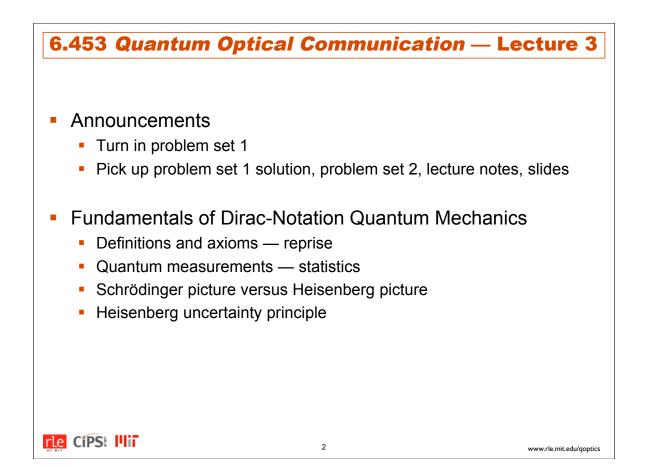
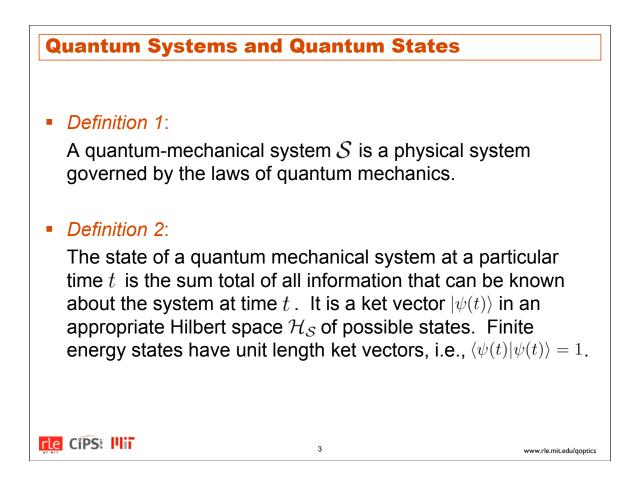
6.453 Quantum Optical Communication Spring 2009

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Time Evolution via the Schrödinger Equation

• Axiom 1:

For $t \ge 0$, an isolated system with initial state $|\psi(0)\rangle$ will reach state

$$|\psi(t)\rangle = U(t,0)|\psi(0)\rangle$$

where $\hat{U}(t,0)$ is the unitary time-evolution operator for the system \mathcal{S} . $\hat{U}(t,0)$ is obtained by solving

$$j\hbar \frac{\mathrm{d}\hat{U}(t,0)}{\mathrm{d}t} = \hat{H}\hat{U}(t,0), \quad \text{for } t \ge 0, \text{ with } \hat{U}(0,0) = \hat{I}$$

where \hat{H} is the Hamiltonian (energy) operator for S. Equivalently, we have the Schrödinger equation

$$j\hbar \frac{\mathrm{d}|\psi(t)\rangle}{\mathrm{d}t} = \hat{H}|\psi(t)\rangle, \text{ for } t \ge 0, \text{ with } |\psi(0)\rangle \text{ initial condition}$$

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Quantum Measurements: Observables• Axiom 2:
An observable is a measurable dynamical variable of the
quantum system S. It is represented by an Hermitian
operator which has a complete set of eigenkets.• Axiom 3:
For a quantum system S that is in state
$$|\psi(t)\rangle$$
 at time t,
measurement of the observable
 $\hat{O} \equiv \sum_{n} o_n |o_n\rangle \langle o_n|$
yields an outcome that is one of the eigenvalues, $\{o_n\}$, with
 $\Pr(outcome = o_n) = |\langle o_n | \psi(t) \rangle|^2$ Image: Ima

Quantum Measurements: Observables

Projection postulate:

Immediately after a measurement of an observable \hat{O} , with distinct eigenvalues, yields outcome o_n the state of the system becomes $|o_n\rangle$.

• Axiom 3a:

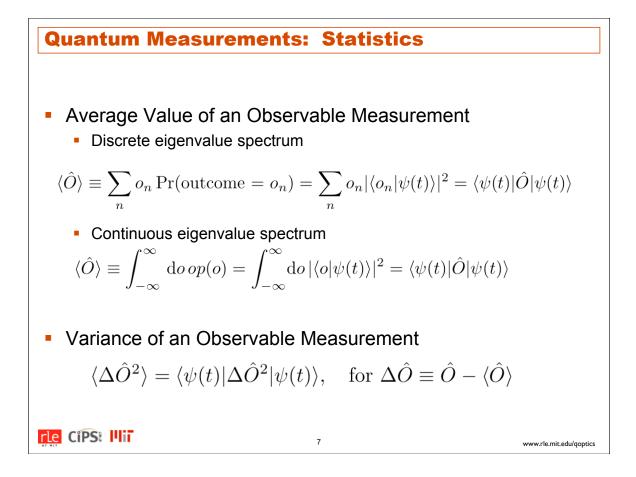
For a quantum system ${\mathcal S}$ that is in state $|\psi(t)\rangle$ at time t , measurement of the observable

 $\hat{O} = \int_{-\infty}^{\infty} \mathrm{d} o \, o |o\rangle \langle o|$

yields an outcome that is one of the eigenvalues, o, with

$$p(o) = |\langle o|\psi(t)\rangle|^2$$

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Schrödinger versus Heisenberg Pictures

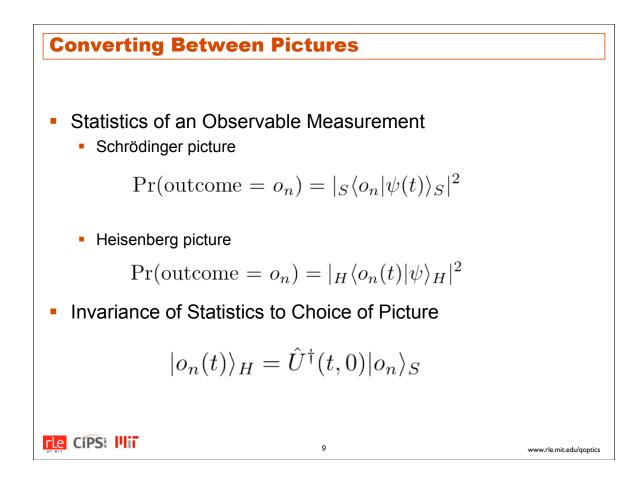
- Schrödinger Picture
 - Observables are time-independent operators
 - Between measurements, states evolve according to the Schrödinger equation

$$\{ |\psi(t)\rangle_S, \hat{O}_S, \hat{H}_S : t \ge 0 \}$$

- Heisenberg Picture
 - Between measurements, states are constant
 - Observables evolve in time according to appropriate equations of motion

$$\{ |\psi\rangle_H, \hat{O}_H(t), \hat{H}_H(t) : t \ge 0 \}$$

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Heisenberg Equations of Motion • Transforming an Observable between Pictures $\hat{O}_H(t) = \hat{U}^{\dagger}(t,0)\hat{O}_S\hat{U}(t,0)$ • Equation of Motion for $\hat{O}_H(t)$ $j\hbar \frac{d\hat{O}_H(t)}{dt} = [\hat{O}_H(t),\hat{H}], \text{ for } t \ge 0, \text{ with } \hat{O}_H(0) = \hat{O}_S$ • Commutator Brackets $[\hat{O}_H(t),\hat{H}] \equiv \hat{O}_H(t)\hat{H} - \hat{H}\hat{O}_H(t)$

