2.25 PROBLEM 1.9: DICUSSION AND SOLUTION

Ain Sonin, MIT

The net upward lift force on a helium-filled balloon under quasi-static conditions is

$$L = (\rho_{air} - \rho_{He})gV - mg \qquad (1)$$

where ρ_{air} is the density of the ambient air around the balloon, ρ_{He} is the density of the helium inside the balloon, V is the balloon's volume, and m is the total mass associated with the balloon, cargo, personnel, equipment, etc. In what follows, we assume that the atmosphere and balloon are at constant absolute temperature T, which is given, and the gases are perfect. Pressure and density are then related by

$$\rho = \frac{pM}{RT} \qquad (2)$$

where R is the universal gas constant (8.31 J/mol K) and M is the molar mass of the gas being referred to (0.029 kg/mol for air, 0.004 for helium).

The hole at the bottom of the balloon prevents balloon rupture by ensuring that the helium pressure inside is the same as the air pressure just outside. Using (2) and pressure equality, we find the density of the helium inside the balloon as

$$\rho_{He} = \frac{M_{He}}{M_{air}}\rho_{air} \qquad (3)$$

For a balloon filled with helium at the local atmospheric pressure, (1) and (3) yield

$$L = \left(1 - \frac{M_{He}}{M_{air}}\right)\rho_{air}gV - mg \qquad (4)$$

The atmospheric density distribution is

$$\rho_{air}(z) = \rho_0 e^{-z/H} \qquad (5)$$

where ρ_0 is the air density at sea level, z=0, and

$$H = \frac{RT}{M_{air}g} \qquad (6)$$

is the e-folding distance—a characteristic thickness—of the atmosphere. Inserting (5) into (4), we find that for a *helium-filled* balloon, that net lift is

$$L = \left(1 - \frac{M_{He}}{M_{air}}\right)\rho_0 e^{-z/H} gV - mg \quad (7)$$

where z is the balloon's altitude.

Equation (7) provides the lift force at sea level, where z=0, and also the altitude z_0 at which the net upward force on the ascending balloon is zero:

$$z_0 = H \ln\left[\left(1 - \frac{M_{He}}{M_{air}}\right)\frac{\rho_0 V}{m}\right]. \quad (8)$$

As the balloon rises, the local ambient pressure drops and the pressure inside the balloon follows it. The helium inside expands as the pressure drops, and the excess flows out via the hole at the bottom and is lost to the atmosphere. Since the rising balloon has upward momentum, it will generally overshoot the equilibrium altitude z_0 somewhat and find itself at the lower pressures where the net lift is negative. The balloon slows, stops, and begins descending. At the moment descent begins, the helium inside the balloon is trapped. The pressure increases as balloon descends and the helium trapped inside the balloon shrinks in volume, accumulating at the top of the balloon due to its buoyancy, and outside air is drawn in through the hole at the bottom to fill the void. The incoming air has the same density as the ambient atmosphere and is neutrally buoyant, causing neither upward nor downward net force. The trapped helium is the sole source of upward force, compensating in part for the downward force mg.