Fall Term 2002
Introduction to Plasma Physics I

22.611J, 6.651J, 8.613J

Problem Set #1

1. **Distribution functions and averages:** The average of a quantity, $G(\mathbf{v})$, over a distribution function, $f(\mathbf{v})$, is defined as,

$$\langle G \rangle = \frac{\int d^3v G(\mathbf{v}) f(\mathbf{v})}{\int d^3v f(\mathbf{v})}$$

The Maxwellian distribution, in three dimensions, is,

$$f(\mathbf{v}) = f(v_x, v_y, v_z) = n \left(\frac{m}{2\pi T}\right)^{3/2} \exp(-\frac{mv^2}{2T})$$

Note that the isotropy of this distribution means that all Cartesian coordinates are equivalent; there is no preferred direction.

   (a) Prove that the form of f is correctly normalized, i.e. that, $\int f d^3v = n$.
   Evaluate averages of the following:

   (b) A specific Cartesian velocity direction: $\langle v_x \rangle$

   (c) The square velocity: $\langle v^2 \rangle$, and hence the average particle energy, $\left\langle \frac{1}{2}mv^2 \right\rangle$

   (d) The average speed, $\langle |v| \rangle$

2. Basic facts you need to know. Find out, write down, and memorize (to 2 significant figures) the values of the following quantities: (*You may use either SI or CGS units, but I will use CGS for lecture, so these are recommended. . .*)

   (a) The speed of light

   (b) The charge on the electron

   (c) The mass of the electron, $m_e c^2$, in MeV.

   (d) The mass of the proton, $m_p c^2$, in MeV.

   (e) The temperature in Kelvins, equal to 1 eV.

   (f) The particle density of the air you breathe (cm^{-3}).

   (g) The density of particles in water.

(h) The ionization potential of the hydrogen atom: calculate, $E_I = \frac{1}{2}\frac{m_e c^2 e^4}{\hbar^2 c^2}$.

(i) The relationship between magnetic units of Gauss and Tesla.

(j) The relationship between particle density in units of cm^{-3} and m^{-3}.

Here are two more fundamental physics constants you might find useful:

(k) Planck's constant in Atomic units: $\hbar c \cong 1970\ eV - \AA = 1.97 \times 10^{-5}\ eV - cm$

(l) The fine structure constant: $\frac{e^2}{\hbar c} = \frac{1}{137}$

3. Suppose the degree of ionization of a gas discharge is governed by the Saha equation,

$$\frac{n_e n_i}{n_0} = \frac{4}{(4\pi)^{5/2}} \left(\frac{m_e c^2}{\hbar c}\frac{e^2}{\hbar c}\right)^3 \left(\frac{T}{E_I}\right)^{3/2} \exp\left(-\frac{E_I}{T}\right)$$

and the Debye length is small relative to the discharge size. Calculate approximately the temperature at which the gas is 50% ionized if, $E_I = 13.6\ eV$, and its total pressure is equal to one atmosphere.

4. Consider a plasma in which both electrons and (singly-charged) ions adopt thermal distributions with Boltzmann factors governed by the respective temperatures, T_e, and, T_i, which are different, in general. Show that a point charge, q, immersed in this plasma gives rise to a potential as a function of distance, r, from the charge:

$$\phi = \frac{q}{r}\exp\left(-r/\lambda\right)$$

in the approximation, $e\phi \ll T_e, T_i$. Obtain an expression for λ. Does the situation of cold ions, $T_i \ll T_e$, correspond to the idealized case of "immobile" ions sometimes referred to in textbooks?

5. Consider a charged sphere of radius, a, located far from all other objects in a plasma that has immobile ions and mobile electrons of temperature, T_e. The electrons can be assumed to adopt a Boltzmann distribution, $n = n_0 \exp\left(e\phi/T_e\right)$, where, ϕ, is the electrostatic potential and, n_0, is the background density of the singly charged ions.

(a) Calculate the potential distribution in the plasma when the potential on the sphere is, ϕ_s, in the approximation, $e\phi_s \ll T_e$.

(b) Sketch the form of, ϕ, as a function of radius, r, in the two cases, $\lambda_D \ll a$, and, $a \ll \lambda_D$

(c) Calculate the charge on the sphere, and hence its capacitance in the presence of the plasma. (*The charge can be determined from the surface electric field*).

(d) Evaluate the capacitances when, $a = 10\ cm$, for the case, $T_e = 1\ KeV$, and (i) $n_0 = 10^{14} cm^{-3}$, and (ii) .$n_0 = 10^6 cm^{-3}$, and compare with the capacitance in vacuum.