

Massachusetts Institute of Technology  
Department of Electrical Engineering and Computer Science  
6.685 Electric Machines

Problem Set 10

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Due November 23, 2005

**Problem 1: Brushless Machine** A permanent magnet synchronous machine with surface magnets and a conventional three-phase stator structure has the following dimensions and parameters:

Rotor Radius	60	mm
Active Length	180	mm
Relative Rotation (magnetic) Gap	1.5	mm
Magnet Height	3.5	mm
Number of pole pairs	2	
Number of stator slots	36	
Number of stator turns/phase	20	
Magnet Angular Width	150	electrical degrees
Winding format	5/6 pitch	

Note: “rotor radius” is to the outer surface of the magnets, and “gap” is the total dimension between the magnets and stator surface, which may include some (non-magnetic) structure which is not of interest to us here. Assume the magnets are made of NdFeB material with a peak ‘energy product’ of 42 MG-Oe and an incremental permeability of  $\mu_0$ .

This machine is turned through some external mechanism at a speed of 6,000 RPM. The terminals are left open-circuited.

- What are the magnitude and frequency of the fundamental and first four higher order harmonic voltages? Note that in order to do this right you will need to use the ‘gap coefficient’ derived in the notes (see Page 31 of Chapter 7). You will need to augment the expression in the notes to account for space harmonics.
- Plot the terminal voltage waveform for one revolution of the rotor.

**Problem 2: More Brushless Machines** Now we will explore some aspects of the rating of this machine. Assume that the stator slots have radial extent of 15 mm. The slot tops have a ‘depression’ (the opening to the air gap) which is 1 mm deep and 1 mm wide. The slots are 5 mm wide at their top (just below the depression and tooth tips) but are wider at the back. This is a three-phase machine.

1. Assuming that the mechanism for cooling the stator is sufficient to permit a slot current density of  $J_a = 2 \times 10^6$  A/square meter (RMS), what is the *current* rating of the machine?
2. Operating as a motor, what is the *torque* rating of the machine?
3. What is the ‘air-gap’ component of inductance?

4. What is the 'slot leakage' component of inductance?
5. Assuming that the slot packing factor (ratio of conductor area to slot area) is 25%, estimate the stator resistance of the machine. For this calculation, you must make an estimate of end turn length. You may estimate that the end turns follow a path which is maps to a half circle on the cylinder that cuts halfway up the stator slots. Assume the winding is made of copper with bulk conductivity of  $6 \times 10^6$  S/m.
6. Now assume the machine is short-circuited and the shaft is turned by the same mechanism as in Problem 1. Calculate and plot the magnitude of time fundamental current as a function of machine speed. Plot this from zero to 12,000 RPM.
7. If the machine is operated as a motor at 9,000 RPM with rated current and at a phase angle to produce maximum torque, what *is* the rating of the motor? What is the terminal power factor? What is the efficiency?

**Problem 3: Experiment (!)** This is not only an experiment for you, it is an experiment for me, too. We have obtained two small experimental 'dynamometer sets' that have a DC machine as the prime mover and three other machines that can be easily connected mechanically to that machine: another DC machine, a 'brushless DC' (permanent magnet synchronous) machine and an induction motor. For this experiment you will use the 'brushless DC' machine, which is really a smaller version of the machine you are considering here. The objective of this experiment is to see if you can obtain consistency between internal voltage and torque production. The work in the lab should not take a long time.

1. First, find the experimental machines. They are located at the west end of Room 38-600 on either side of the penultimate bench. By the time you attempt this we should have the needed equipment to operate at least one of them. The brushless DC machine (which we will call the 'test machine') should be connected to the dynamometer machine with a torquemeter on between.
2. Connect an oscilloscope to the terminals of the test machine and a power supply to the dynamometer machine. Put enough voltage on the dynamometer machine to make the assembly turn at a convenient speed and record the voltage waveform at the terminals of the test machine. Note how fast the machine is going. Is there consistency between electrical frequency and speed?
3. Now, with a power supply in current limiting mode put some current into the test machine (you will have to drive line-line). The rotor should turn to one of the 'zero torque' points. By putting current through the dynamometer machine you will torque the whole apparatus. The machine should rotate to a new equilibrium position. With enough current in the dynamometer machine the machine under test will have rotated to its maximum torque position and will 'slip' a pole. Being careful (gentle) in raising the current in the dynamometer machine, note the torque at which the test machine breaks away.
4. Now you should be able to derive the motor constant from both measurements (voltage and torque). Are they consistent?