Problems for Recitation 13

1 Asymptotic Notation

Which of these symbols

$$\Theta \qquad O \qquad \Omega \qquad o \qquad \omega$$

can go in these boxes? (List all that apply.)

$$2n + \log n \quad = \quad \square \; (n)$$

$$\log n \quad = \quad \square \; (n)$$

$$\sqrt{n} \quad = \quad \square \; (\log^{300} n)$$

$$n2^n \quad = \quad \square \; (n)$$

$$n^7 \quad = \quad \square \; (1.01^n)$$

2 Asymptotic Equivalence

Suppose $f, g : \mathbb{Z}^+ \to \mathbb{Z}^+$ and $f \sim g$.

1. Prove that $2f \sim 2g$.
2. Prove that $f^2 \sim g^2$.
3. Give examples of f and g such that $2^f \not\sim 2^g$.
4. Show that $\sim$ is an equivalence relation
5. Show that Θ is an equivalence relation

3 More Asymptotic Notation

1. Show that
$$(an)^{b/n} \sim 1.$$
where a, b are positive constants and $\sim$ denotes asymptotic equality. Hint $an = a2^{\log_2 n}$.
2. You may assume that if $f(n) \geq 1$ and $g(n) \geq 1$ for all n, then $f \sim g \Rightarrow f^{\frac{1}{n}} \sim g^{\frac{1}{n}}$. Show that
$$\sqrt[n]{n!} = \Theta(n).$$

6.042J / 18.062J Mathematics for Computer Science
Fall 2010