$\boxed{\text{Today}}$: (I) Recap EQS ⇒ Steady Conduction 10/18

(Bound. Conditions ; Separation of Var. Sol'n of Laplace)

(II) $\boxed{\text{Non-Equilibrium X-port in Electrolyte Media}}$

"Charge Relaxation:" When can diffusion compete w/ elec. migration??

(III) Electrical double layers and Debye Length @ cell & molecular surfaces.

⇒ Charged Nano-continuum vs neutral Macro-continuum

Last Time:

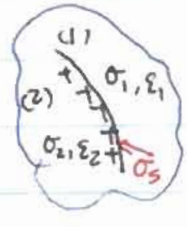
$$\left.\begin{array}{l} (1)\ +\underline{N}_i = -D_i \nabla c_i + \dfrac{z_i}{|z_i|}\mu_i c_i \underline{E} \\[2ex] (2)\ \dfrac{\partial c_i}{\partial t} = -\nabla\cdot\underline{N}_i + \cancel{R_i}^{\,0} \text{ today.} \end{array}\right\} \text{Boundary Cond. } (\checkmark)$$

(3) $\nabla\cdot\varepsilon E = \rho_e = \sum z_i F c_i$

(4) $\nabla\times E \simeq 0 \Rightarrow E = -\nabla\Phi$

(5) $\nabla\cdot\underline{J} = -\dfrac{\partial\rho_e}{\partial t}$

(6) $\underline{J} = \sigma\underline{E} + (\quad)\nabla c_i = \sum z_i F N_i$

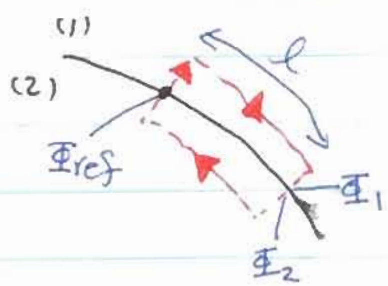


$\underline{n}\cdot(\varepsilon_1 E_1 - \varepsilon_2 E_2) = \sigma_s$

$n\times(E_1 - E_2) = 0$

$\underline{n}\cdot(\sigma_1 E_1 - \sigma_2 E_2) = -\dfrac{\partial}{\partial t}\sigma_s$

B.C. on (4): "Φ is continuous"



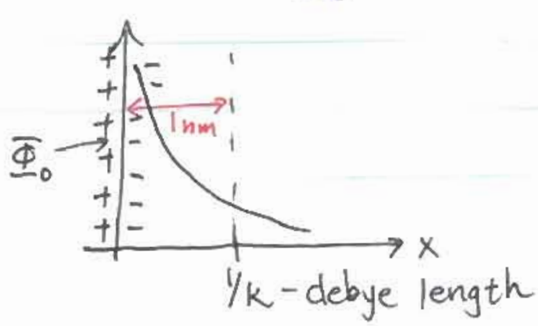
$$\lim_{h\to 0}\oint_C \underline{E}\cdot d\underline{s} = 0$$

$\ell E_1{}_{tan} - \ell E_2{}_{tan} = 0$

$E_1^{tan} = E_2^{tan}$

$\therefore \Phi_{ref} - \Phi_1 = \Phi_{ref} - \Phi_2$

$\boxed{\Phi_1 \equiv \Phi_2}$

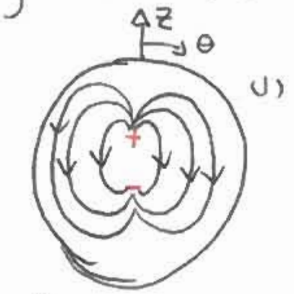


Note: surface current cause discontinuity in E, not in Φ

Steady Conduction: Boundary Value Problem

P4.2.

$\sigma = 0$
ε_0
(air)



$$\left(\frac{\partial}{\partial t} \equiv 0!\right)$$

$$\nabla \cdot J = 0$$

$$\sigma \nabla \cdot (-\nabla \Phi) = \boxed{0 = \nabla^2 \Phi}$$ Laplace's Equation.

Find Φ given B.C.

$$E = -\nabla \Phi$$

$$\boxed{\underline{n} \cdot \underbrace{\cancel{\sigma_1}}_{0} E_1 = n \cdot \sigma_2 E_2 = 0}$$ ✓✓

Spherical Coordinates

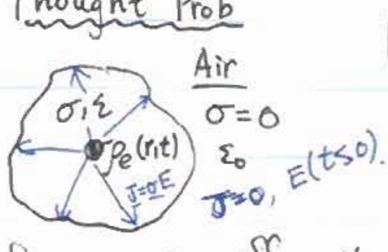
(axis symmetric indep. of ϕ) } Try $\Phi(r, \theta) = R(r)\, \Theta(\theta)$

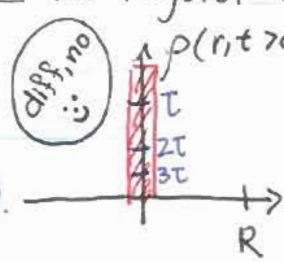
$$\boxed{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \Phi}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \Phi}{\partial \theta}\right) = 0} \equiv \nabla^2 \Phi$$

$$\Phi(r, \theta) = \underbrace{Ar\cos\theta}_{\text{uniform}} + \underbrace{\frac{B\cos\theta}{r^2}}_{\text{dipole}} + \underbrace{\cancel{\frac{C}{r}}}_{\text{pt. charge}} \to 0$$

II. <u>Non-Equil</u> <u>Transport</u> in Physiol Systems $\left(\frac{\partial}{\partial t} \neq 0 !!\right)$ "τ charge relax"

<u>Thought Prob</u>





for $t > 0$, turnoff ρ_e!!

$$(5)\quad \frac{\partial \rho_e}{\partial t} + \nabla \cdot J = 0$$

$$\frac{\partial \rho_e}{\partial t} + \frac{\sigma}{\varepsilon} \nabla \cdot \varepsilon E = 0$$

$$\boxed{\frac{\partial \rho_e}{\partial t} + \frac{\sigma}{\varepsilon} \rho_e = 0} \Rightarrow \rho = \rho_0 e^{-t/\tau}$$

$\tau_{ch,rel} = \frac{\varepsilon}{\sigma}$ In any position where ρ_e is $\underline{0}$ @ $t=0$, then ρ_e is 0 for all time!

$$\tau = \frac{\varepsilon}{\sigma} \sim \frac{10^{-9}\,(\text{farad}/m)}{1\,(\text{mho}/m)}$$ like "RC" time constant.

~ 1 ns.

$$\tau_{diff} \sim \frac{L^2}{(\,)D_i}$$

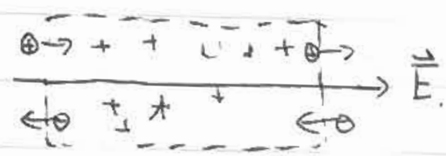
$\left(\sim 10^{-5} \frac{cm^2}{s}\right)$

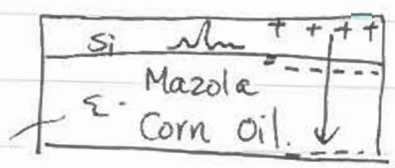
Na^+, Cl^-

L	1 cm	1 mm	1 μm	1 nm
τ_{diff}	10^5	10^3	1 ms	1 nanosec.
$\tau^{ch.}_{rel.}$	1 ns	1 ns	1 ns	1 ns

Can

~~In any~~





$$\tau_{mazola} = \frac{\varepsilon \sim 10^{-11}}{\sigma \times 10^{-11}} \sim 1 \text{ sec.}$$

$$\tau_{Si} \sim \frac{\varepsilon \sim 10^{-11}}{\sigma \sim 10^{-15}} \sim 100\,s.$$