Quiz 1 Solutions   **October 24, 2005**
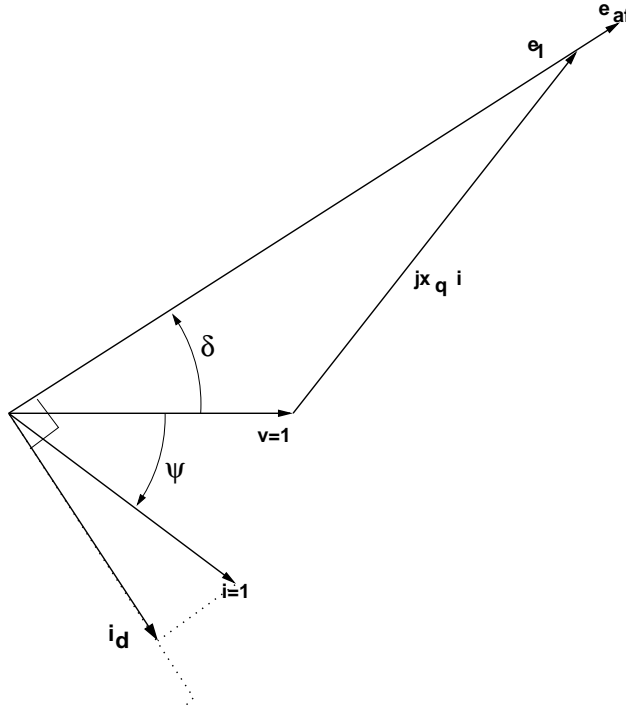

Figure 1: Phasor Diagram for Problem 1

Problem 1: 1. With the machine operating at rated terminal voltage and current ($|v_a| = 1.0$, $|i_a| = 1.0$) at power factor of 0.8, real power P=200 MW and reactive power Q=150 MVA, the phasor diagram is as shown (very nearly to the correct scale) in Figure 1. Remember that we can find a point on the q-axis:

$$\underline{e}_1 = \underline{v} + jx_q\underline{i}$$

The magnitude and angle of that point are:

$$
\begin{aligned}
|e_1|^2 &= (v + x_qi\sin\psi)^2 + (x_qi\cos\psi)^2 \\
\delta &= \tan^{-1}\frac{x_qi\cos\psi}{v + x_qi\sin\psi}
\end{aligned}
$$

Here, $v = 1$, $i = 1$, $x_q = 1.6$, $\cos\psi = 0.8$ and $\sin\psi = 0.6$. $|e_1 = 2.34$ and $\delta = 33.15°$. To get required field current we need

$$e_{af} = e_1 + (x_d - x_q)i_d$$

1

Direct axis armature current is
$$i_d = i \sin \psi + \delta$$
Since $\psi = \cos^{-1} 0.8 \approx 36.87°$, $i_d \approx \sin 70° \approx 0.94$. Then:
$$e_{af} = 2.34 + 0.4 \times 0.94 \approx 2.716$$

Required field current is thus 2,716 A.

2. Reactive power is,for a generator in per-unit:
$$q = \frac{ve_{af}}{x_d} \cos \delta + \frac{v^2}{2} \left(\frac{1}{x_q} - \frac{1}{x_d} \right) \cos 2\delta - \frac{v^2}{2} \left(\frac{1}{x_q} + \frac{1}{x_d} \right)$$

With the machine operating at zero real power, the angle δ is zero and this expression becomes:
$$q = \frac{v}{x_d} (e_af - v)$$

For the field current limit suggested above, this becomes:
$$q = \frac{1.716}{2} \approx 0.858$$

This is just about 214 MVAR and looks like about what the capability curve suggests. For $I_f = 2,600$ A, $e_a f = 2.6$ per-unit, so
$$q = \frac{1.6}{2} = 0.8$$

This is just 200 MVAR.

3. The stability limit is achieved when the torque-angle curve is flat. Noting power to be:
$$= \frac{ve_{af}}{x_d} \sin \delta + \frac{v^2}{2} \left(\frac{1}{x_q} - \frac{1}{x_d} \right) \sin 2\delta$$

the stability point is when
$$\frac{\partial p}{\partial \delta} = \frac{ve_{af}}{x_d} \cos \delta + v^2 \left(\frac{1}{x_q} - \frac{1}{x_d} \right) \cos 2\delta = 0$$

This means that
$$e_{af} = -v \left(\frac{x_d}{x_q} - 1 \right) \approx 0.25$$

And that implies that field current is about -250 A. Then reactive power at that limit is
$$q = -\frac{1.25}{2} \approx -0.625$$

This is just about -156.25 MVAR

Problem 2: 1. The field winding produces a radial flux density of

$$B_r = \mu_0 \frac{N_f I_f}{2g} \approx \frac{1.2566 \times 10^{-7} \times 7958}{.02} \approx 0.5T$$

And this is a square wave riding along with the rotor. To get voltage we can easily use the 'BLV rule', noting that if the two sides of a coil see flux density of opposite sign each of the two stator coils will produce a voltage that is:

$$2 \times N \times \ell \times R_r \times \Omega$$

With flux density of ±0.5 T, this means that each coil produces

$$V_c = \pm 209 \times \frac{1}{4} \times 20 \times \frac{1}{2}$$

or just about 523.5 volts. Note that when the rotor is at an angle between -30 and +30 degrees, neither coil is generating any voltage because both sides of both coils see the same flux. Note that at 2,000 RPM a cycle is 30'Ms, and it takes 2.5 mS (1/12 of 30 mS) to rotate 30 degrees. So we can now construct the waveform: starting at time equals zero and the angle of zero, voltage is zero until thirty degrees (time is 2.5 mS). At this point the voltage is negative (flux linked by the coils is going down). All the rest is symmetry.

2. This one is almost like the voltage induced in the first part, except the torque produced is, at peak:

$$T = 2(\text{coils}) \times 2(\text{sides}) \times 10(turns) \times 100(\text{amperes}) \times .5(\text{Tesla}) \times 1(\text{m}) \times .25(\text{m}) \approx 500\text{Nm}$$

This has the same characteristic: it is zero when the two coil sides see the same rotor flux density, negative in the first part and positive in the second.

3. For the last part, see that the voltage is just the complement of the first part. The only difference is when the voltages add and when they cancel.
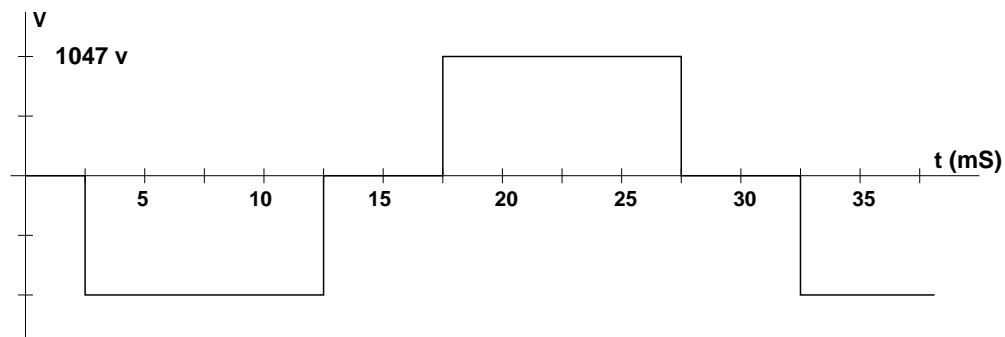
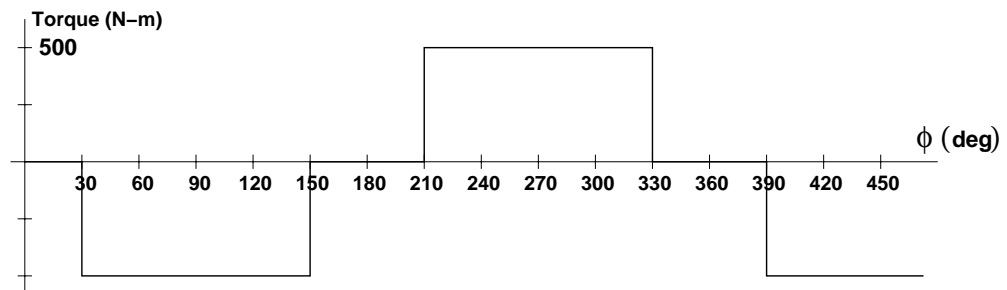

Figure 2: Induced Voltage: Series Coils, Same Sense

Torque (N–m)

500

φ (deg)

30 60 90 120 150 180 210 240 270 300 330 360 390 420 450
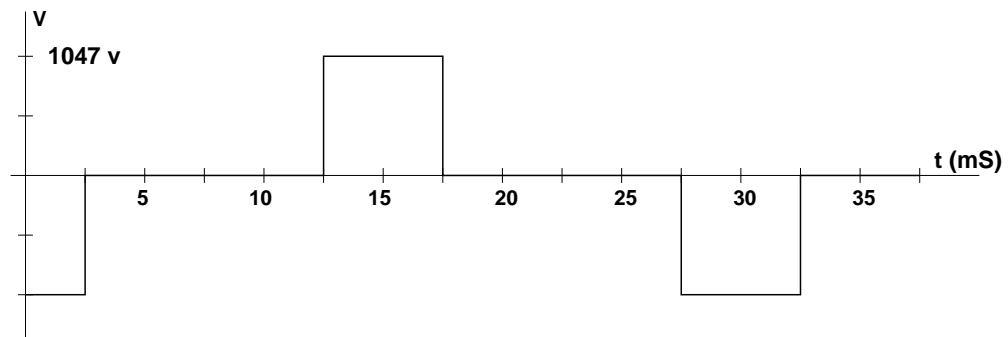
Figure 3: Torque

V

1047 v

t (mS)

5 10 15 20 25 30 35

Figure 4: Induced Voltage: Series Coils, Reverse Sense