Navier-stokes Equation

Navier–Stokes Equation for an Incompressible Fluid

Rectangular coordinates

***x* direction**

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right]$$

***y* direction**

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right]$$

Shear-Stress Tensor for an Incompressible Newtonian Fluid

Rectangular coordinates

$$\tau_{xx} = 2\mu\frac{\partial v_x}{\partial x}$$

$$\tau_{yx} = \tau_{yx} = \mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$$

$$\tau_{zx} = \tau_{xz} = \mu\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)$$

$$\tau_{yy} = 2\mu\frac{\partial v_y}{\partial y}$$

$$\tau_{zy} = \tau_{yz} = \mu\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)$$

$$\tau_{zz} = 2\mu\frac{\partial v_z}{\partial z}$$

Mass conservation:

$$\frac{d}{dt}\int_V \rho dV + \int_S \rho\vec{v}\cdot\hat{n}dS = 0$$

Peclet = Pe = VL/D

Reaction rate modulus = $R^* = \Phi^2 = RL^2/(C_oD)$

Convection-diffusion Eq:

$$\frac{\partial C}{\partial t} + (\vec{v}\cdot\vec{\nabla})C = D\nabla^2 C + R$$

Equations

Maxwell's Equations

$$\nabla \cdot (\varepsilon \vec{E}) = \rho_e \qquad \frac{1}{\mu}\nabla \times \vec{B} = \vec{J}_e + \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{\mathrm{E}} = -\frac{\partial \vec{\mathrm{B}}}{\partial \mathrm{t}} \qquad \nabla \cdot \vec{\mathrm{B}} = 0$$

Lorentz Force Law : $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Charge continuity: $\nabla \cdot J_e + \frac{\partial \rho_e}{\partial t} = 0$

Ohm's law: $\vec{J}_e = \sigma \vec{E}$ Conductivity of buffer σ ~1 S/m

Electrical permittivity: $\varepsilon_o = 8.85 \times 10^{-12} (F/m)$

$\varepsilon_r \cong 80$ for water (buffer)

μ_o : magnetic permeability ($4\pi\times10^{-7}$ H/m) in vacuum

Debye length $\lambda_D = \kappa^{-1} = \sqrt{\frac{\varepsilon RT}{2z^2F^2c_o}}$

Charge relaxation time : $t_r \sim \varepsilon/\sigma$

Faraday's constant : *F*= 96500 C/mol
Gas constant R= 8.314472 J · K^{-1} · mol^{-1}

Assume room temperature for all calculations

DEP forces $\vec{F}_{DEP} = 2\pi\varepsilon R^3 \left(\frac{\sigma_i - \sigma_o}{\sigma_i + 2\sigma_o} \right) \nabla \left| \vec{E} \right|^2$