12/1/04

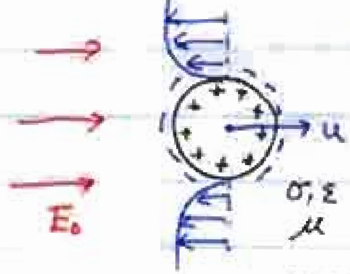
Today: "Levich Model" for Electrophoresis

- Implications for MEMS; NEMs; and molecular Transport

$$T_{ij}^{visc} = 2\mu \mathring{\varepsilon}_{ij} + \left(\lambda + \tfrac{1}{3}\mu\right)\cancel{\mathring{\varepsilon}_{kk}}^{\,0} \quad \text{(incompressible fluid)}$$

$$\sigma_{ij}^{stress} = -p\,\delta_{ij} + T_{ij}^{visc}; \quad \mathring{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$$

"Free Electrophoresis"

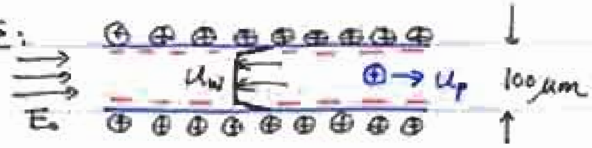


$$\left(\frac{u}{E_0}\right) = \frac{\left(\dfrac{\varepsilon\zeta}{\mu}\right)^{\checkmark\,\text{(today)}}}{1 + \left[\left(\dfrac{\varepsilon}{\sigma}\right)\left(\dfrac{\zeta\sigma_m}{R\mu}\right)\right] + \left[\begin{array}{l}\text{double layer}\\ \text{surface}\\ \text{conductivity}\end{array}\right] + \left\{\begin{array}{l}\text{particle}\\ \text{cond.}\end{array}\right\} + \ldots}$$

Charge relaxation / Convective time constant

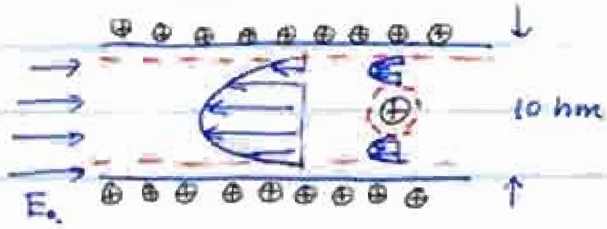
Levich: Analytical Model (1950s, 1960s)

Capillary electrophoresis:

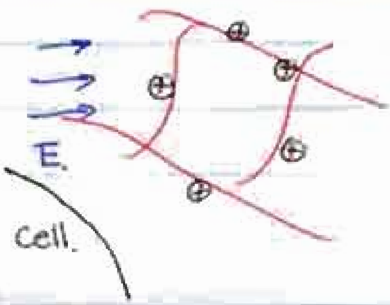


particle is too small compare to capillary.

NEMS. ⇒



Extra-intra-cellular networks



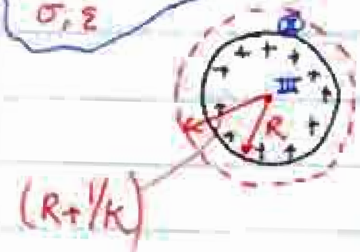
$$E \begin{cases} \text{applied field} \\ \quad -\text{OR}- \\ \text{self field} \Leftarrow \nabla c_i \end{cases}$$

- ⇒ electrokinetic effects
- hindred transport
- convective diffusion.

Levich Approach.

Assumes $R \gg 1/\kappa$

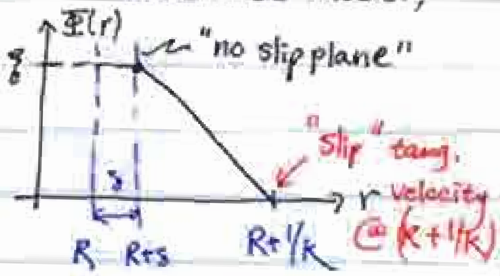
(I) fluid μ, σ, ε



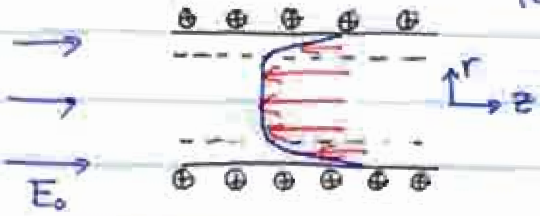
(III) First: $\sigma_{III} = 0$; ε (solid, insulating)

Eventually: σ_{III}, ε_{III}, even μ_{III} → "cell"

(II) Double Layer: "Helmholtz model"



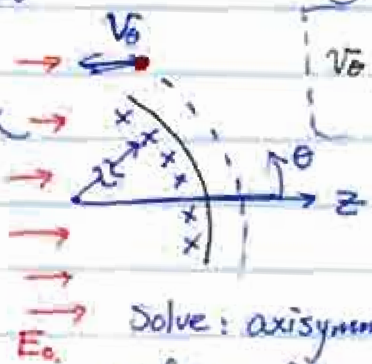
In Region (II) (Remember Smoluchowski) (was known)



$$V_z(r) = -\frac{\varepsilon}{\mu}\left(\zeta - \Phi(r)\right) E_0$$

$\boxed{\text{Helmholtz:}}$ $V_z(R + 1/\kappa) = -\left(\frac{\varepsilon\zeta}{\mu}\right) E_0$ channel.

$\boxed{\text{Levich:}}$ as long as $R \gg 1/\kappa$



(1)
$$v_\theta\left(R + \tfrac{1}{\kappa}, \theta\right) = -\left(\frac{\varepsilon\zeta}{\mu}\right) E_0 \left(R + \tfrac{1}{\kappa}, \theta\right)$$

? ?

1 equation, 2 unknowns

Eq. (1) became a B.C. on fluids problem.

Solve: axisymmetric fluid velocity profile self-consistent with soln of elec. field near/around particle

Method of Solution.

(1) $0 = -\nabla p + \mu \nabla^2 \underline{v} + \cancel{\rho_e \underline{E}} \to 0$

(2) $\nabla \cdot \underline{v} = 0$

(3) $\nabla \cdot \varepsilon \underline{E} = \cancel{\rho_e} \to 0$

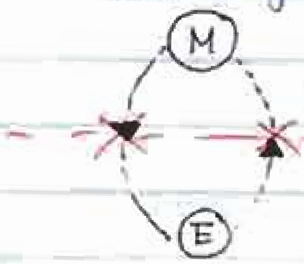
(4) $\underline{E} = -\nabla \Phi$

(3), (4) $\Big\}\ \nabla^2 \Phi = 0$ §1.3.

(5) $\nabla \cdot \underline{J} = -\left(\frac{\partial \rho_e}{\partial t}\right) \simeq 0$ ← charge relaxation

(6) $\underline{J} = \sigma \underline{E} + \cancel{\rho_e \underline{v}_{sl}} \to 0$

In region (I) (fluid): Helmholtz double layer assumption ⇒ $\rho_e = 0$



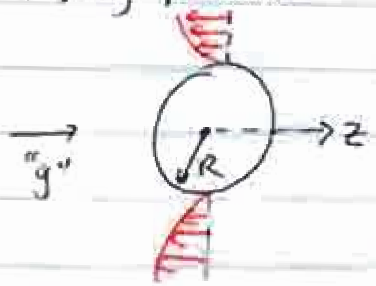
⇒ decouple mechanical & electrical system in the bulk.

$\boxed{A}$ Solve (1) & (2) subject to BC's:

(a) @ $r=R$, $v_r=0$ (no penetration)

(b) @ $r=(R+1/\kappa)\simeq R$, $v_\theta(R,\theta)=-\left(\frac{\varepsilon\zeta}{\mu}\right)E_0(R,\theta)$

Stoke's Drag Problem



$F_{tot\,z}=0$ particle falling w/ constant velocity

$=(mg-6\pi R\,U\mu)$

B.C's on $v(r,\theta)$

(1) $r\to\infty$, $v=-U\hat{i}_z$

(2) $R=0$, $v_r=0$, $v_\theta=0$ (no slip).

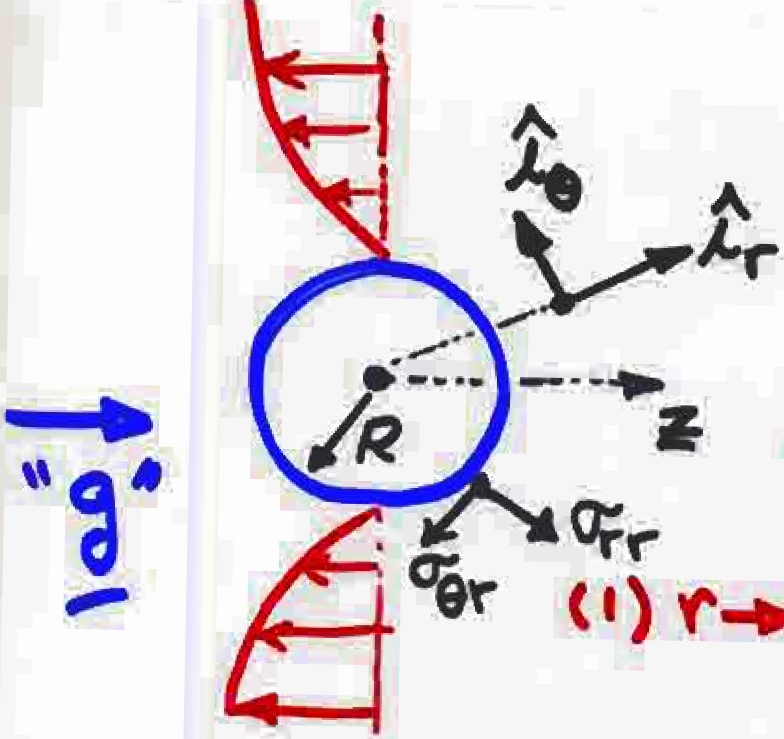
solve Stoke's Equations subject to above boundary conditions.

Electrophoresis

$F_{tot\,z}=0=\left(4\pi R\tilde{v}_\theta-6\pi R\,U\mu\right)$ where $v_\theta=\tilde{v}_\theta\sin\theta$??

② ↑ electrophoretic speed.

Now, solve the electrical subsystem.



"Uniform Flow past solid sphere" ("Stoke's Drag")

B.C.'s on $\underline{v}(r,\theta)$:

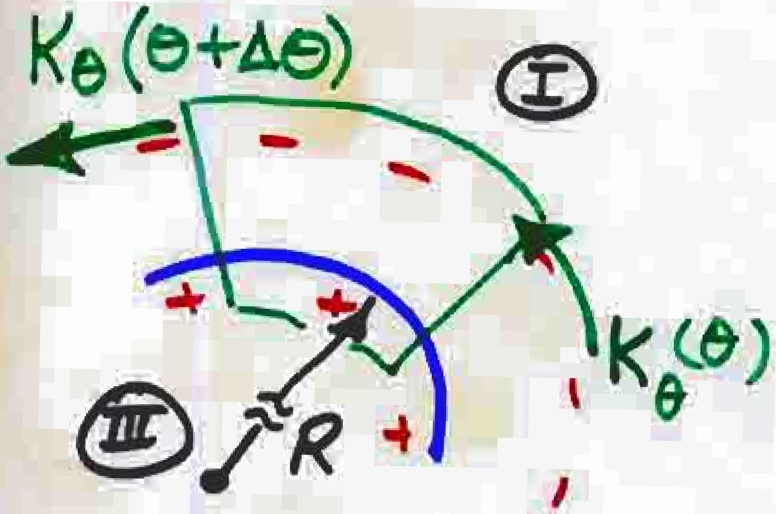
(1) $r \to \infty$, $\underline{v} = -U\hat{\lambda}_z$
$$= -U(\hat{\lambda}_r \cos\theta - \hat{\lambda}_\theta \sin\theta)$$

(2) $r=R$, $\underline{v_r = 0}$; $\underline{v_\theta = 0}$ ($\underline{\text{no slip}}$)

- From (1)+(2): Find $\underline{v_r}(r,\theta)$; $\underline{v_\theta}(r,\theta)$; $\underline{p}(r,\theta)$
- Find total stress at surface $\sigma_{rr}(R,\theta)$, $\sigma_{\theta r}(r,\theta)$
- $$F_z^{drag} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \left[\sigma_{rr}(R,\theta)\cos\theta - \sigma_{\theta r}(R,\theta)\sin\theta\right] R^2 \sin\theta \, d\theta \, d\phi$$

$$\boxed{F_z^{drag} = -6\pi R \mu U}$$

Elec. B.C. at $r = (R + 1/k) \simeq R$



$$\underline{n}\cdot(\underline{J}_I - \underline{J}_{III}) + \nabla_\Sigma \cdot \underline{K} = -\frac{\partial \sigma_s}{\partial t}$$

charge relax. 0

$\underline{K} = \hat{i}_\theta K_\theta$ surface current

$$= \hat{i}_\theta \left[(-\sigma_m)\, \underbrace{v_\theta(R+\tfrac{1}{k}, \theta)}_{\tilde{v}_\theta(R)\sin\theta} \right]$$

B.C.

$$\Rightarrow \sigma\left(-\frac{\partial \Phi}{\partial r}\right)\bigg|_{r=R} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left((-\sigma_m)\,\tilde{v}_\theta \sin^2\theta\right) = 0$$

where $\boxed{\Phi = -E_0 r\cos\theta + \frac{B\cos\theta}{r^2}}$

3 EQNS. in 3 UNKNOWNS: U, $\tilde{v}_\theta$, B

(E_θ or Φ)