MASSACHUSETTS INSTITUTE OF TECHNOLOGY

15.082J | Fall 2010

Midterm 2 Review

1. Prove the max-flow min-cut Theorem.

2. Consider some instance of the maximum s-t flow problem. Suppose that in some feasible flow there exists a cut C all of whose forward arcs are saturated. What does this imply? What is the relationship between saturated cuts, minimum cuts, and maximum flows?

3. What does it mean for a distance function to be valid for a given flow? What does it mean for an arc (or path) to be admissible in a residual network? Explain why these notions are important for proving runtime bounds for max-flow algorithms (give at least one example and the associated proofs to back it up).

4. Explain the difference between a 'saturating push' and 'non-saturating push' in the context of max-flow algorithms. What proof techniques are used to bound the number of each kind of push (respectively)? Give proof sketches to support your arguments.

5. What does it mean for an algorithm to use scaling? Why does scaling make an algorithm's runtime easier to analyze? Justify your arguments by analyzing the excess scaling algorithm.

6. State and prove the optimality conditions for the min-cost flow problem. Based on your optimality condition, describe an algorithm for min-cost flow problem and analyze its runtime. If your algorithm is not polynomial time, describe a sequence of instances on which your algorithm takes 'exponential' time.

7. Give the reduction from the family of linear programs with the 'consecutive '1's property' (Application 9.6 of the book) to the min-cost-flow problem.

8. Exercise 9.39

9. What is a spanning tree solution? What is a basic feasible solution in a linear program? What is the relation between these notions in the network simplex algorithm?

10. Describe how a pivot is performed in the network simplex algorithm. Discuss the role of node potentials and reduced costs. What is the role of linear independence during a pivot?

11. Exercise 11.29

12. Given a spanning tree solution during an iteration of network simplex, explain how to compute the corresponding node potentials.

13. Given a spanning tree solution during an iteration of network simplex, explain how to compute the corresponding flows.

14. Given a spanning tree solution during an iteration of network simplex, explain how to determine if the solution is optimal.

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