MASSACHUSETTS INSTITUTE OF TECHNOLOGY

6.436J/15.085J — Fall 2008

Problem Set 6 — due 10/17/2008

Readings: Notes for lecture 10.

Recommended readings: Sections 3.6, 4.2 of [BT, 1st edition], or Section 4.1 of [BT, 2nd edition]; Sections 4.7-4.8 of [GS].

Exercise 1. Suppose that X and Y are independent and uniformly distributed on $[0, 1]$. Find the PDF of $Z = \max\{X^2, Y\}$.

Exercise 2. Consider two independent and identically distributed discrete random variables X and Y. Assume that their common PMF, denoted by $p(x)$, is symmetric around zero, i.e., $p(x) = p(-x)$ for all x. Show that the PMF of $X + Y$ is also symmetric around zero and is largest at zero. *Hint:* Use the Schwarz inequality: $\sum_k (a_k b_k) \leq \big(\sum_k a_k^2\big)^{1/2}\big(\sum_k b_k^2\big)^{1/2}$.

Exercise 3. Assuming that $X_1, \ldots, X_n$ are independent with common density function f, establish that the joint distribution of $X^{(1)}, \ldots, X^{(n)}$ is given by

$$f_{X^{(1)},\ldots,X^{(n)}}(x_1, \ldots, x_n) = n! f(x_1) \cdots f(x_n), \qquad x_1 < x_2 < \cdots < x_n,$$

and $f_{X^{(1)},\ldots,X^{(n)}}(x_1, \ldots, x_n) = 0$, otherwise. Use this to derive the densities for $\max_j X_j$ and $\min_j X_j$.

Exercise 4. Let X and Y be independent exponential random variables with parameter $\lambda = 1$. That is, $f_X(t) = f_Y(t) = e^{-t}$, for $t \geq 0$. Let

$$U = X^2, \qquad V = X^2 + Y.$$

Find the joint PDF of U and V.

Exercise 5. Let X and Y be independent exponential random variables with parameters λ and μ, respectively. Find the joint PDF of $U = X + Y$ and $V = X/(X + Y)$, and then the marginal PDF of V.

Exercise 6. Suppose that X and Y are independent and identically distributed, and not necessarily continuous random variables. Show that $X + Y$ cannot be uniformly distributed on $[0, 1]$.

Exercise 7. Let X and Y be independent exponential random variables with parameter equal to 1. Let $S = X + Y$, $D = X - Y$, and $R = X/Y$.

(a) Find the conditional PDF $f_{X|D}(x \mid 0)$.

(b) Find the conditional PDF $f_{X|R}(x \mid 1)$.

(c) Note that the events $\{D = 0\}$ and $\{R = 1\}$ coincide. In view of this should the answers to (a) and (b) be the same? Are they?

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