MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.097 (UG) Fundamentals of Photonics
6.974 (G) Quantum Electronics

Spring 2006

Quiz I

Time: March *10, 2006, 2-3:30pm*

Problems marked with (grad) are for graduate students only.

- This is a closed book exam, but one 81/2"x11" sheet (both sides) is allowed.
- At the end of the booklet there is a collection of equations you might find helpful for the exam.
- Everything on the notes must be in your original handwriting (i.e. material cannot be Xeroxed).
- You have 90 minutes for this exam.
- There are 4 problems on the exam with the number of points for each part and the total points for each problem as indicated. Note, that the problems do not all have the same total number of points.
- Some of the problems have parts for graduate students only. Undergraduate students solving these problems can make these additional points and compensate eventually for points lost on other problems.
- Make sure that you have seen all 21 numbered sides of this answer booklet.
- The problems are not in order of difficulty. We recommend that you read through all the problems, then do the problems in whatever order suits you best.
- We tried to provide ample space for you to write in. However, the space provided is not an indication of the length of the explanation required. Short, to the point, explanations are preferred to long ones that show no understanding.
- Please be neat-we cannot grade what we cannot decipher.

All work and answers must be in the space provided on the exam booklet. You are welcome to use scratch pages that we provide but when you hand in the exam we will not accept any pages other than the exam booklet.

Exam Grading:

In grading of the exams we will be focusing on your level of understanding of the material associated with each problem. When we grade each part of a problem we will do our best to assess, from your work, your level of understanding. On each part fo an exam question we will also indicate the percentage of the total exam grade represented by that part, and your numerical score on the exam will then be calculated accordingly.

Our assessment of your level of understanding will be based upon what is given in your solution. A correct answer with no explanation will not receive full credit, and may not receive much-if any. An incorrect final answer having a solution and explanation that shows excellent understanding quite likely will receive full (or close to full) credit.

This page is intentionally left blank. Use it as scratch paper.

No work on this page will be evaluated.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.097 (UG) Fundamentals of Photonics
6.974 (G) Quantum Electronics

Spring 2006

Quiz I

Time: March *10, 2006, 2-3:30pm*

Full Name: ____

Are you taking 6.097: ____

Are you taking 6.974: ____

	Points
1	
2	
3	
4	
Total	

Problem 1: Susceptibility and Absorption

1. The equation of motion of a one-dimensional electron oscillator in a highly damping medium due to an electric field can be expressed as:

$$m\gamma \frac{dx}{dt} \; - eE$$

where the electric field at the point where the electron is situated, E, is $E_o \exp\left(j(\omega t - kz + \phi) \right)$, γ is the damping coefficient, m is the electron mass, e the electron charge and we assume that N of these oscillators exist per unit volume.

(a) (10 points) Using the above expression, find the complex susceptibility of a highly damping material.

(b) (5 points) Sketch the real and imaginary parts of the complex susceptibility versus ω.

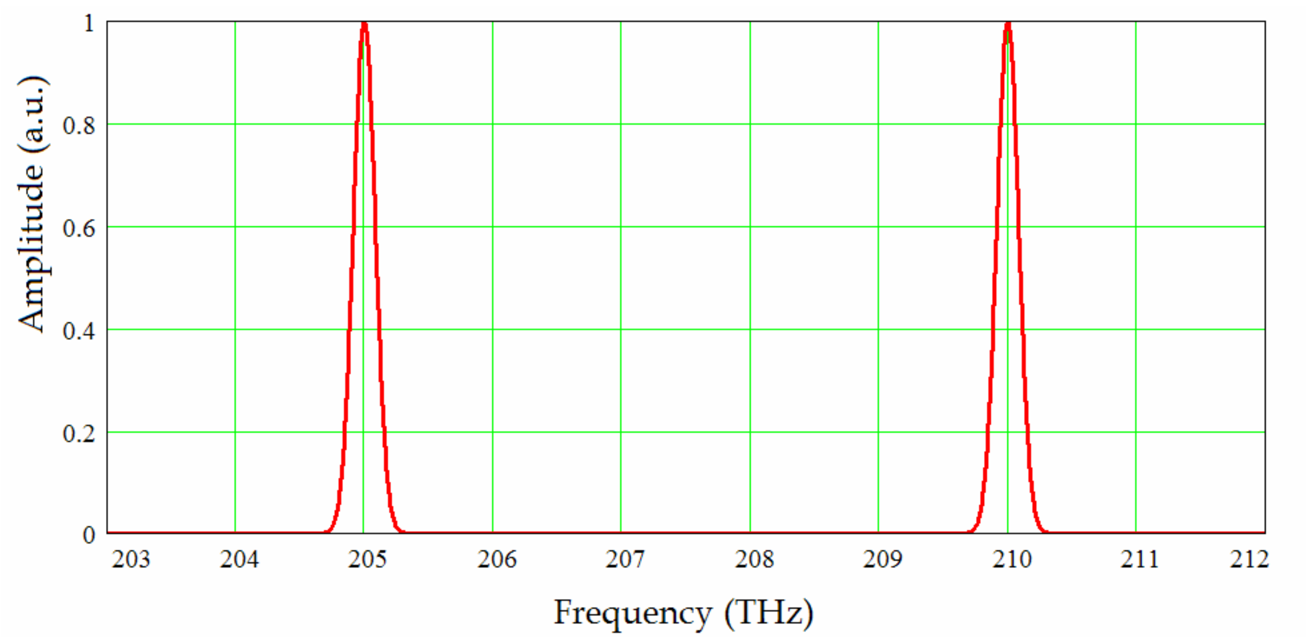
(c) (10 points) Find the complex index of refraction of this material. Interpret your expression physically in the limits of small angular frequency (i.e. $\omega \to 0$) and of large angular frequency (i.e. $\omega \to \infty$).

Problem 2: Pulse Propagation, Group and Phase Velocity

The input of a fiber contains two Gaussian pulses with spectra (see figure below):

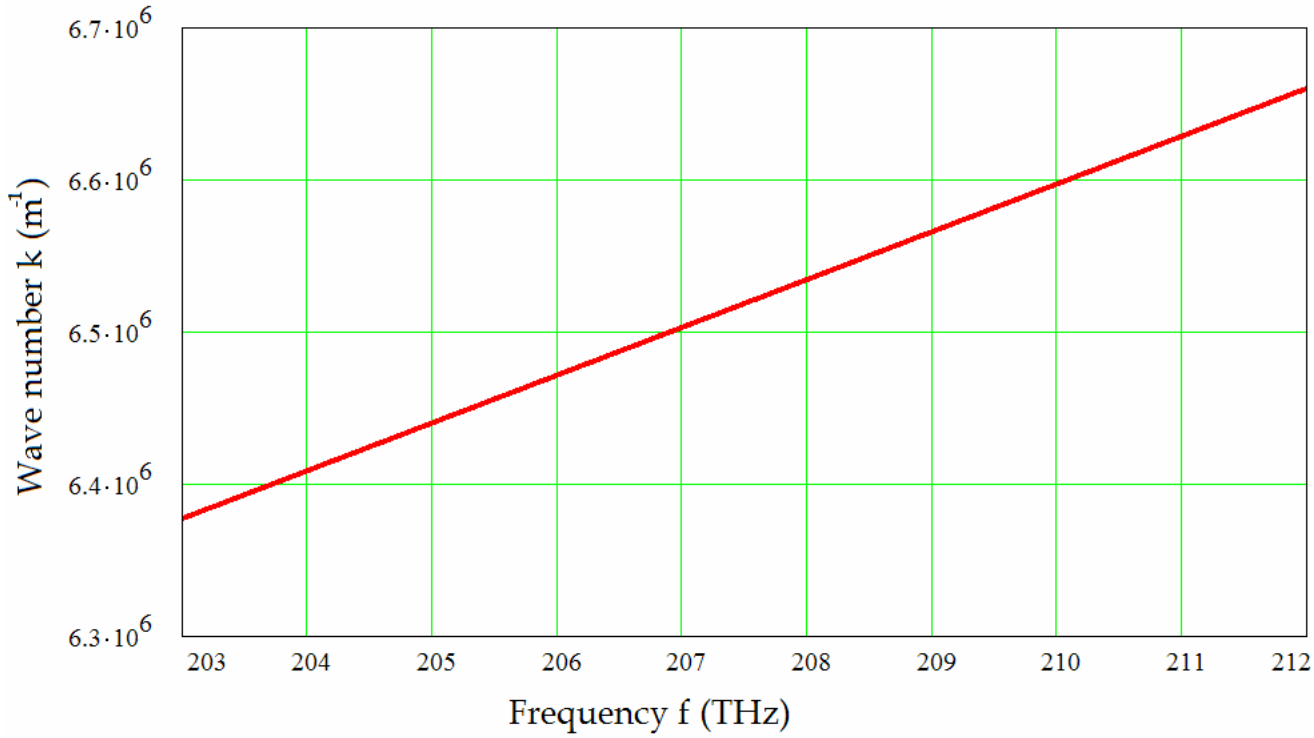
$$E_1(f) = E_0 \exp\left[-\frac{(f-f_1)^2}{\Delta f^2}\right] \qquad\qquad E_2(f) = E_0 \exp\left[-\frac{(f-f_2)^2}{\Delta f^2}\right]$$

The center frequencies of these pulses are $f_1 = 205\,THz$ and $f_2 = 210\,THz$ (1 THz = 10^{12} Hz). All other parameters of the pulses are identical. Dispersion of the fiber can lead to pulse broadening as well as time delay between pulses as they propagate through the fiber. You are asked to estimate the pulse broadening and time delay from the dispersion characteristics of the fiber.



(see next page)

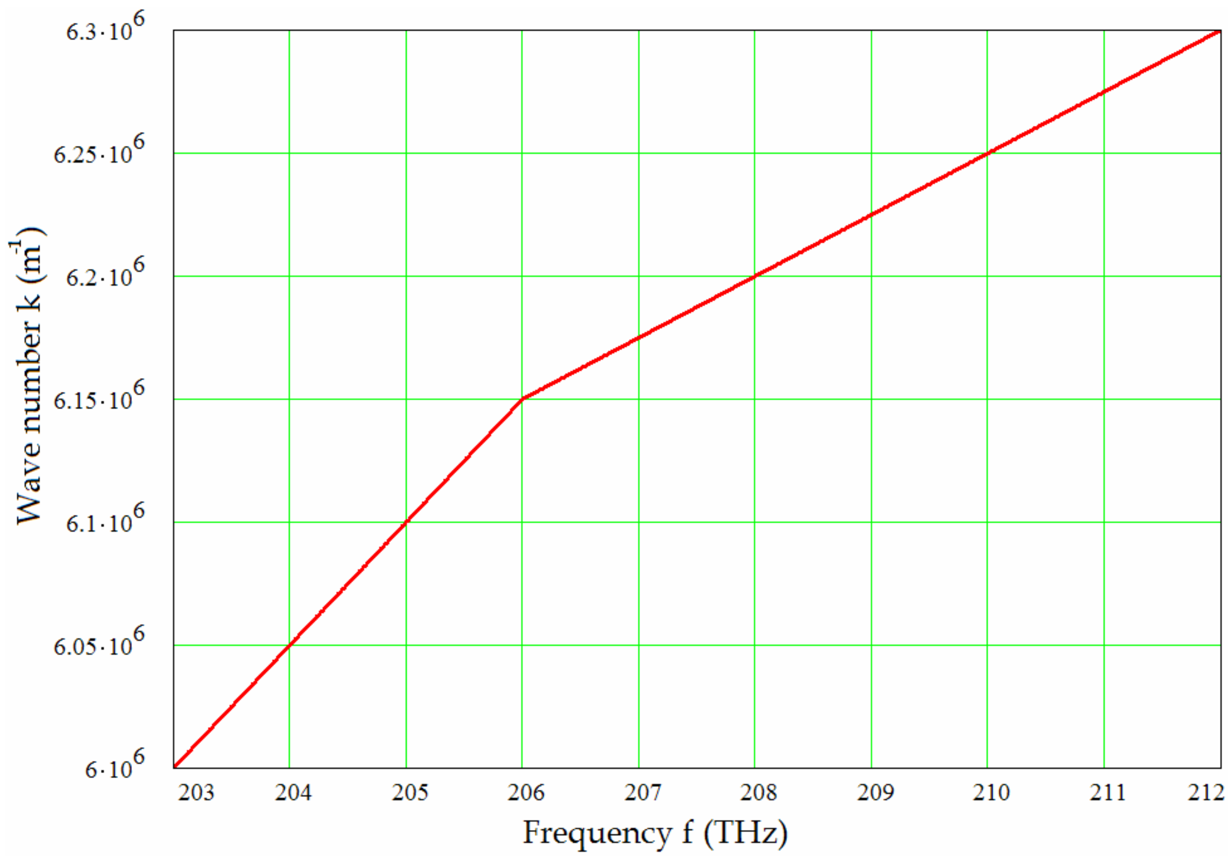
(a) (5 points) The dependence of ***wave number k*** on frequency is *linear* as shown below.



- Which pulse arrives at the output first? Give a brief justification.

- Which pulse is broadened by fiber dispersion more? Give a brief justification.

(b) (5 points) The dependence of ***wave number k*** on frequency is composed of *linear segments* as shown below.

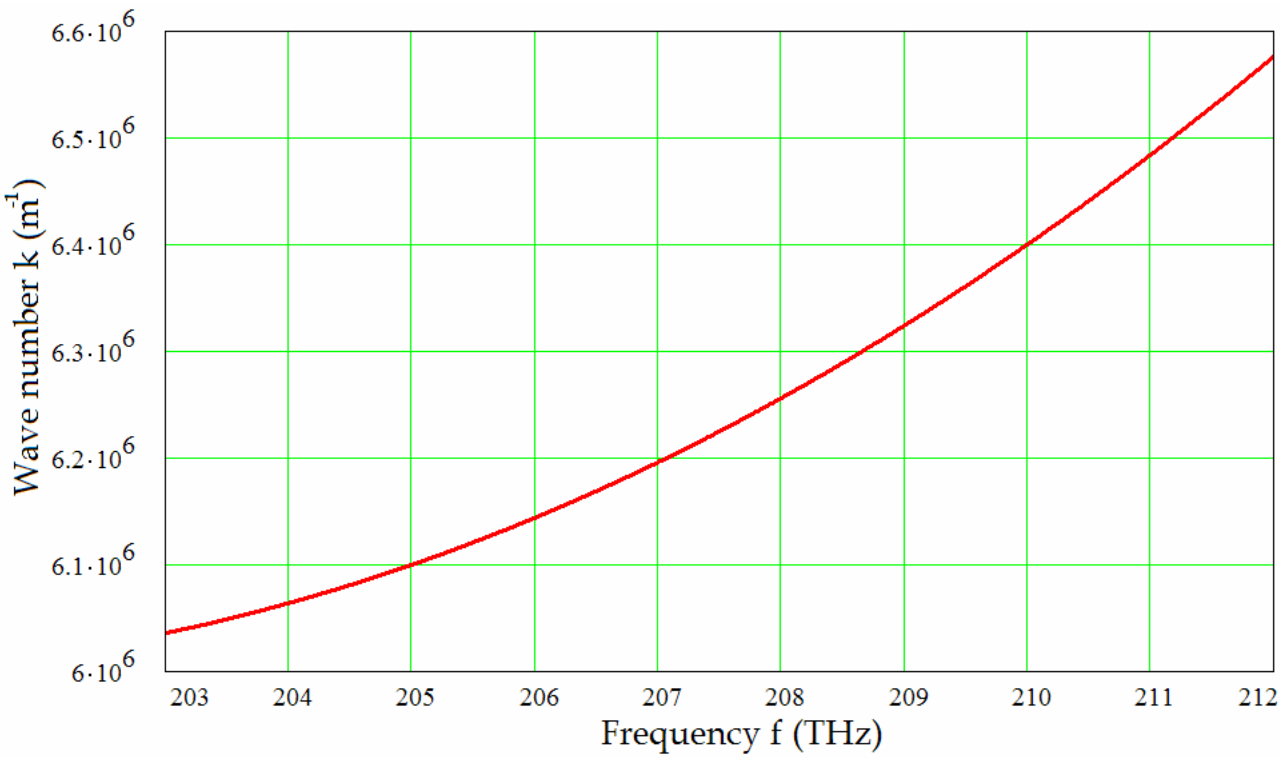


- Which pulse arrives at the output first? Give a brief justification.

- Calculate an *approximate* value for the time delay between the two pulses at the output of the fiber given the fiber length is 25 km.

- Which pulse is broadened by fiber dispersion more? Give a brief justification.

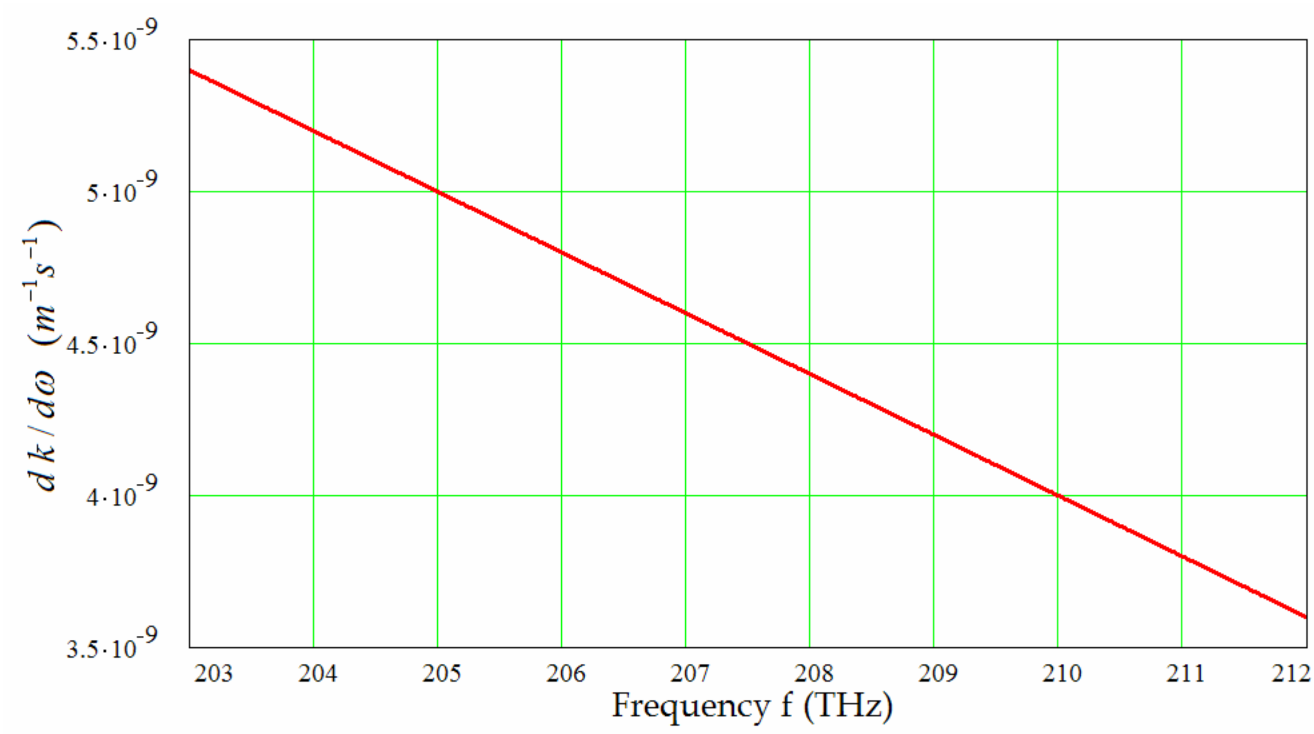
(c) (5 points) The dependence of ***wave number k*** on frequency is *parabolic* as shown below.



- The pulse at 205 THz travels through the fiber in $3\cdot10^{-4}s$. How long does it a*pproximately* take the pulse at 210 THz to travel through this fiber?

- Which pulse is more broadened by fiber dispersion? Give a brief justification.

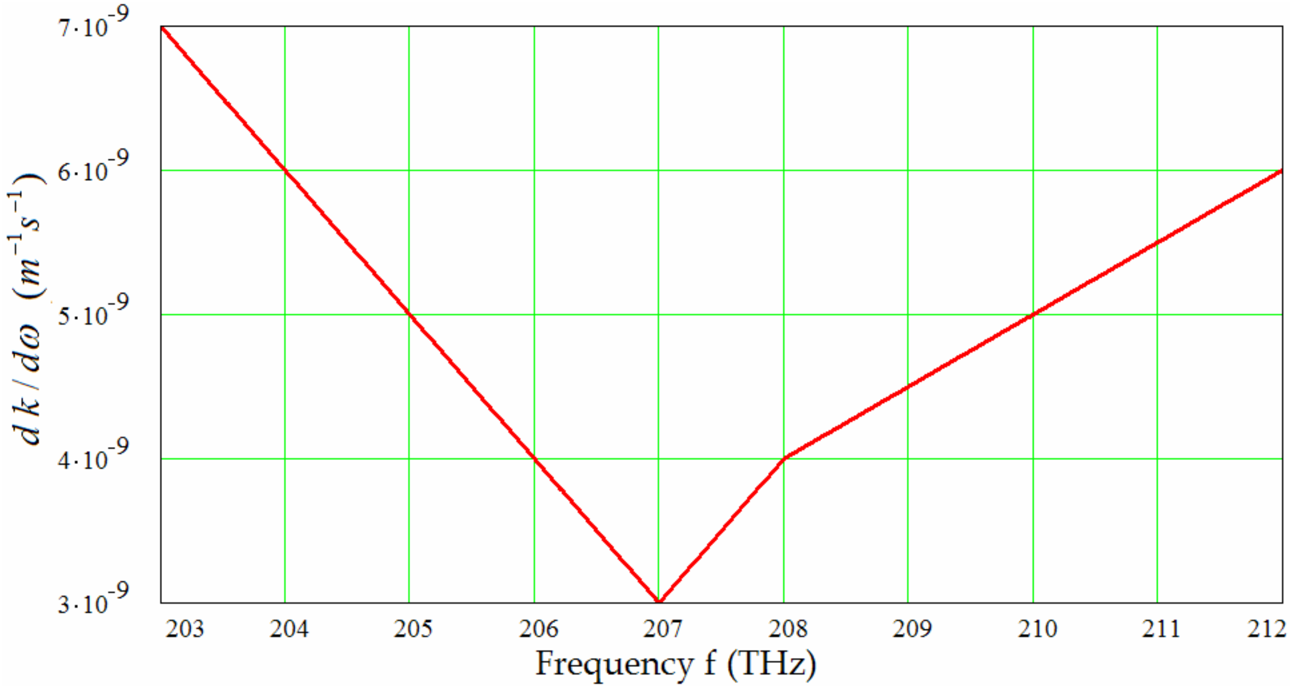
(d) (5 points) The plot below shows the ***first derivative*** of the wave number with respect to the *angular frequency* ω, i.e. $dk/d\omega$ versus frequency f. The dependence is linear.



- Which pulse arrives at the output first? Give a brief justification.

- The FWHM of the pulse at $205\,THz$ is increased by $\sqrt{2}$ after propagation through the fiber. By which factor is the FWHM of the pulse at $210\,THz$ increased?

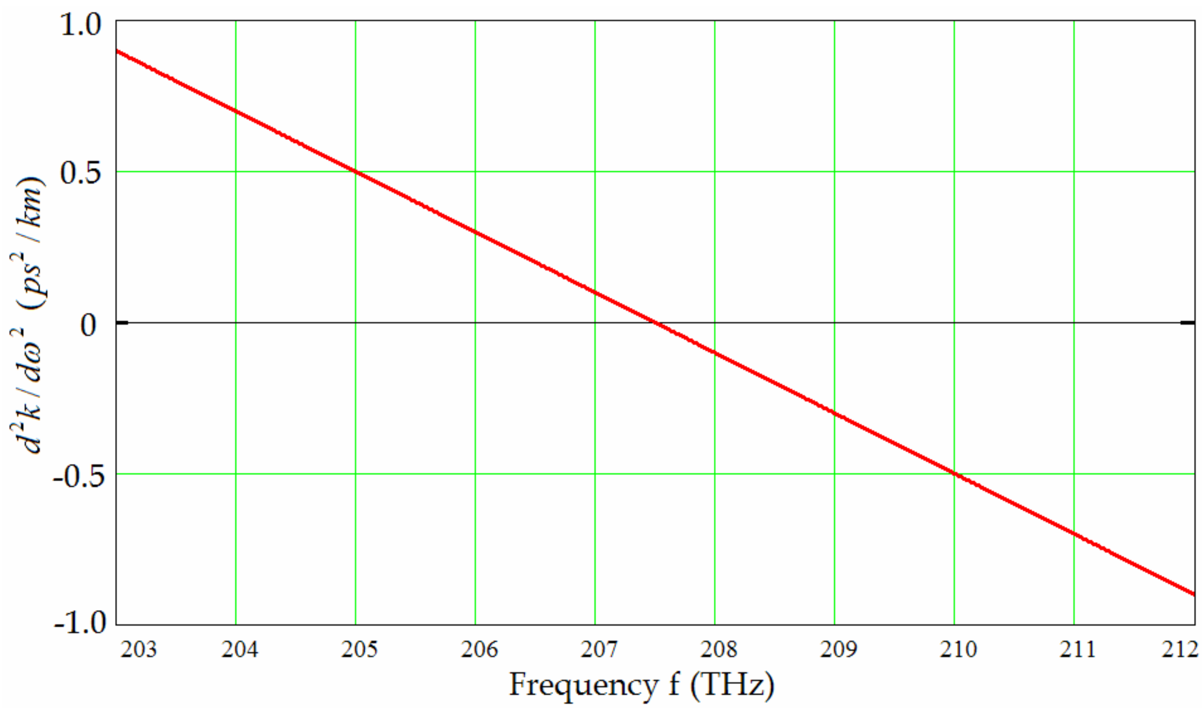
(e) (5 points) The plot below shows the ***first derivative*** of the wave number with respect to the *angular frequency* ω, i.e. $dk/d\omega$ versus frequency f. All curve segments are straight lines.



- Which pulse arrives at the output first? Give a brief justification.

- The FWHM of a Gaussian pulse at $205\,THz$ has increased by $\sqrt{2}$ after propagation through the fiber. By which factor is the FWHM of the pulse at $210\,THz$ has increased?

(f) **(<u>grad</u>)** (5 points) The plot below shows the *group velocity dispersion*, i.e. the ***<u>second derivative</u>*** $d^2k/d\omega^2$ of wave number with respect to the *angular frequency* ω, versus frequency f. The dependence is linear.

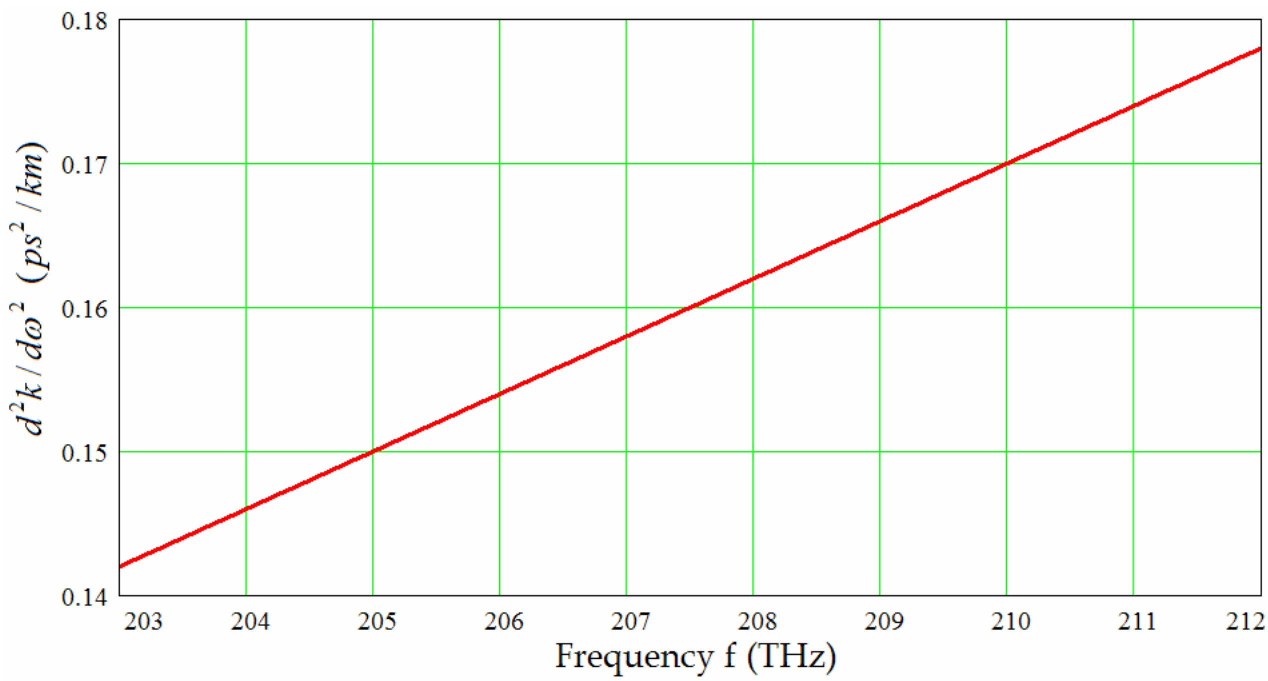


- Which pulse arrives at the output first? Give a brief justification.

- Which pulse is more broadened by fiber dispersion? Give a brief justification.

- How are the output pulses different?

(g) **(grad)** (5 points) The plot below shows the *dispersion*, i.e. the ***second derivative*** $d^2k/d\omega^2$ of the wave number with respect to the *angular frequency* ω, versus frequency f. The dependence is linear. The units of dispersion are ps^2/km.



- Group velocity of the pulse at $205\,THz$ is $2\cdot10^8\ m/s$. What is the *approximate* group velocity of the pulse at $210\,THz$?

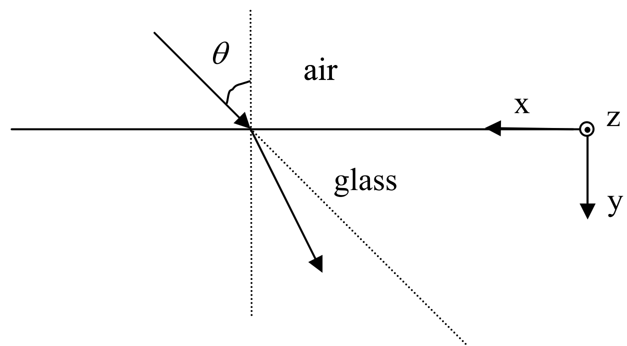
- Which pulse is more broadened by the fiber dispersion? Give a brief justification.

Problem 3: Reflection of EM-waves at Interfaces

An electric field with strength of $\sqrt{2}$ V/m is incident upon an air glass (n=1.5) interface at an angle of incidence $\theta = 30°$. The incident electric field can be described by:

$$\vec{E} = \left(\frac{\sqrt{3}}{2}\vec{e}_x + \frac{1}{2}\vec{e}_y + \vec{e}_z \right) \cdot \cos(\omega t - \vec{k} \cdot \vec{r}) \text{ , where } \vec{k} = -\frac{1}{2}\vec{e}_x + \frac{\sqrt{3}}{2}\vec{e}_y$$

The x, y and z directions are depicted in the figure.



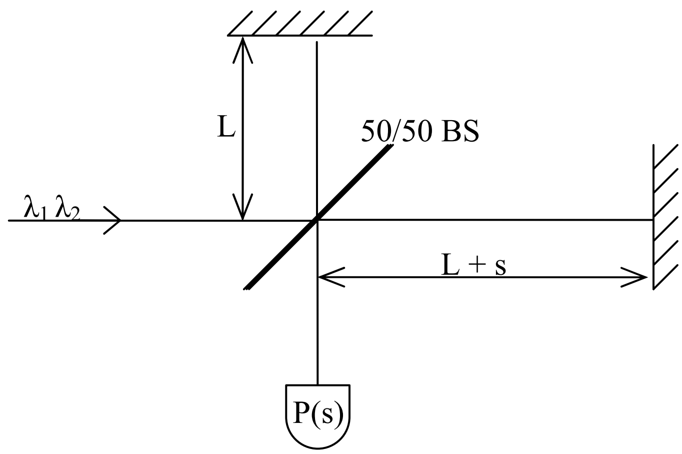
a) (5 points) What fraction of the input power is in the TE component, and what fraction is in the TM component of the input wave?

b) (10 points) What fraction of the incident wave power is transmitted?

c) (5 points) What angle of incidence would you choose so that the wave reflected from the surface is polarized along the z-axis?

Problem 4: Fourier-Spectrometer

A Fourier-Spectrometer is composed of a Michelson Interferometer using a 50/50 beam splitter and a detector P that measures the output power of the interferometer, see figure below. One arm of the interferometer is moveable by a distance, s. The light at the input of the interferometer is composed of two monochromatic plane waves at wavelengths λ_1 and λ_2 with power $P_1 = P_2 = P_o$, respectively. The optical phase between the two waves is assumed to be random and, therefore, the interference effects between the waves λ_1 and λ_2 can be neglected. For all parts of this question, assume the paths of the system to be lossless.



Fourier-Spectrometer

(a) (10 points) Using the scatting matrix for a 50/50 beam splitter, see equation sheet at the end of the booklet, derive an expression for the detected signal power at the output of the Michelson Interferometer if only one wave (λ_1) is present. Additional work space is provided on the following page.

(a) (continued from previous page)

(b) (5 points) Sketch the output power you found in part (a) as a function of the delay path, s. Mark significant numbers on both axis.

(c) (10 points) Now consider the case where both waves (λ_1 and λ_2) are present. Derive an expression for the detected output power that results from having both of these waves present. How is your answer related to your answer from part (a)? *Remember: The optical phase between the two waves is assumed to be random.*

(d) **<u>(grad)</u>** (5 points) The output power of the interferometer is measured over the interval $0 < s < s_{max}$. Compute the Fourier transform of the output power with respect to s and sketch it. Give important numbers on the spatial frequency axis and ordinate.

(e) **<u>(grad)</u>** (5 points) How large must the scanning range, s_{max}, of the interferometer be to resolve two lines at wavelength $\lambda_1 = 1\mu m$ and $\lambda_2 = 1\mu m + 1pm$? ($1pm = 10^{-12}$ m)

Quiz 1 Equation Sheet

Maxwell's Equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$\nabla \cdot \vec{D} = \rho \qquad \nabla \cdot \vec{B} = 0$$

Material Equations

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon \vec{E} \qquad \vec{P} = \varepsilon_0 \chi_e \vec{E} \qquad \varepsilon = \varepsilon_0 (1 + \chi_e)$$

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M} = \mu \vec{H} \qquad \vec{M} = \chi_m \vec{H} \qquad \mu = \mu_0 (1 + \chi_m)$$

Index of Refraction

$$n^2 = 1 + \chi\,, \qquad \text{for } \chi << 1: \quad n \approx 1 + \chi / 2$$

Poynting Vector

$$\vec{S} = \vec{E} \times \vec{H} \qquad \vec{T} = \frac{1}{2} \underline{\vec{E}} \times \underline{\vec{H}}^*$$

Energy density

$$w_e = \frac{1}{2} \varepsilon \vec{E}^2 \qquad w_m = \frac{1}{2} \mu \vec{H}^2 \qquad w = w_e + w_m$$

Snell's Law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Brewster's Angle

$$\tan \theta_B = \frac{n_2}{n_1}$$

Reflectivity

$$r^{TE} = \frac{Z_2^{TE} - Z_1^{TE}}{Z_1^{TE} + Z_2^{TE}} \qquad r^{TM} = \frac{Z_1^{TM} - Z_2^{TM}}{Z_1^{TM} + Z_2^{TM}}$$

$$Z_{1/2}^{TE} = \sqrt{\frac{\mu_{1/2}}{\varepsilon_{1/2}}} \frac{1}{\cos \theta_{1/2}} \qquad Z_{1/2}^{TM} = \sqrt{\frac{\mu_{1/2}}{\varepsilon_{1/2}}} \cos \theta_{1/2}$$

$$r^{TE} = \frac{n_1 \cos \theta_1 - n_2 \cos \theta_2}{n_1 \cos \theta_1 + n_2 \cos \theta_2} \qquad r^{TM} = \frac{\dfrac{n_2}{\cos \theta_2} - \dfrac{n_1}{\cos \theta_1}}{\dfrac{n_2}{\cos \theta_2} + \dfrac{n_1}{\cos \theta_1}}$$

Transmitivity

$$t^{TE} = \frac{2 Z_2^{TE}}{Z_1^{TE} + Z_2^{TE}} \qquad t^{TM} = \frac{2 Z_1^{TM}}{Z_1^{TM} + Z_2^{TM}}$$

$$t^{TE} = \frac{2 n_1 \cos(\theta_1)}{n_1 \cos(\theta_1) + n_2 \cos(\theta_2)} \qquad t^{TM} = \frac{2 \dfrac{n_2}{\cos(\theta_2)}}{\dfrac{n_2}{\cos(\theta_2)} + \dfrac{n_1}{\cos(\theta_1)}}$$

Power Refl. Coef.

$$R^{TE} = \left| r^{TE} \right|^2 \qquad R^{TM} = \left| r^{TM} \right|^2$$

Power Transm. Coef.

$$T^{TE} = \left| t^{TE} \right|^2 \frac{Z_1^{TE}}{Z_2^{TE}} = \frac{4 Z_1^{TE} Z_2^{TE}}{\left| Z_1^{TE} + Z_2^{TE} \right|^2} \qquad T^{TM} = \left| t^{TM} \right|^2 \frac{Z_2^{TM}}{Z_1^{TM}} = \frac{4 Z_1^{TM} Z_2^{TM}}{\left| Z_1^{TM} + Z_2^{TM} \right|^2}$$

(continued on the next page)

Pulse Dispersion $\quad \dfrac{\partial A(z,t')}{\partial z} = j\dfrac{k''}{2}\dfrac{\partial^2 A(z,t')}{\partial t'^2}$

Gaussian Pulse $\quad \tau(L) = \tau\sqrt{1+\left(\dfrac{k''L}{\tau^2}\right)^2} \qquad \tau_{FWHM} = 2\sqrt{\ln 2}\,\tau$

Fabry Perot $\quad \left|S_{21}\right|^2 = \dfrac{(1-R)^2}{(1-R)^2 + 4R\sin^2(\phi/2)}$ where $\phi = 2kL$, $k = \dfrac{2\pi f}{c_0}n$

Beam Splitter S-matrix $S = \begin{pmatrix} r & jt \\ jt & r \end{pmatrix}$, with $r^2 + t^2 = 1$

Constants

$\varepsilon_o = 8.85 \times 10^{-12}$ C^2/Nm^2	permittivity of free space
$\mu_o = 4\pi \times 10^{-7}$ N/A^2	permeability of free space
m = 9.11×10^{-31} kg	mass of an electron
e = 1.60×10^{-19} C	charge of an electron