

15.081 Fall 2009
Recitation
for Lectures {2,3,4}
Geometry of Linear Programming

Note: In what follows, I will present the material that was covered in the recitations corresponding to the lectures 2, 3 and 4 which dealt with the geometry of linear programming. The material will be somewhat a superset of what I covered in the recitations. These material are intended to cover the most important principles of Chapter 2 from the book. These notes should be viewed as a summary of the chapter and not as a reading material (for the exam).

1 Definitions

1. Polyhedron : Intersection of a finite number of half spaces ($a'x \leq b$). “Intersection” and “finite” are the keywords.
2. Convex Set : If the average of the elements of a set also belongs to the set, then the set is convex.
3. Extreme point, Vertex and BFS : The definitions are different (although equivalent for a polyhedron). Extreme point is defined in terms of the inability to express the point as an average of two other points in the set. Vertex is defined in terms of the point being an optimal solution for some cost vector c .
4. Adjacent basic solutions and adjacent bases and the correspondence between a basis and a basic solution: These definitions should be clear, as they play a major role in the understanding of degeneracy.
5. Degeneracy : If more than n constraints are active at a basic solution, it is degenerate. For a standard form, this reduces to the existence of more than $n - m$ components of a point being zero. As one can imagine, Degeneracy is not a geometric property. This is because if we know the object and if we take a minimal (with respect to the number of constraints) representation of the polyhedron, we do not get degeneracy.

2 Important Theorems

The following theorems are important based on either what they say or the proof technique that was used in the proof.

1. Theorem 2.2 : States the equivalence of existence of a unique solution to a set of linear equations and the row vectors being spanning R^n . This theorem is the basis for construction of basic feasible solutions for simplex method.
2. Equivalence of a vertex, and extreme point and a bfs : The equivalence being obvious, the proof is the best part of this theorem. In particular, two of the directions have a constructive proof which allows us to use

the construction in other proofs. Given a bfs, the construction of a cost vector that is optimal at that bfs is particularly interesting. And if we are given a point which is not a bfs, the construction of two points in the set whose average is the given point, is also interesting.

3. Equivalence of

- (a) Existence of an extreme point
- (b) Non-existence of a line
- (c) Existence of enough number of linearly independent vectors.

The proof is constructive, please go through it.

- 4. Optimality of extreme points - The statement being very clear, the proof is pretty non-trivial and cute.
- 5. Non-empty bounded polyhedron is the convex hull of its extreme points.- Proof is constructive.

3 Problems and Exercises

Will present the problems that are “important”.

- 1. 2.6 - Caratheodory’s theorem - Shows the existence of a minimal set of extreme points that contains a given point in its convex hull. The proof uses the existence of a bfs.
- 2. Fourier -Motzkin Elimination (2.20) - Shows that we could obtain exponentially many constraints.
- 3. Exercise 2.22 : Uses the fact that a projection of a polyhedron is a polyhedron.

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6.251J / 15.081J Introduction to Mathematical Programming
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