6.334 Power Electronics
Spring 2007

Power Electronics Notes - D. Perreault

★★ 3 Phase Systems

3 sources (v_a, v_b, v_c) separated by $120°$ in phase ($\frac{2\pi}{3}$ rad)

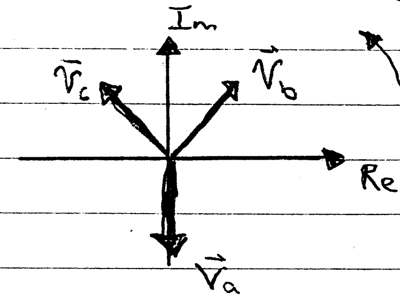
e.g. $v_a = V_s \sin(\omega t)$

$v_b = V_s \sin(\omega t + \frac{2\pi}{3})$

$v_c = V_s \sin(\omega t - \frac{2\pi}{3})$

Represented as phasors:

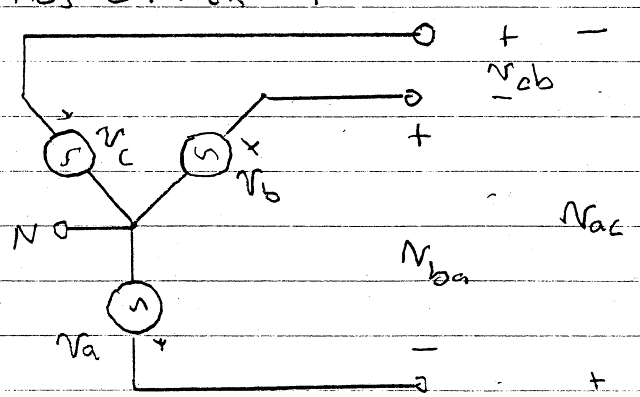
$$v_x(t) = Re\{\vec{V}_x e^{j\omega t}\}$$



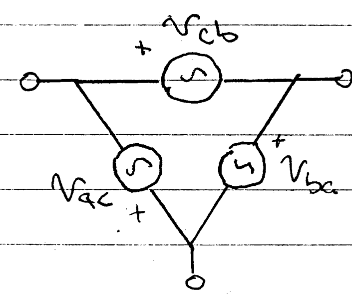
$\times e^{j\omega t}$ = rotate by angle ωt

Connection of 3ϕ sources

most common: Y

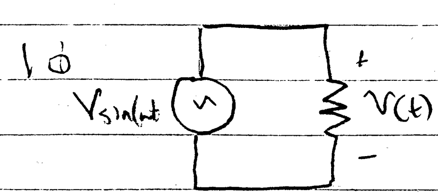


Δ Connected (no neutral)



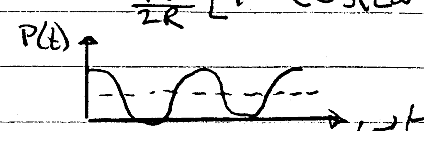
why 3ϕ?

1. Constant Power Sourcing



$$P(t) = \frac{v^2(t)}{R} = \frac{V_s^2}{R}\sin^2(\omega t)$$

$$= \frac{V_s^2}{2R}[1-\cos(2\omega t)]$$



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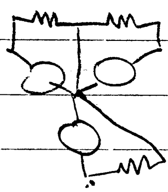
1Φ:

Even at unity power factor, instantaneous power fluctuates between zero + twice average at double the line frequency (no const. power out). Makes sense: can't get power when voltage is zero. This is bad for supplying power to machines, rectifiers, etc. which would like to draw const power, + must buffer the fluctuations (via. inertia, capacitance, etc.)



3Φ solves this: $P_{TOT} = \frac{V_s^2}{R}\left[\sin^2(\omega t) + \sin^2(\omega t + \frac{2\pi}{3}) + \sin^2(\omega t - \frac{2\pi}{3})\right]$

$$P_{TOT} = \frac{V_s^2}{2R}\left[3 + \underbrace{\cos(2\omega t) + \cos(2\omega t - \tfrac{4\pi}{3}) + \cos(2\omega t + \tfrac{4\pi}{3})}_{3\phi \text{ set cancels}}\right]$$

$$P_{TOT} = \frac{3V_s^2}{2R}$$

$\therefore$ 3Φ sets can deliver constant total output power without fluctuations!

(note: 2Φ sets can as well, since $\sin^2(\omega t) + \cos^2(\omega t) = 1$!)

2. Neutral wire return not needed: unlike 2Φ power, one can deliver 3Φ power w/o the need for a neutral return. So 3 wires are needed for both 2Φ + 3Φ, but can deliver more power for same amount of wire cabling in 3Φ. $\therefore$ Important for cost of utility lines.

3. 3Φ systems allow cancellation of all triplen harmonics (harmonics that are multiples of 3). How?

If I take a waveform (not necessarily sinusoidal

$f(t) = \sum a_n \sin(n\omega t + \phi_n)$ and shift it by $\pm\frac{T}{3}$ ($\pm\frac{2\pi}{3}$ radians of fundamental $= 120°$)

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The +/- shifted waveforms will have fundamentals that differ by $120^\circ = \frac{2\pi}{3}$ radians. 3n harmonics will be shifted by $3n \times 120^\circ = n \times 360^\circ$. Thus, if we take the difference of the shifted waveforms, (e.g. l-l), the 3n harmonics will drop out!

Thus, no even harmonics, 3n's gone, 5^{th}, 7^{th}, 11^{th}, 13^{th} lowest harmonics left. → great!

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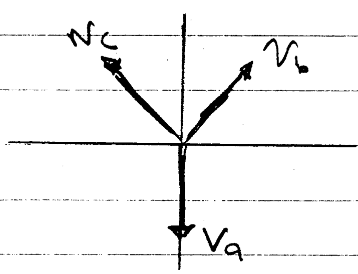
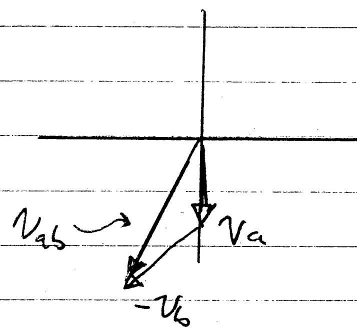
line-line voltages can be vector constructed from line-neutral





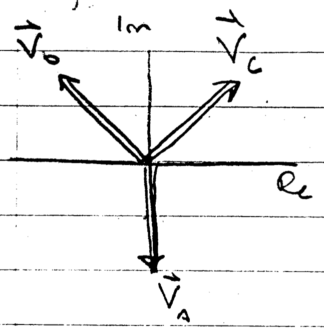
e.g. $v_{ab} = v_a - v_b = \sqrt{3}\, V_s \sin(\omega t - \pi/6)$

120V l-n → 208V l-l (rms)

- So l-l magnitude is scaled by $\sqrt{3}$
- l-l phase is shifted by $\frac{\pi}{6}$ (30°) from l-n
  → phase shift can be useful for converters fed from Δ/Δ, ΔY transformers, e.g. 12-pulse rectifiers

Given a 3Φ set of voltages, we can create a set with any phase relation we desire.

e.g.

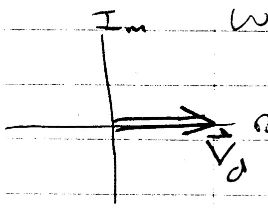


$$V_A(t) = Re\left\{ \bar{V}_s e^{-j\pi/2} e^{j\omega t} \right\}$$
$$V_b(t) = Re\left\{ \bar{V}_s e^{j5\pi/6} e^{j\omega t} \right\}$$
$$V_c(t) = Re\left\{ \bar{V}_s e^{j\pi/6} e^{j\omega t} \right\}$$

in rectangular $\begin{bmatrix} x \\ y \end{bmatrix}$ coordinates, we could represent $\vec{V}_a = \begin{bmatrix} \cos(-\frac{\pi}{2}) \\ \sin(-\frac{\pi}{2}) \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \bar{V}_s$

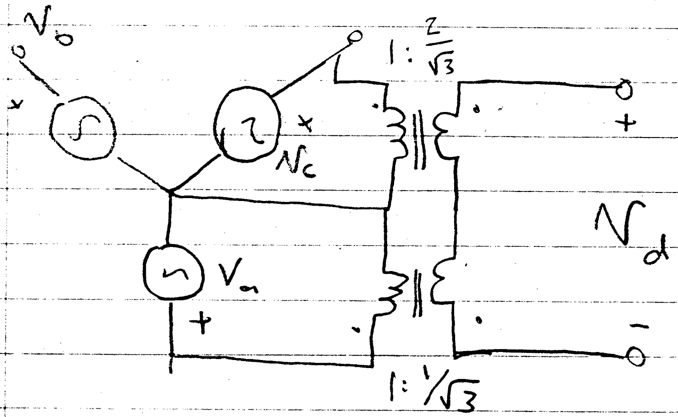
$$\vec{V}_c = \begin{bmatrix} \cos(\frac{\pi}{6}) \\ \sin(\frac{\pi}{6}) \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} \bar{V}_s$$

If we wanted to synthesize a phasor $\vec{V}_d$ with $\measuredangle = 0$, $\| \| = \bar{V}_s$ we could do this by summing parts of $\vec{V}_a, \vec{V}_c$



$$\vec{V}_d = \alpha \vec{V}_a + \beta \vec{V}_c$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{\sqrt{3}}{2} \\ -1 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \Rightarrow \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{2}{\sqrt{3}} \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} \\ \frac{2}{\sqrt{3}} \end{bmatrix}$$



- we could similarly synthesize any other angle to other phases
- There are multiple ways to do this, since $\vec{V}_a, \vec{V}_b, \vec{V}_c$ are a linearly dependent set

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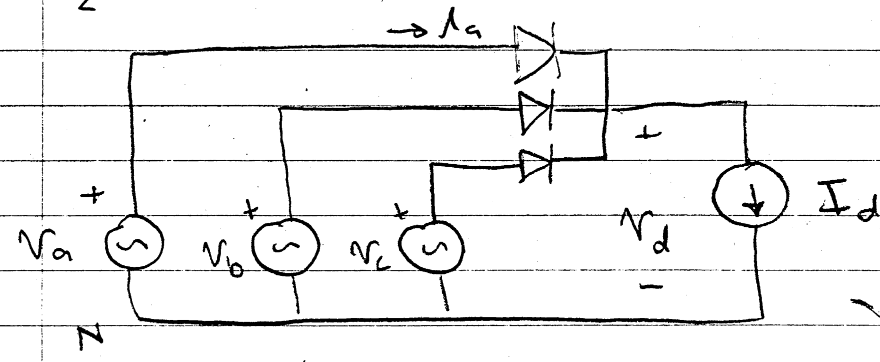
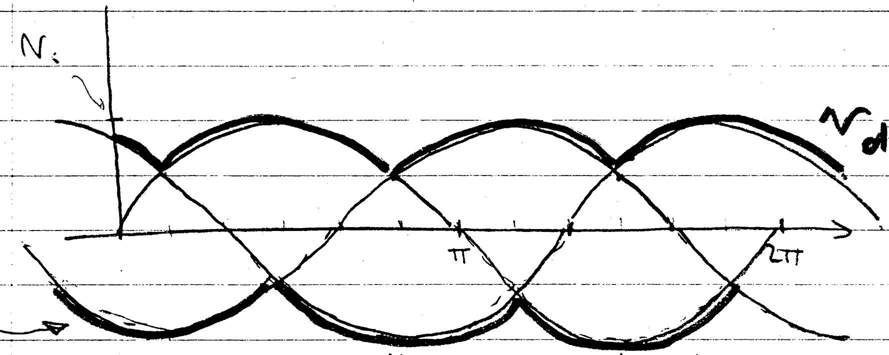
★ 3 Phase Rectification

$\frac{1}{2}$ wave rectifier



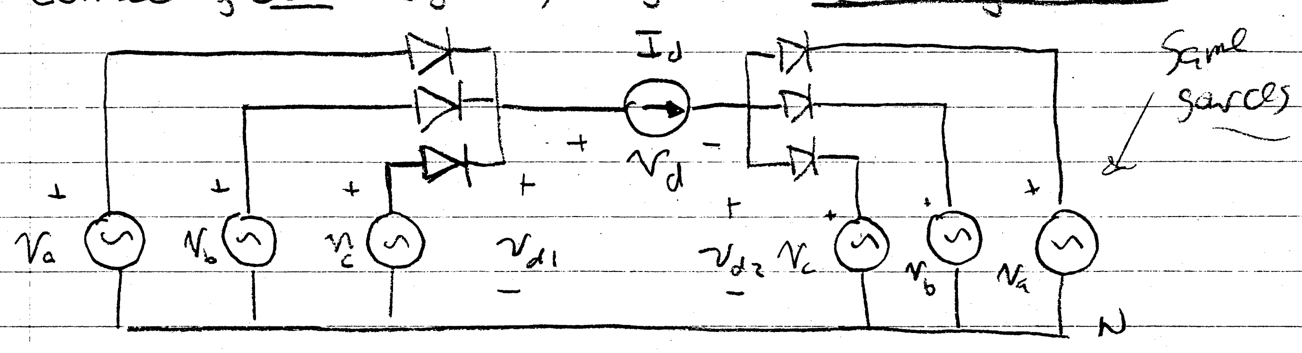
V_d is the diode "or" of the 3 voltages



3 V_{out} pulses/cycle

If we connect things negatively, we get other halves

Connecting both together, we get the full-bridge rectifier:



V_d is "most positive of $\{V_a, V_b, V_c\}$ - most negative of $\{V_a, V_b, V_c\}$

This is the same as the largest of $\{V_{ab}, V_{ac}, V_{bc}, V_{ba}, V_{ca}, V_{cb}\}$

This "full-bridge" connection operates on line-to-line voltages!

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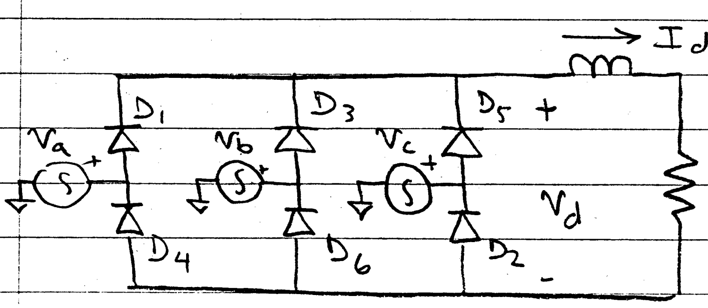
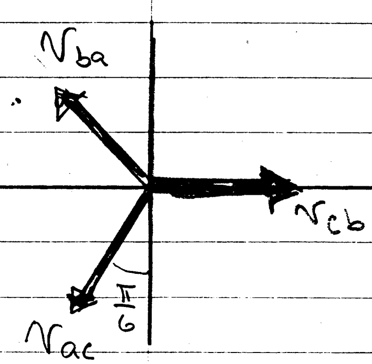
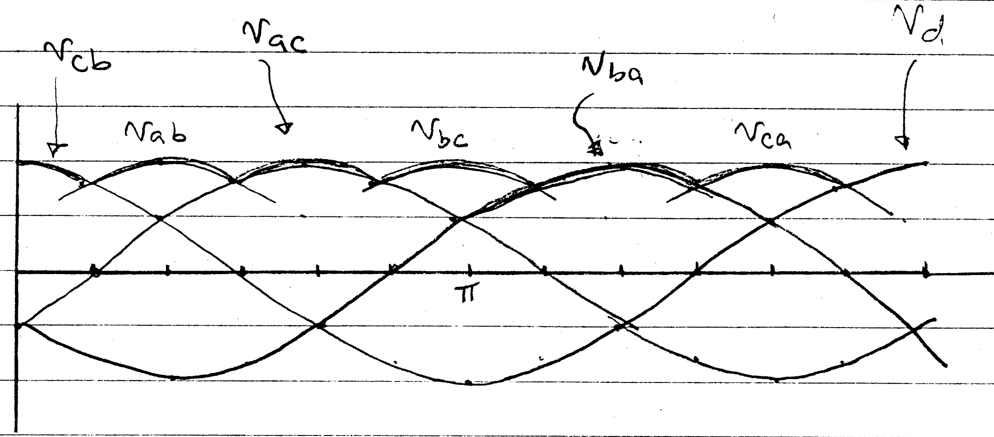
The full-bridge rectifier can be drawn as follows:



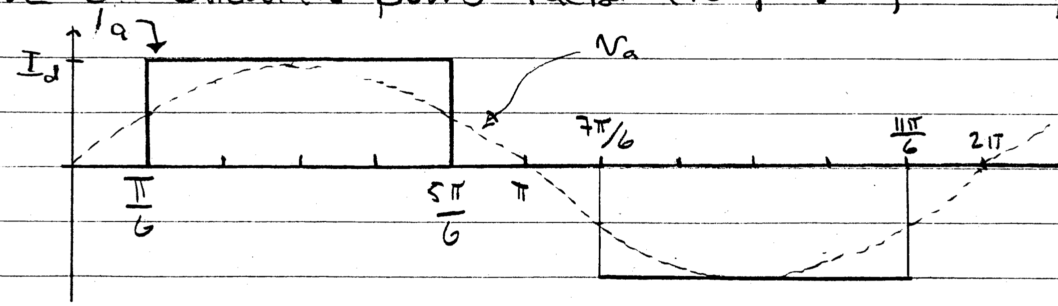
3Φ l-l phasors





NOTE: The bridge diodes are numbered in the order in which they conduct over the cycle. (eg: $D_1D_2, D_2D_3, D_3D_4, D_4D_5, D_5D_6, D_6D_1 \ldots$)

We can calculate power factor (for phase A, for example)



$v_{a,rms} = \frac{V_s}{\sqrt{2}} \qquad i_{a,rms} = \sqrt{\frac{2}{3}}\, I_d$

$$\langle P \rangle = \frac{1}{\pi}\int_{\frac{\pi}{6}}^{5\pi/6} V_s I_d \sin(\varphi)\,d\varphi = \frac{\sqrt{3}}{\pi} V_s I_d$$

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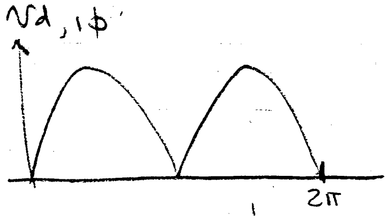
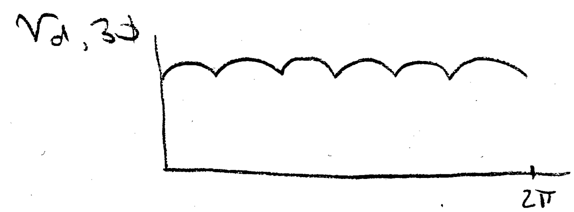
$$K_p = \frac{\langle P \rangle}{V_{rms} I_{rms}} = \frac{\sqrt{3}}{\pi} V_s I_d \cdot \left(\frac{\sqrt{3}}{V_s I_d} \right) = \frac{3}{\pi} \simeq .96$$

(compare to 0.91 for a 1ϕ bridge)

also 1ϕ bridge $\rightarrow$ 4 diodes
3ϕ bridge $\rightarrow$ only 6 diodes

ripple voltage is at $6 f_{line}$ not $2 f_{line}$ $\Rightarrow$ easier to filter





Single phase rectifier

3 phase rectifier

the ripple-voltage <u>magnitude</u> is also smaller $\Rightarrow$ easier to filter

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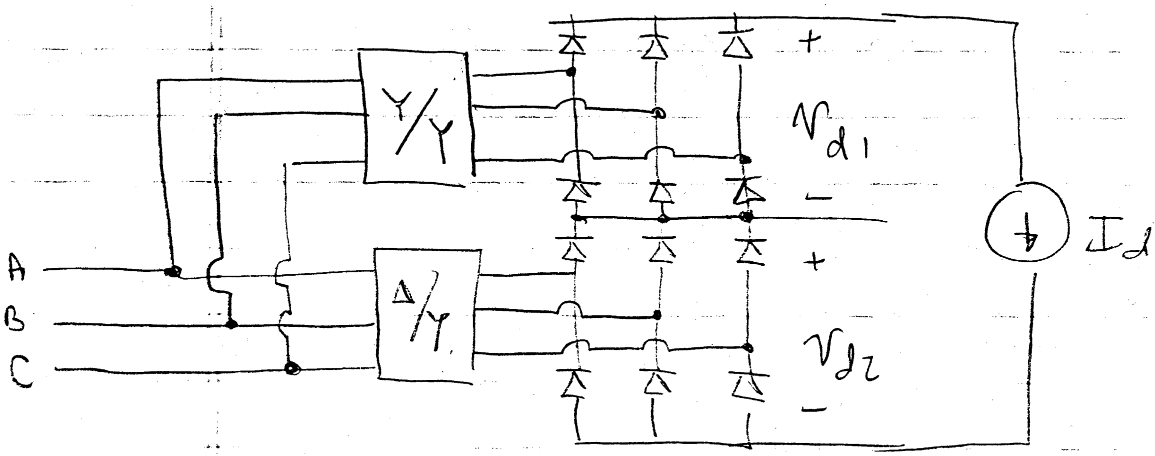
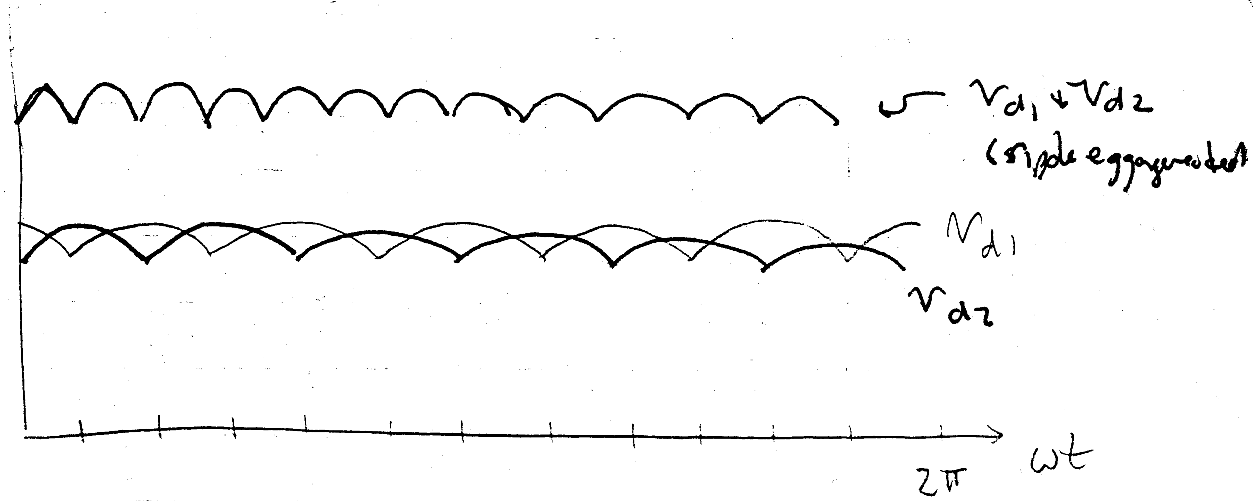
Higher-Order rectifiers

Suppose we use two phase-shifted transformer sets on series-stacked six-pulse bridges



The Y/Y, Δ/Y transformer sets generate equal voltage magnitudes, with a 30° phase difference between their 3φ outputs

Since all voltages are isolated, constant current in the bridges → the two six pulse bridges act independently.
Since input waveforms shifted by 30° $\left(\frac{T}{12}\right)$ and output ripple is at 6× input frequency $\left(T_{V_{OUT}} = \frac{T}{6}\right)$
Output ripple voltages are shifted by $T_{Vout}/2$ (30° fund. is 180° 6th)



We have a 12-pulse rectifier! (fundamentals of ripple terms cancel)

- → smaller ripple magnitude
- → higher ripple frequency

} easier output filter

→ net input current + power factor also improves

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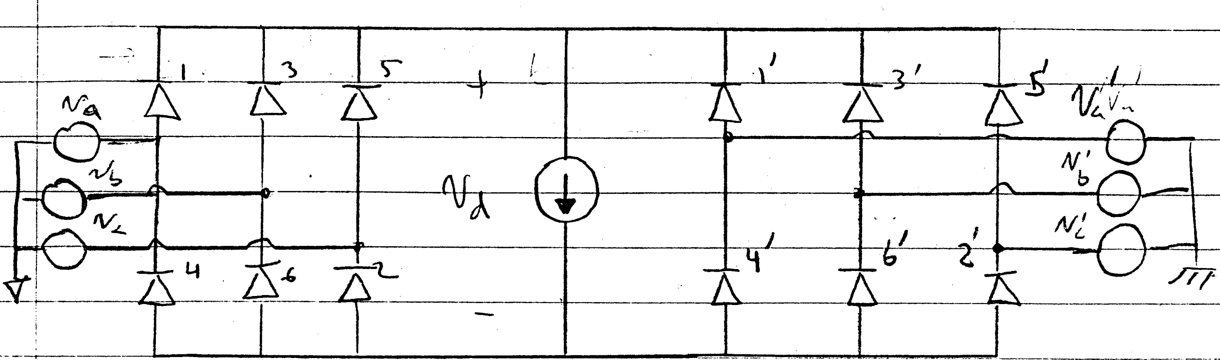
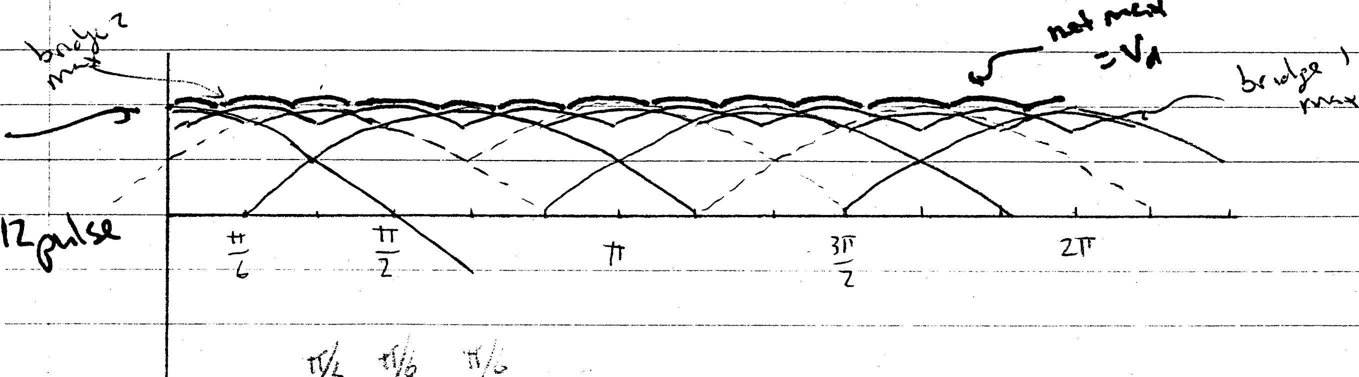
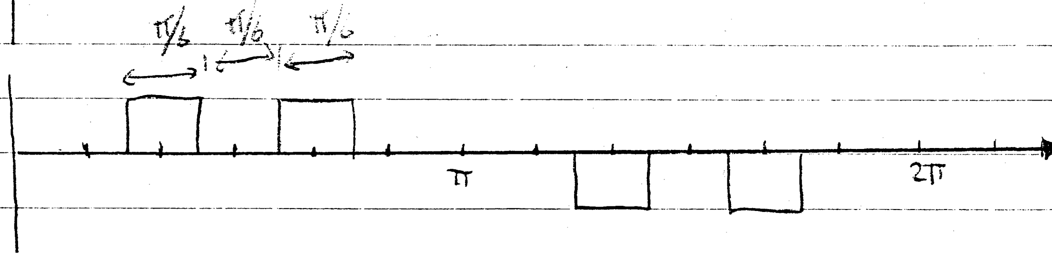
consider parallel case (direct connection



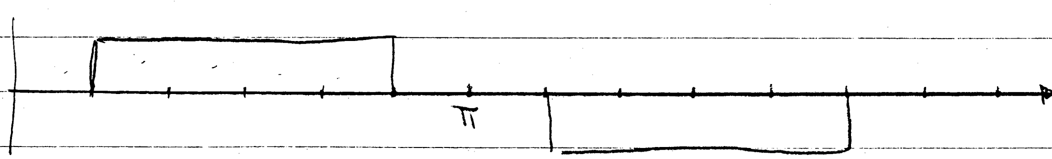


again, V_d is 12 pulse

12-pulse i_a



six-pulse i_a



12 pulse $\quad I_{d1,rms} = \sqrt{\frac{2\pi}{6} \cdot \frac{1}{2\pi} \cdot I_d^2} = \frac{I_d}{\sqrt{6}}$

6 pulse $\quad I_{d1,rms} = \sqrt{\frac{4\pi}{6} \cdot \frac{1}{2\pi} I_d^2} = \frac{I_d}{\sqrt{3}}$

So 12 pulse, $I_{rms} \downarrow$ by $\sqrt{2}$, but twice as many devices

→ each device carries the full current for $\frac{1}{2}$ the time!

(ohmic loss in devices, xformers, lines depend on RMS!)

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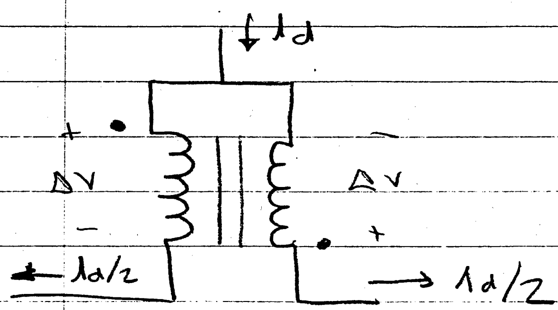
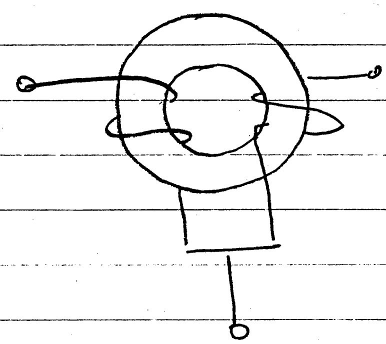
what we would like is to get the 12-pulse waveform with 2 bridges acting independently so each carries half the current





Interphase transformers

Ideal : • forces the current to split evenly
• forces the voltage at the output to be the average of 2 input voltages

What happens? → each bridge operates independently

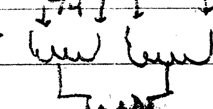
$$I_{d1,rms} = \sqrt{\frac{4\pi}{6} \cdot \frac{1}{2\pi} \left(\frac{I_d}{2}\right)^2} = \frac{I_d}{2\sqrt{3}}$$

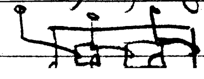
• So we have 2 x as many devices as a 6-pulse rectifier each of which carry half the rms current

• The voltage is the average of 2 shifted 6-pulse waveforms, giving harmonic cancellation in the output voltage and the input current (as w/ series connection)

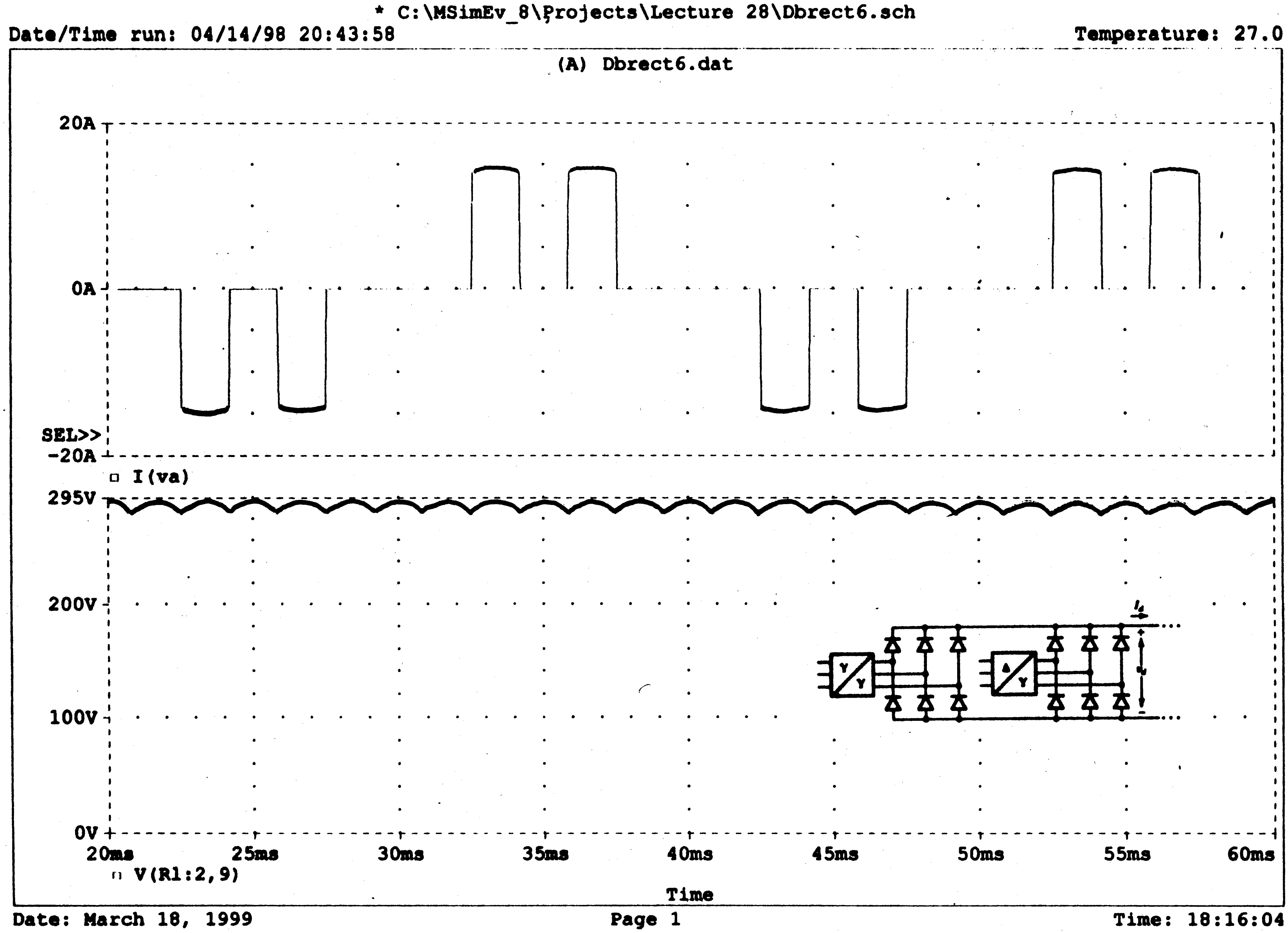
• This connection is often used for high-current systems (MIT tokamak, Railroad converters, etc.)

• Can extend to 18, 24, 36 pulse etc .wiffle trees





← multilevel interphase xformers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20A
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Page 1
Time: 18:16:04

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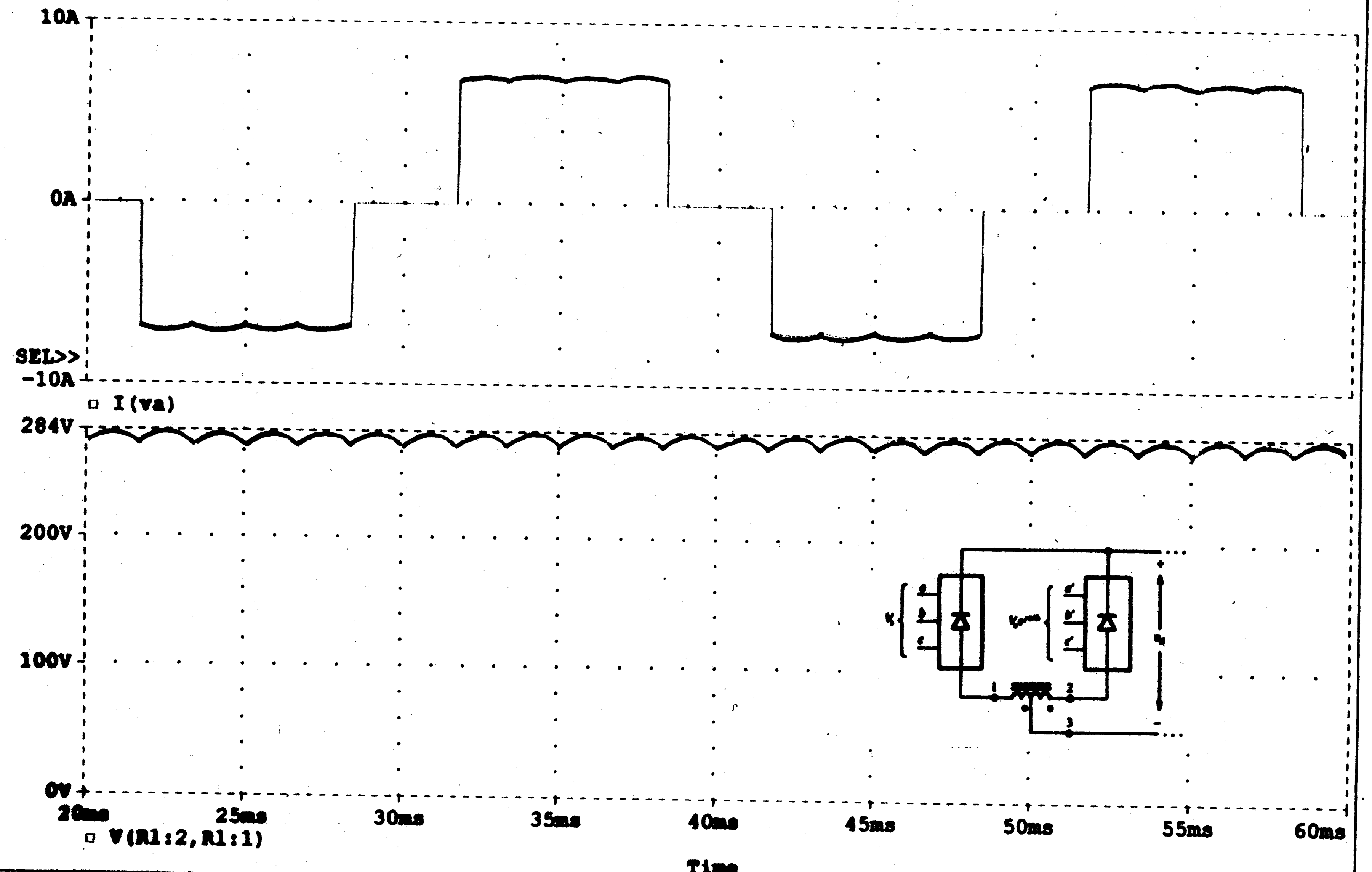
Temperature: 27.0

(B) Dbrect6_Int.dat



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