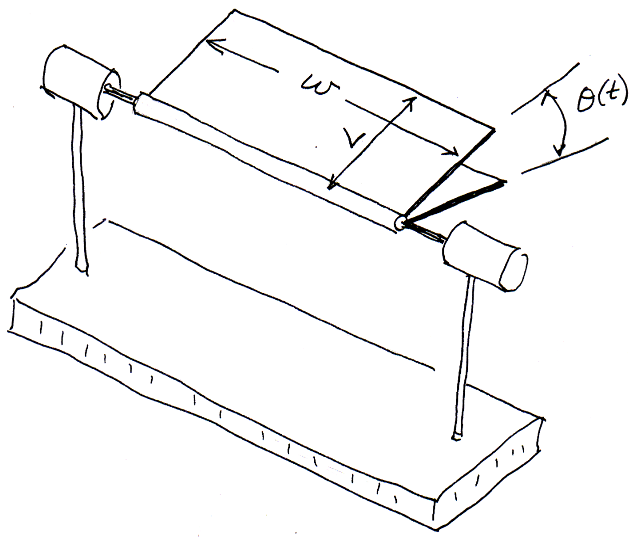
Thursday, November 13, 2003

7:00-9:00 p.m.

You are allowed to bring in two 8.5x11 sheets (4 pages) of notations, typed or handwritten.

There are two problems.

PROBLEM 1



An inventor proposes to propel a small, low-speed underwater vehicle with a device that successively closes and opens two rigid flaps as shown above. The sketch shows a scale model being tested in a large, deep pool containing stationary water (density ρ). In one cycle, the system starts at its maximum open angle θ_1, closes at a uniform angular speed

$$\frac{d\theta}{dt} = -\omega$$

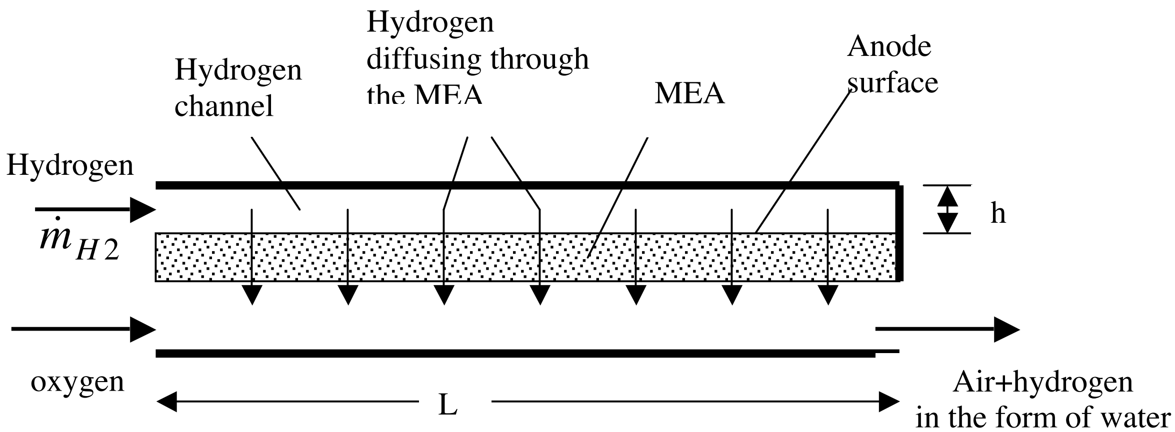
until it reaches a smaller angle θ_2, and then opens at the same angular speed ω until it returns to its original angle.

Note that, unlike the depiction in the sketch, $w >> L$ *and* $\theta << 1$. <u>*Simplify your analysis accordingly.*</u>

(a) *Using a control volume approach*, derive an expression for the thrust force exerted by the device (on its supports) as a function of time during the closing stroke, measuring time from stroke initiation. You may assume incompressible, inviscid flow.

(b) In what way(s) will the opening stroke differ from the closing stroke? Will this device produce net forward propulsion in one full cycle? Explain.

PROBLEM II



A fuel cell is a device used to convert chemical energy stored in hydrogen (and possibly other fuels) into electricity directly. Hydrogen flows in a very narrow channel over the surface of an anode while diffusing through the anode, an electrolyte and an electrode (the MEA in the figure) in the form of ions, to react with oxygen on the cathode side. The reaction forms water, which flows out on the cathode side. For a small fuel cell used to provide electricity for a hand-held device, the hydrogen flow channel thickness, *h*, is about 1 mm, while its length, *L*, and width, *W*, are about 10 cm. The hydrogen channel is closed at the end, i.e., all the hydrogen flows through the MEA. For the case we analyze here, we assume that hydrogen has a constant dynamic viscosity μ, while its density, ρ, changes with the pressure *p* according to the relation: $\rho = \alpha\ p$ (the flow is isothermal) where α is a known constant. Moreover, the rate at which hydrogen diffuses through the MEA is $m_a = \beta\ p^2$ where *m* is the mass flux (mass flow rate per unit area) and β is a known constant.

Calculate hydrogen pressure at the entry of the hydrogen channel necessary to deliver a total mass flow $\dot{m}_{H2}$ of hydrogen through the MAE. You may assume that the hydrogen pressure at the closed end of the channel is very near zero, and that the velocity of hydrogen in the channel is small.

Note that the solution of differential equation: $\frac{d}{dx}\left(x\frac{dy}{dx}\right) - a^2 xy = 0$, with the boundary conditions: $at\ x = 0,\ y = y_0,\ and\ at\ x \to \infty,\ y \to 0$, is $y = y_0 e^{-ax}$.