6.453 Quantum Optical Communication
Spring 2009

November 20, 2008

6.453 *Quantum Optical Communication* Lecture 19

Jeffrey H. Shapiro

Optical and Quantum Communications Group

RESEARCH LABORATORY OF ELECTRONICS
Massachusetts Institute of Technology

6.453 *Quantum Optical Communication* - Lecture 19

- Announcements
  - Pick up lecture notes, slides
- Continuous-Time Photodetection
  - Noise spectral densities in direct detection
  - Semiclassical theory of coherent detection
  - Quantum theory of coherent detection
  - Coherent-detection signatures of non-classical light

Semiclassical versus Quantum Photodetection

- Semiclassical Theory: Given $\{\, P(t) : -\infty < t < \infty \,\}$
  - $\{\, i(t) : -\infty < t < \infty \,\}$ is an inhomogeneous Poisson Impulse Train
  - Rate function $\lambda(t) \equiv \eta P(t)/\hbar\omega_o$
- Quantum Theory:

$$\hat{E}'(t) \equiv \sqrt{\eta}\,\hat{E}(t) + \sqrt{1-\eta}\,\hat{E}_\eta(t)$$

$$i(t) \leftrightarrow \hat{i}(t) \equiv q\hat{E}'^\dagger(t)\hat{E}'(t)$$

Stationary Statistics for Continuous-Wave Sources

- Stationary Mean and Covariance Functions:

$$\langle x(t)\rangle = \text{constant} \equiv \langle x\rangle$$

$$\langle \Delta x(t+\tau)\Delta x(t)\rangle = \text{function of } \tau \text{ only} \equiv K_{xx}(\tau)$$

- Semiclassical Photodetection:

$$\langle i\rangle = q\frac{\eta\langle P\rangle}{\hbar\omega_o} \quad \text{and} \quad K_{ii}(\tau) = q\langle i\rangle\delta(\tau) + q^2\frac{\eta^2 K_{PP}(\tau)}{(\hbar\omega_o)^2}$$

- Quantum Photodetection:

$$\langle i\rangle = q\eta\langle \hat{E}^\dagger(0)\hat{E}(0)\rangle$$

$$K_{ii}(\tau) = q\langle i\rangle\delta(\tau) + q^2\eta^2[\langle \hat{E}^\dagger(\tau)\hat{E}^\dagger(0)\hat{E}(\tau)\hat{E}(0)\rangle - \langle \hat{E}^\dagger(0)\hat{E}(0)\rangle^2]$$

Photocurrent Noise Spectral Density

- Photocurrent Noise Spectral Density:

$$\mathcal{S}_{ii}(\omega) \equiv \int_{-\infty}^{\infty} \mathrm{d}\tau\, K_{ii}(\tau) e^{-j\omega\tau}$$

- Propagation Through a Linear Time-Invariant Filter:

$$i(t) \longrightarrow \boxed{h(t), H(\omega)} \longrightarrow i'(t)$$

$$\mathcal{S}_{i'i'}(\omega) = \mathcal{S}_{ii}(\omega)|H(\omega)|^2$$

- Physical Interpretation: $\mathcal{S}_{ii}(\omega)$ = mean-squared fluctuation strength per unit bilateral bandwidth (in Hz) in frequency ω components of $i(t)$

Direct-Detection Signatures of Non-Classical Light

- Semiclassical Theory:

$$\mathcal{S}_{ii}(\omega) = q\langle i \rangle + q^2 \frac{\eta^2 \mathcal{S}_{PP}(\omega)}{(\hbar\omega_o)^2} \geq q\langle i \rangle$$

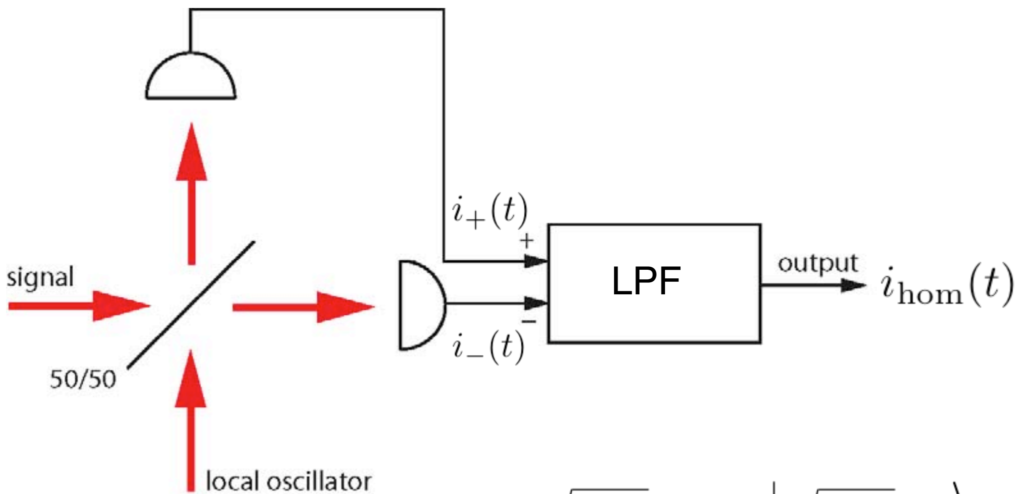
- Quantum Theory:

$$\mathcal{S}_{ii}(\omega) \geq 0$$

- Sub-Shot-Noise Non-Classical Signature:

$$\mathcal{S}_{ii}(\omega) < q\langle i \rangle$$

Balanced Homodyne Detection



- Signal = $E(t)$ or $\hat{E}(t)$, LO = $\sqrt{\frac{P_{\text{LO}}}{\hbar\omega_o}}\,e^{j\theta}$ or $\left|\sqrt{\frac{P_{\text{LO}}}{\hbar\omega_o}}\,e^{j\theta}\right\rangle$
- Low-Pass Filter: $H(\omega) = 1, \quad \text{for } |\omega| \leq \frac{\Delta\omega}{2}$

Balanced Homodyne Detection (Within Passband)

- Semiclassical Statistics in Strong Local Oscillator Limit:

$$i_{\text{hom}}(t) = 2q\eta\sqrt{\frac{P_{\text{LO}}}{\hbar\omega_o}}\text{Re}(E(t)e^{-j\theta}) + i_{\text{LO}}(t)$$

  - Gaussian-process local oscillator shot noise:

$$\langle i_{\text{LO}}\rangle = 0 \quad \text{and} \quad S_{i_{\text{LO}}i_{\text{LO}}}(\omega) = q^2\frac{\eta P_{\text{LO}}}{\hbar\omega_o}$$

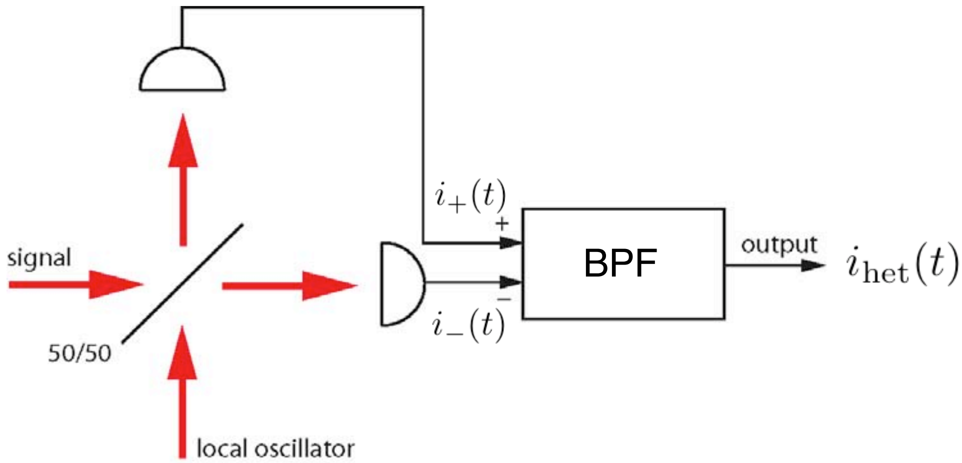
- Quantum Statistics in Strong Local Oscillator Limit:

$$i_{\text{hom}}(t) \leftrightarrow \hat{i}_{\text{hom}}(t) = 2q\eta\sqrt{\frac{P_{\text{LO}}}{\hbar\omega_o}}\text{Re}(\hat{E}(t)e^{-j\theta}) + i_\eta(t)$$

  - Gaussian-process sub-unity quantum efficiency noise:

$$\langle i_\eta\rangle = 0 \quad \text{and} \quad S_{i_\eta i_\eta}(\omega) = q^2(1-\eta)\frac{\eta P_{\text{LO}}}{\hbar\omega_o}$$

Balanced Heterodyne Detection



- Signal = $E_S(t)e^{-j\omega_{\text{IF}}t}$ or $\hat{E}_S(t)e^{-j\omega_{\text{IF}}t} + \hat{E}_I(t)e^{j\omega_{\text{IF}}t}$
- LO = $\sqrt{P_{\text{LO}}/\hbar\omega_o}$ or $|\sqrt{P_{\text{LO}}/\hbar\omega_o}\rangle$
- Band-Pass Filter: $H(\omega) = 1, \quad \text{for } |\omega \pm \omega_{\text{IF}}| \leq \dfrac{\Delta\omega}{2}$

Balanced Heterodyne Detection (Within Passband)

- Semiclassical Statistics in Strong Local Oscillator Limit:

$$i_{\text{het}}(t) = 2q\eta\sqrt{\frac{P_{\text{LO}}}{\hbar\omega_o}}\text{Re}(E_S(t)e^{-j\omega_{\text{IF}}t}) + i_{\text{LO}}(t)$$

  - Gaussian-process local oscillator shot noise:

$$\langle i_{\text{LO}}\rangle = 0 \quad \text{and} \quad S_{i_{\text{LO}}i_{\text{LO}}}(\omega) = q^2\frac{\eta P_{\text{LO}}}{\hbar\omega_o}$$

- Quantum Statistics in Strong Local Oscillator Limit:

$$i_{\text{het}}(t) \leftrightarrow \hat{i}_{\text{het}}(t) = 2q\eta\sqrt{\frac{P_{\text{LO}}}{\hbar\omega_o}}\,\text{Re}[(\hat{E}_S(t) + \hat{E}_I^\dagger(t))e^{-j\omega_{\text{IF}}t}] + i_\eta(t)$$

  - Gaussian-process sub-unity quantum efficiency noise:

$$\langle i_\eta\rangle = 0 \quad \text{and} \quad S_{i_\eta i_\eta}(\omega) = q^2(1-\eta)\frac{\eta P_{\text{LO}}}{\hbar\omega_o}$$

Coherent-Detection Non-Classical Light Signatures

- Semiclassical Theory (within filter passband):

$$\mathcal{S}_{i_{\rm hom}i_{\rm hom}}(\omega) \geq \mathcal{S}_{i_{\rm LO}i_{\rm LO}}(\omega) = q^2\frac{\eta P_{\rm LO}}{\hbar\omega_o}$$

$$\mathcal{S}_{i_{\rm het}i_{\rm het}}(\omega) \geq \mathcal{S}_{i_{\rm LO}i_{\rm LO}}(\omega) = q^2\frac{\eta P_{\rm LO}}{\hbar\omega_o}$$

- Quantum Theory (within filter passband):

$$\mathcal{S}_{i_{\rm hom}i_{\rm hom}}(\omega) \geq \mathcal{S}_{i_\eta i_\eta}(\omega) = q^2(1-\eta)\frac{\eta P_{\rm LO}}{\hbar\omega_o}$$

$$\mathcal{S}_{i_{\rm het}i_{\rm het}}(\omega) \geq q^2\eta^2\frac{P_{\rm LO}}{2\hbar\omega_o} + \mathcal{S}_{i_\eta i_\eta}(\omega) = q^2(1-\eta/2)\frac{\eta P_{\rm LO}}{\hbar\omega_o}$$

- Sub-Shot-Noise Spectra are Non-Classical:

$$\mathcal{S}_{i_{\rm hom}i_{\rm hom}}(\omega) \text{ or } \mathcal{S}_{i_{\rm het}i_{\rm het}}(\omega) < q^2\frac{\eta P_{\rm LO}}{\hbar\omega_o}$$

Coming Attractions: Lecture 20

- Lecture 20:
  Nonlinear Optics of $\chi^{(2)}$ Interactions
  - Maxwell's equations with a nonlinear polarization
  - Coupled-mode equations for parametric downconversion
  - Phase-matching for efficient interactions