$\boxed{\text{Today:}}$ Ⓘ Recap Maxwell's Eqns L9 10/6

Ⓘ ElectroQuasiStatic (EQS) subset:
when applicable (always for us)

Ⓘ Define Electrical Potential $\Phi(r,t)$ (volts)

Ⓘ Constitutive Law for $\underline{J}\left(\frac{A}{m^2}\right) \triangleq \sum_i z_i F N_i$

$\underline{J} = \text{"}\sigma\underline{E}\text{"} + (\quad)\nabla c + (\quad) c\underline{v}_{fluid}$

Ⓘ Maxwell Eqns:

① $\nabla \cdot \varepsilon\underline{E} = \rho_e(\underline{r}, t)$ Gauss

② $\nabla \times \underline{E} = -\frac{\partial B}{\partial t} = -\frac{\partial}{\partial t}\mu\underline{H}$ Faraday.

③ $\nabla \times \underline{H} = \underline{J} + \frac{\partial}{\partial t}\underline{D} = \underline{J}(r,t) + \frac{\partial}{\partial t}\varepsilon\underline{E}(r,t)$

④ $\nabla \cdot \mu\underline{H} = 0$

⑤ $\nabla \cdot \underline{J} = -\frac{\partial}{\partial t}\rho_e$

⑥ $F_e\ \cancel{f^e} = \rho_e\ \cancel{q}(\underline{E} + \underline{v} \times \mu\underline{H})$ free charge
force density

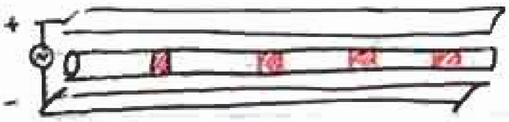
⑦ $f = m\frac{d\underline{v}}{dt}$

- ①–⑦ constituted complete description of fields $(\underline{E}, \underline{H})$ given sources $(\rho_e, \underline{J})$
- written for linear, isotropic, uniform medium where properties ε, μ, σ

$\underline{D} = \varepsilon\underline{E}$ "displacement current density"

$\underline{B} = \mu\underline{H}$ "mag. flux density"

$\underline{J} = \sigma\underline{E}$ Ohm's Law

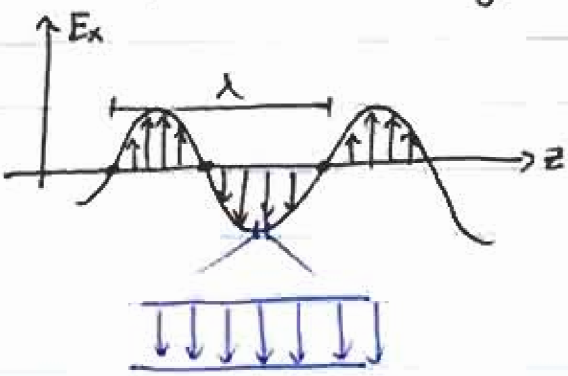


$f = 200\text{ MHz}$

$\lambda = 1.5\text{ m}$. for $L^{char} \ll \lambda$

E looks like uniform "static" field

intensity $\propto |E|^2$ "Standing wave"



cell
$10\,\mu m - L$
$L^{char} \sim 10\,\mu m$

For cases: $\underline{H}$ not important, or f low:

<u>EQS:</u> ① $\nabla \cdot \varepsilon E = \rho_e$

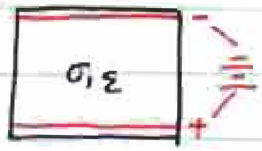
*②′ $\nabla \times E \simeq 0$ "decoupled" $\bar{H}$ from $\bar{E}$; Define voltage only in EQS.

③ $\nabla \times H = J + \frac{\partial}{\partial t} \varepsilon E$

④ $\nabla \cdot \mu H = 0$

⑤ $\nabla \cdot \underline{J} = -\frac{\partial \rho_e}{\partial t}$

Where does this go: <u>Ex ①:</u>



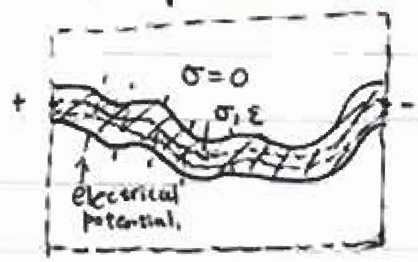
protein (−)
↓

<u>air</u> $\sigma = 0$
$\varepsilon = \varepsilon_0$

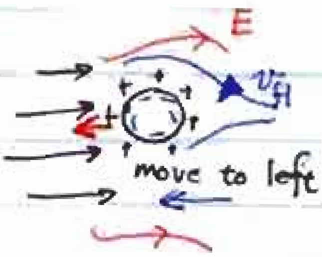
Find $\underline{E}$, $\underline{J}$, given B.C.'s @ 4 sides.

② Heart "Dipole"

③



microfluidics chip



(cell) electrophoresis

How good are we after throwing out $\frac{\partial H}{\partial t}$ ⇒ EQS

<u>Suppose</u>: $\rho_e = 0$, $J = 0$, find error in $\frac{\partial H}{\partial t}$

• Start with ① + ②′ ⇒ find E_0

• In ③: $\nabla \to \frac{1}{L^{char}}$; $\frac{\partial}{\partial t} \to \omega = 2\pi f$; ③ $\frac{H_1}{L} \Leftarrow \omega \varepsilon E_0$

• From ②′ $\left(\frac{1}{L^{char}}\right) E_{error} = \omega \mu H_1 = \omega^2 \mu \varepsilon E_0 L$ $\quad f = \left(\frac{3 \times 10^8 \, m/s}{10^{-2} \, m}\right) = 30\,GHz!$

$$\frac{|Error|}{E_0} = \frac{\omega^2 \mu \varepsilon L^2}{c^2} = \frac{L^2}{\lambda^2}$$ to within $(2\pi)^2$ for $L^2 \ll \lambda^2$; $(10\,\mu m) \ll 1\,cm$ all

$\mu \varepsilon = \frac{1}{c^2}$