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| Problem Set #9	Issued: 11/9/05
Fall Term 2005	Due: 11/23/05

Suggested Reading Assignment: Chapters 4 and 8.

Quiz 2 will be on Thursday, November 17 at 10-11 a.m. It will cover material through P. S. #8, with a focus on sinusoidal steady state and transient waves on transmission lines; parallel plate, rectangular, and dielectric waveguides. Quiz 2 is a closed book exam. **Quiz 2 Formula Sheets** (as attached to this problem set) will be provided. No individually prepared formula sheets will be allowed.

Problem 9.1

A coaxial cylinder inductor is dipped into a magnetizable fluid with permeability μ and mass density ρ_m. A current I flows down the center conductor and returns up the outer cylinder. The total height of the cylindrical inductor is $\ell + s$ where ℓ is the inductor height above the outside fluid level and s is the inductor length below the outside fluid level.
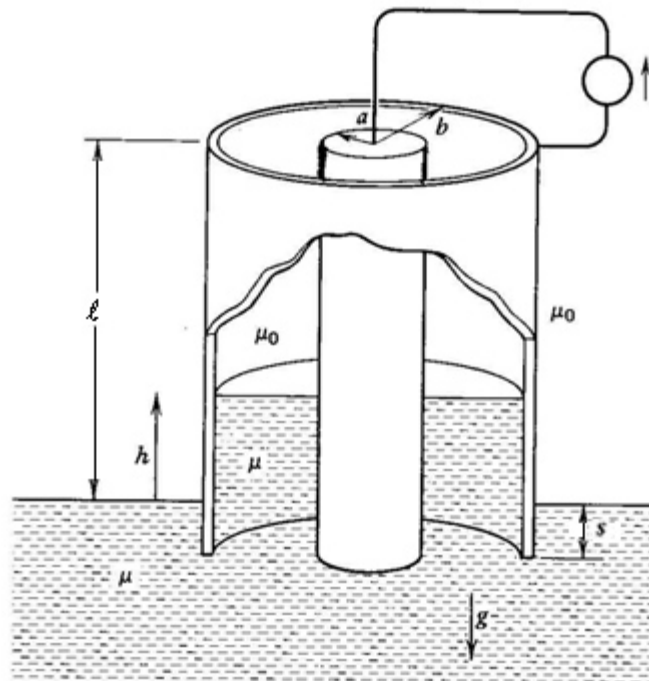

a) Calculate $\bar{B}$ and $\bar{H}$ in the annulus between cylinders in the free space and magnetic fluid regions. Neglect fringing field effects.

b) Calculate the self-inductance of the coaxial inductor as a function of the magnetic fluid height h above the outside fluid level.

c) What is the magnetic force on the magnetic fluid?

d) How high h does the fluid rise within the cylinder against the gravitational acceleration downwards of g ?

Adapted from Problem 6.37 in *Electromagnetic Field Theory: A Problem Solving Approach*, by Markus Zahn, 1987. Used with permission.

The figure below shows a diagrammatic cross section of a two-phase, salient-pole machine. The windings in an actual machine are distributed in many slots
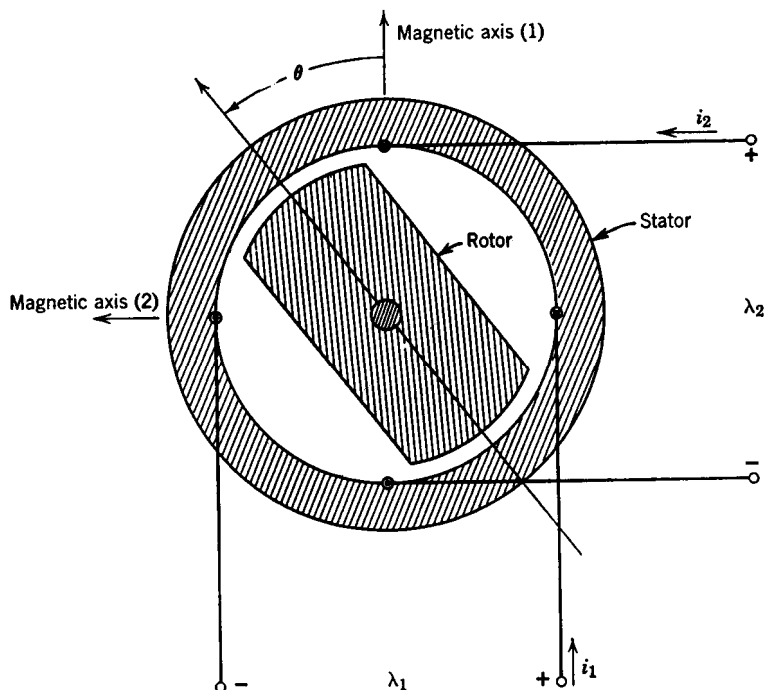
along the periphery of the stator, rather than as shown. The rotor is made of magnetically soft iron which has no permanent magnetism. The electrical terminal relations are given by

$$\lambda_1 = (L_0 + M \cos 2\theta)i_1 + M \sin 2\theta i_2, \quad \lambda_2 = M \sin 2\theta i_1 + (L_0 - M \cos 2\theta)i_2$$

(a) Determine the torque of electrical origin $T^e(i_1, i_2, \theta)$.

(b) Assume that the machine is excited by sources such that $i_1 = I \cos \omega_s t$, $i_2 = I \sin \omega_s t$, and the rotor has the constant angular velocity ω_m such that $\theta = \omega_m t + \gamma$. Evaluate the instantaneous torque T^e. Under what conditions is it constant?

(c) The rotor is subjected to a mechanical torque (acting on it in the $+\theta$-direction): $T = T_0 + T'(t)$, where T_0 is a constant. The time-varying part of the torque perturbs the steady rotation of (b) so that $\theta = \omega_m t + \gamma_0 + \gamma'(t)$. Assume that the rotor has a moment of inertia J but there is no damping. Find the possible equilibrium angles γ_0 between the rotor and the stator magnetic field. Then write a differential equation for $\gamma'(t)$, with $T'(t)$ as a driving function.

(d) Consider small perturbations of the rotation $\gamma'(t)$, so that the equation of motion found in (c) can be linearized. Find the response to an impulse of torque $T'(t) = I_0 u_0(t)$, assuming that before the impulse in torque the rotation velocity is constant.

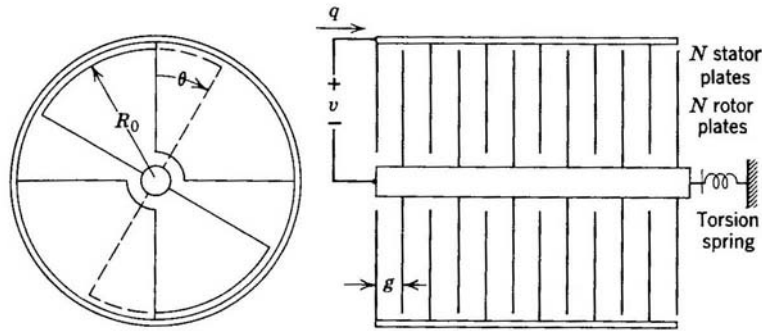(e) Which of the equilibrium phase angles γ_0 found in (c) is stable?

Fig. 3P.13a

Adapted from Problem 3.13 in *Electromechanical Dynamics*, by Herbert H. Woodson and James R. Melcher, 1968.

Electrostatic voltmeters are often constructed as shown in the figure above. N pairs of pie-shaped plates form the stator and rotor of a variable capacitor (the figure shows six pairs of rotor plates and six pairs of stator plates). The rotor plates are attached to a conducting shaft that is free to rotate through an angle θ. In the electrostatic voltmeter a pointer is attached to this shaft so that the deflection θ is indicated on a calibrated scale (not shown).

a) Determine $q(v,\theta)$, where q is the charge on the stator and v is the voltage applied between the rotor and the stator. The device is constructed so that fringing fields can be ignored and the area of the plates is large compared with the cross section of the shaft. In addition, it is operated in a region of θ in which the plates overlap but not completely.

b) Find the torque of electrical origin on the rotor.

c) The shaft is attached to a torsional spring with torque-angle relationship

$$T = -K\theta$$

What is the static angular deflection θ as a function of voltage?
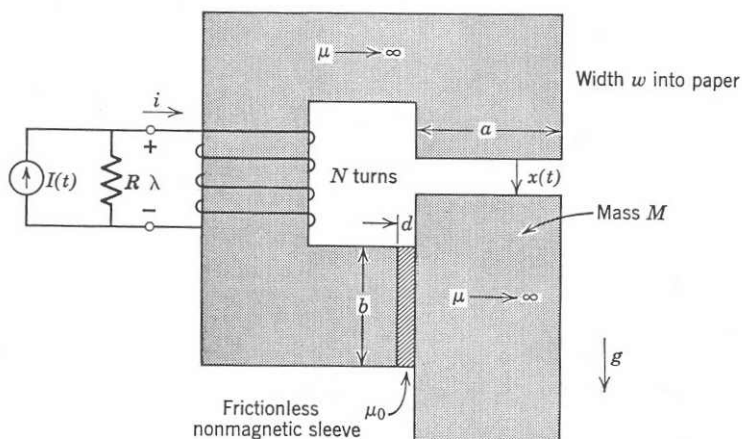
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Problem 9.4

Fig. 3P.4

A magnetic circuit, including a movable plunger of mass m, is shown in the above figure. The circuit is excited by an N-turn coil with current i and consists of a perfectly permeable yoke and plunger $\mu \to \infty$ with a variable air gap $x(t)$ and a fixed non-magnetic gap d. The system, with the cross section shown, has a width w into the paper.

a) Find the terminal relation for the flux $\lambda(i, x)$ linked by the electrical terminal pair. Ignore fringing in the nonmagnetic gaps. Note that the coil links the flux through the magnetic material N times.

b) Find the energy $W_m(\lambda, x)$ stored in the electromechanical coupling. This should be done by making use of part (a).

c) Use the energy function $W_m(\lambda, x)$ to compute the force of electrical origin f^e acting on the plunger.

d) Write an electrical (circuit) equation of motion involving λ and x as the only dependent variables and $I(t)$ as a driving function.

e) Write the mechanical equation of motion for the mass. This differential equation should have λ and x as the only dependent variables, hence taken with the result of (d) should constitute a mathematical formulation appropriate for analyzing the system dynamics.

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