Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.685 Electric Machines

Problem Set 10 Solutions November 25, 2005

Problem 1: Brushless Machine Note that we can write Magnetization as a Fourier Series:

$$M(\theta) = \sum_{n=1}^{\infty} M_n \sin np\theta$$

where the harmonic components are:

$$M_n = \frac{B_r}{\mu_0}\frac{4}{n\pi}\sin n\frac{\theta_{me}}{2}\sin n\frac{\pi}{2}$$

To get the remanent flux density B_r, note that the energy product is $W = \frac{B_r^2}{4\mu_0}$, or

$$B_r = \sqrt{4 \times 42 \times 10^6} = 12,961\text{gauss} = 1.2961\text{T}$$

Then the Fourier series for flux density at the rotor surface is

$$B_s = \sum_n B_n \sin p\theta$$

where

$$B_n = \mu_0 M_n k_{gn}$$

and the expression for the 'gap factor' is a straightforward extrapolation of what is in the notes:

$$k_{gn} = \frac{1}{1-\left(\frac{R_i}{R_s}\right)^{2np}}\left(\frac{np}{np+1}\left[\left(\frac{R}{R_s}\right)^{np+1} - \left(\frac{R_i}{R_s}\right)^{np+1}\right] + \frac{np}{np-1}\left(\frac{R_i}{R_s}\right)^{np+1}\left[1-\left(\frac{R_i}{R}\right)^{np-1}\right]\right)$$

Voltage is (to within a fixed sign), using VLB:

$$V_n = 2R\ell\Omega k_{bn}k_{pn}B_n$$

and the winding factors are the well known pitch and breadth factors. Details of the calculations are in the attached script. The results are shown in Figure 1, which shows the magnetization reconstructed from the Fourier Series (in this case, I have taken the series out to 35th space harmonic: your answer may have fewer or more terms). Figure 2 shows the magnetic flux density at the surface of the stator bore and Figure 3 shows the induced voltage.

The harmonic amplitudes of the first five interesting harmonics are:

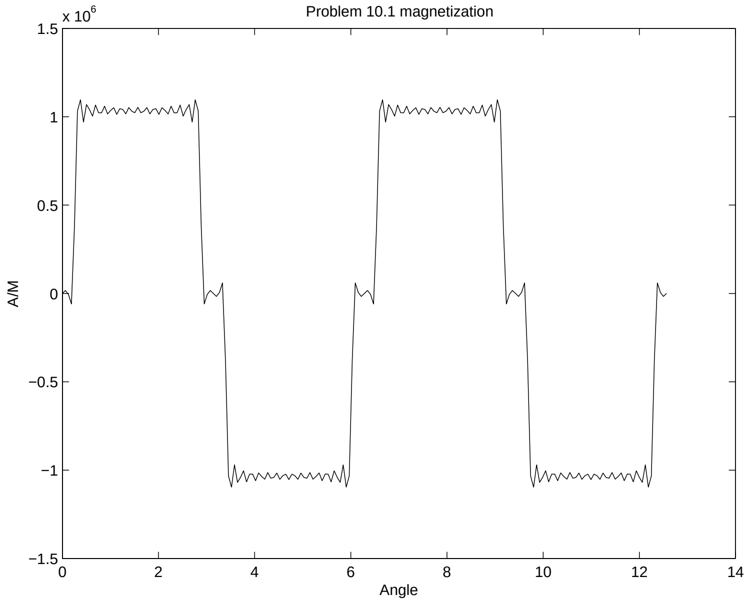

Figure 1: Magnetization reconstructed from Fourier Series

```
6.685  Problem  Set  10,  Problem  1
Harmonic  Number      1
Gap  Factor         =        0.6696      Magnetization   =     1.272e+06
Pitch  Factor       =        0.9659      Breadth  Factor  =       0.9598
Frequency          =         1257      Flux  Component  =        1.071
Voltage  Component  =         242.4
Harmonic  Number      3
Gap  Factor         =        0.6573      Magnetization   =     3.105e+05
Pitch  Factor       =        0.7071      Breadth  Factor  =       0.6667
Frequency          =         3770      Flux  Component  =       0.2564
Voltage  Component  =         29.53
Harmonic  Number      5
Gap  Factor         =        0.6341      Magnetization   =     6.818e+04
Pitch  Factor       =        0.2588      Breadth  Factor  =       0.2176
Frequency          =         6283      Flux  Component  =      0.05433
Voltage  Component  =        0.7474
Harmonic  Number      7
Gap  Factor         =        0.6026      Magnetization   =     -4.87e+04
Pitch  Factor       =       -0.2588      Breadth  Factor  =      -0.1774
Frequency          =         8796      Flux  Component  =     -0.03688
Voltage  Component  =       -0.4136
Harmonic  Number      9
Gap  Factor         =        0.5655      Magnetization   =    -1.035e+05
Pitch  Factor       =       -0.7071      Breadth  Factor  =      -0.3333
Frequency          =     1.131e+04      Flux  Component  =     -0.07354
```

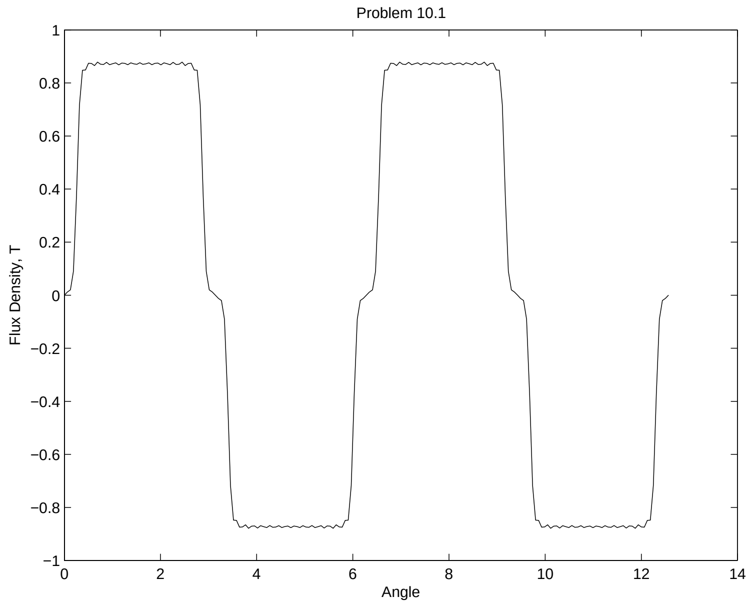

Figure 2: Flux Density at the Surface of the Stator

```
Voltage  Component  =        -4.234
```

Problem 2: More Brushless Machines To get the useful current capability of this machine we first need to estimate slot area. This is done by, first, computing tooth width:

$$w_t = \frac{2\pi\left(R + h_d\right)}{6pm} - w_{st}$$

where w_{st} is the 'slot top' width. Slot bottom width is then:

$$w_{sb} = \frac{2\pi\left(R + h_d + h_s\right)}{6pm} - w_t$$

Slot area is just

$$A_s = h_s \frac{w_{st} + w_{sb}}{2}$$

Total armature ampere-turns per phase is:

$$NI = J_a pm A_s$$

Internal flux per turn (fudamental) is:

$$\Phi = \frac{2R_s \ell B_{s1} k_{p1} k_{b1}}{p}$$

and since this is a round rotor machine, torque rating is:

$$T_e = \frac{3}{2} pNI\Phi$$

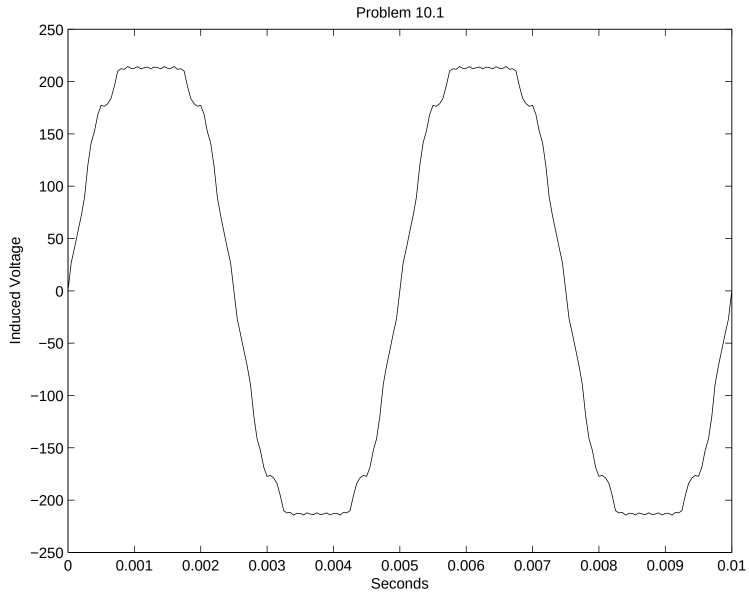

Figure 3: Induced Voltage

The two components of inductance are air gap, for which we use the fundamental:

$$L_{ag} = \frac{3}{2}\frac{4}{\pi}\frac{\mu_0 R\ell N_a^2 k_p^2 k_b^2}{p^2\left(g+h_m\right)}$$

and slot leakage:

$$L_{s\ell} = \mathcal{P}_{\text{slot}} 2\ell p N_c^2 \left(4N_{\text{self}} + N_{\text{mut}}\right)$$

where the numbmer of slots per pole per phase with both coil halves in the same slot is

$$N_{\text{self}} = m - N_{sp}$$

and the number of slots per pole per phase with coil halves in slots with other phases is:

$$N_{\text{mut}} = 2N_{sp}$$

and of course slot permeance per unit length is:

$$\mathcal{P}_{\text{slot}} = \mu_0\left(\frac{h_d}{w_d} + \frac{1}{3}\frac{h_s}{w_{st}}\right)$$

To find winding resistance we need, first, area for each wire. This is:

$$A_w = \frac{\lambda_a A_s}{2N_c}$$

where λ_s is the winding space factor. We also need the length of each wire, which is

$$\ell_w = 2N_a\left(\ell + \ell_e\right)$$

The length of the end winding is taken as half the circumference of a circle: $\ell_e = \pi C$ where the diameter of that circle is the distance along a cylinder at the midplane of the slot and over the coil throw:

$$C = \frac{5}{6}\frac{\pi}{p}\left(R_s + h_d + \frac{1}{2}h_s\right)$$

Then winding resistance is

$$R_a = \frac{\ell_w}{\sigma_a A_w}$$

To get short circuit current we take fundamental voltage which is just:

$$E_a = \omega N_a \Phi$$

and divide that by impedance of the stator:

$$Z_s = j\omega\left(L_{ag} + Ls\ell\right) + R_a$$

The short circuit current is shown in Figure 4

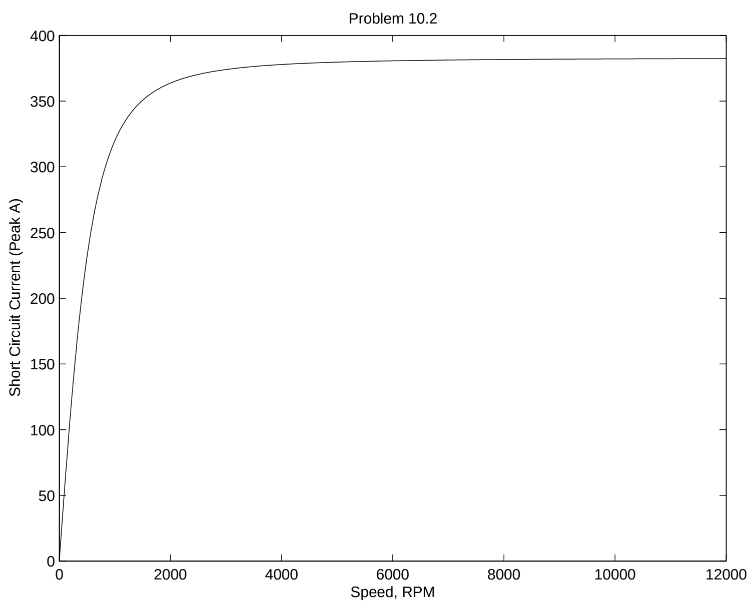

Figure 4: Short Circuit Current

The other elements of rating are contained in the Matlab screen output:

```
 Problem  10.2:  Operating  Details:
Internal  Voltage   =         248.5    Terminal  Voltage  =         261.4
Terminal  Current   =         89.22    Output  Power      =     3.326e+04
Reactance          =        0.6491    Resistance        =       0.07119
Efficiency         =        0.9751    Power  Factor      =        0.9751
Air-Gap  L          =     0.0003699    Slot  L      =     0.0001466
```

```
% Solution to 6.685 Problem Set 10, Problems 1 and 2
% voltage generated by a surface mount machine

% dimensions, etc
R = .06;                    % rotor radius
L = .18;                    % active length
g = .0015;                   % air-gap
hm = .0035;                  % magnet height
p = 2;                      % 4 poles
m = 3;                      % = 36/(4*3) Slots/pole/phase
Na = 18;                    % turns/phase
hd = .001;                  % slot depression height
wd = .001;                  % slot depression width
wst = .005;                 % slot top width
hs = .015;                  % slot height
thme =5*pi/6;               % 150 degrees, electrical
Br = 1.3;                   % moderately good stuff: 42 MG-Oe
alf = pi*5/6;               % this is a 5/6 pitch winding
gama = 2*pi/(3*p*m);          % slot electrical angle
Ri = R - hm;                % inner radius
Rs = R + g;                 % outer radius (for magnetic problem)
N = 6000;                   % RPM
omm = (2*pi/60)*N;          % electrical speed
u = omm*R;                  % surface speed
om = p*omm;
muzero = pi*4e-7;
n = 1:2:35;                 % interesting harmonics
np = p .* n;                % another interesting vector

% kg is the magnetic gap factor

kg1 = 1 ./ (1 - (Ri/Rs) .^ (2 .* np));
kg2 = (np ./ (np + 1)) .* ((R/Rs) .^ (np+1) - (Ri/Rs) .^ (np+1));
kg3 = (np ./ (np - 1)) .* ((Ri/Rs) .^ (np+1));
kg4 = (1 - (Ri/R) .^ (np-1));
kg = kg1 .* (kg2 + kg3 .* kg4);

Mn = (Br/muzero) .* ((4/pi) ./ n) .* sin(n .* thme/2) .* sin(n .* pi/2);  % fi
Bn = muzero .* Mn .* kg;                          % flux density

kp = sin(n .* pi/2) .* sin(n .* alf/2);         % pitch factor
kb = sin(n .* m*gama/2) ./ (m .* sin(n .* gama/2));     % breadth factor

V = 2*L*u*Na .* Bn .* kp .* kb;                  % voltage harmonics
```

```
fprintf('6.685 Problem Set 10, Problem 1\n')
     for i = 1:5
     fprintf('Harmonic Number %5.0f\n',n(i));
     fprintf('Gap Factor       = %12.4g   Magnetization = %12.4g\n', kg(i), Mn(i));
     fprintf('Pitch Factor     = %12.4g   Breadth Factor = %12.4g\n', kp(i), kb(i));
     fprintf('Frequency        = %12.4g   Flux Component = %12.4g\n', n(i)*om, Bn(i));
     fprintf('Voltage Component = %12.4g\n',V(i));
end

omt = 0:pi/50:4*pi;
t = omt ./ om;
v = zeros(size(omt));
B = zeros(size(omt));
M = zeros(size(omt));
for i = 1:length(n);,
  M = M+Mn(i) .* sin(n(i) .* omt);
  v = v+V(i) .* sin(n(i) .* omt);
  B = B+Bn(i) .* sin(n(i) .* omt);
end

  figure(1)
     plot(omt, M)
     title('Problem 10.1 magnetization')
     ylabel('A/M')
     xlabel('Angle')

  figure(2)
     plot(omt, B)
     title('Problem 10.1')
     ylabel('Flux Density, T')
     xlabel('Angle')

  figure(3)
     plot(t, v)
     title('Problem 10.1')
     ylabel('Induced Voltage')
     xlabel('Seconds')

% now machine rating:
Ja = sqrt(2)*2e6;                  % peak slot current density
% need to calculate a few details of slot geometry
wt = 2*pi*(Rs+hd)/(6*p*m) - wst;     % this is tooth width
wsb = 2*pi*(Rs+hd+hs)/(6*p*m) - wt;  % slot bottom width
As = hs*.5*(wst+wsb);              % this is area of each slot
NI = Ja*p*m*As;                    % this is ampere-turns/phase (peak as Ja is peak)
```

```
I = NI/Na;                         % and this is now peak current
Phi = 2*Rs*L*Bn(1)*kp(1)*kb(1)/p;     % this is internal flux per turn (peak)
Torque = 1.5*p*NI*Phi;               % that makes this torque rating
Lag = (6/pi)*(muzero*Rs*L*Na^2*kp(1)^2*kb(1)^2)/(p^2*(g+hm));  % air-gap inductance
Pslot = muzero*(hd/wd+hs/(3*wst));   % slot permeance
% what is below is non generalizable
N_c = 2;                           % slots with both coils the same
N_s = 1;                           % slots short pitched
Nc = 3;                            % turns in each coil
Lslot = Pslot*2*L*p*Nc^2*(4*N_c+N_s);  % this is slot leakage impedance
% now to get resistance
lama = .25;                        % winding space factor
Aw = As*lama/(2*Nc);               % this is area per wire
C = (pi/p)*(Rs+hd+hs/2)*5/6;          % circumferential path at each end
le = pi*C;                         % average length of an end turn
lw = 2*(L+le)*Na;                  % this is length of wire
sig = 6e7;                         % assume copper wire
Ra = lw/(Aw*sig);                  % this is winding resistance
Nrot = 0:10:12000;                 % here is a speed span
omrot = (2*p*pi/60) .* Nrot;
Z = j*(Lag+Lslot) .* omrot + Ra;      % stator impedance
Is = omrot .* Na*Phi ./ Z;           % short circuit current

figure(4)
plot(Nrot, abs(Is))
title('Problem 10.2')
ylabel('Short Circuit Current (Peak A)')
xlabel('Speed, RPM')

% now look at ideal operation
Ea = om*Na*Phi;                       % back voltage at rated speed
Xt = om*(Lag+Lslot);
Zt = j*Xt + Ra;
V = Ea+Zt*I;                        % terminal voltage (current at zero angle
Pm = 1.5*Ea*I;
VA = 1.5*V*I;
Pin = real(VA);
Qin = imag(VA);
eff = Pm/Pin;
pf = Pin/abs(VA);

fprintf(' Problem 10.2: Operating Details:\n')
fprintf('Internal Voltage = %12.4g Terminal Voltage = %12.4g\n', Ea, abs(V));
fprintf('Terminal Current = %12.4g Output Power   = %12.4g\n', I, Pm);
fprintf('Reactance        = %12.4g Resistance     = %12.4g\n', Xt, Ra);
```

```
fprintf('Efficiency         =  %12.4g    Power  Factor      =  %12.4g\n',  eff,  pf);
fprintf('Air-Gap  L          =  %12.4g    Slot  L      =  %12.4g\n',  Lag,  Lslot);
```