Lectures on Dynamic Systems and Control

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Chapter 19

Robust Stability in SISO Systems

19.1 Introduction

There are many reasons to use feedback control. As we have seen earlier, with the help of an appropriately designed feedback controller we can reduce the effect of noise and disturbances, and we can improve the tracking of command signals. Another very important use for feedback control is the reduction of the effects of plant uncertainty. The mathematical models that we use to describe the plant dynamics are almost never perfect. A feedback controller can be designed so as to maintain stability of the closed-loop and an acceptable level of performance in the presence of uncertainties in the plant description, i.e., so as to achieve *robust stability* and *robust performance* respectively.

For the study of robust stability and robust performance, we assume that the dynamics of the actual plant are represented by a transfer function that belongs to some uncertainty set Ω. We begin by giving mathematical descriptions of two possible uncertainty sets. Many other descriptions exist, and may be treated by methods similar to those we present for these particular types of uncertainty sets.

19.2 Additive Representation of Uncertainty

It is commonly the case that the nominal plant model is quite accurate for low frequencies but deteriorates in the high-frequency range, because of parasitics, nonlinearities and/or time-varying effects that become significant at higher frequencies. These high-frequency effects may have been left unmodeled because the effort required for system identification was not justified by the level of performance that was being sought, or they may be well-understood effects that were omitted from the nominal model because they were awkward and unwieldy to carry along during control design. This problem, namely the deterioration of nominal models at higher frequencies, is mitigated to some extent by the fact that almost all physical systems have strictly proper transfer functions, so that the system gain begins to roll off at high frequency.

In the above situation, with a nominal plant model given by the proper transfer function $P_0(s)$, the actual plant represented by $P(s)$, and the difference $P(s) - P_0(s)$ assumed to be stable, we may be able to characterize the model uncertainty via a bound of the form

$$|P(j\omega) - P_0(j\omega)| \leq \ell_a(\omega) \tag{19.1}$$

where

$$\ell_a(\omega) = \begin{cases} \text{“Small”} & ; \quad |\omega| < \omega_c \\ \text{“Bounded”} & ; \quad |\omega| > \omega_c \end{cases} . \tag{19.2}$$

This says that the response of the actual plant lies in a “band” of uncertainty around that of the nominal plant. Notice that no phase information about the modeling error is incorporated into this description. For this reason, it may lead to conservative results.

The preceding description suggests the following simple *additive* characterization of the uncertainty set:

$$\Omega_a = \{P(s) \mid P(s) = P_0(s) + W(s)\Delta(s)\} \tag{19.3}$$

where Δ is an arbitrary *stable* transfer function satisfying the norm condition

$$\|\Delta\|_\infty = \sup_\omega |\Delta(j\omega)| \leq 1, \tag{19.4}$$

and the *stable* proper rational weighting term $W(s)$ is used to represent any information we have on how the accuracy of the nominal plant model varies as a function of frequency. Figure 19.1 shows the additive representation of uncertainty in the context of a standard servo loop, with K denoting the compensator.

When the modeling uncertainty increases with frequency, it makes sense to use a weighting function $W(j\omega)$ that looks like a high-pass filter: small magnitude at low frequencies, increasing but bounded at higher frequencies.

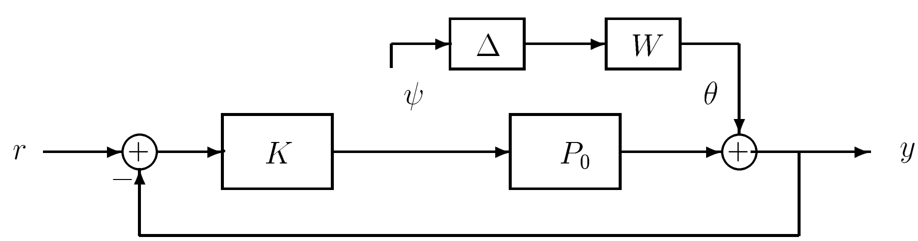

Figure 19.1: Representation of the actual plant in a servo loop via an additive perturbation of the nominal plant.

Caution: The above formulation of an additive model perturbation should *not* be interpreted as saying that the actual or perturbed plant is the *parallel combination* of the nominal system $P_0(s)$ and a system with transfer function $W(s)\Delta(s)$. Rather, the actual plant should be considered as being a *minimal realization* of the transfer function $P(s)$, which happens to be written in the additive form $P_0(s) + W(s)\Delta(s)$.

Some features of the above uncertainty set are worth noting:

- The *unstable* poles of all plants in the set are precisely those of the nominal model. Thus, our modeling and identification efforts are assumed to be careful enough to accurately capture the unstable poles of the system.

- The set includes models of arbitrarily large order. Thus, if the uncertainties of major concern to us were *parametric uncertainties*, i.e. uncertainties in the values of the parameters of a particular (e.g. state-space) model, then the above uncertainty set would greatly overestimate the set of plants of interest to us.

The control design methods that we shall develop will produce controllers that are guaranteed to work for *every member* of the plant uncertainty set. Stated slightly differently, our methods will treat the system as though *every* model in the uncertainty set is a possible representation of the plant. To the extent that not all members of the set are possible plant models, our methods will be conservative.

Suppose we have a set of possible plants Π such that the true plant is a member of that set. We can try to embed this set in an additive perturbation structure. First let $P_0 \in \Pi$ be a certain nominal plant in Π. For any other plant $P \in \Pi$ we write,

$$P(j\omega) = P_0(j\omega) + W(j\omega)\Delta(j\omega).$$

The weight $|W(j\omega)|$ satisfies

$$\begin{aligned} |W(j\omega)| &\geq |W(j\omega)\Delta(j\omega)| = |P(j\omega) - P_0(j\omega)| \\ |W(j\omega)| &\geq \max_{P\in\Pi} |P(j\omega) - P_0(j\omega)| = \ell_a(j\omega). \end{aligned}$$

With the knowledge of the lower bound $\ell_a(j\omega)$, we find a stable system $W(s)$ such that $|W(j\omega)| \geq \ell_a(j\omega)$

19.3 Multiplicative Representation of Uncertainty

Another simple means of representing uncertainty that has some nice analytical properties is the *multiplicative perturbation*, which can be written in the form

$$\Omega_m = \{P \mid P = P_0(1 + W\Delta),\ \|\Delta\|_\infty \leq 1\}. \tag{19.5}$$

W and Δ are stable. As with the additive representation, models of arbitrarily large order

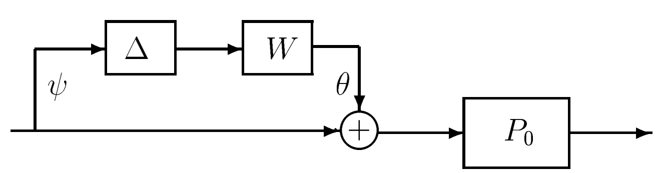

Figure 19.2: Representation of uncertainty as multiplicative perturbation at the plant input.

are included in the above sets.

The caution mentioned in connection with the additive perturbation bears repeating here: the above multiplicative characterizations should *not* be interpreted as saying that the actual plant is the *cascade combination* of the nominal system P_0 and a system $1 + W\Delta$. Rather, the actual plant should be considered as being a *minimal realization* of the transfer function $P(s)$, which happens to be written in the multiplicative form.

Any unstable poles of P are poles of the nominal plant, but not necessarily the other way, because unstable poles of P_0 may be cancelled by zeros of $I + W\Delta$. In other words, the actual plant is allowed to have fewer unstable poles than the nominal plant, but all its unstable poles are confined to the same locations as in the nominal model. In view of the caution in the previous paragraph, such cancellations do *not* correspond to unstable hidden modes, and are therefore not of concern.

As in the case of additive perturbations, suppose we have a set of possible plants Π such that the true plant is a member of that set. We can try to embed this set in a multiplicative perturbation structure. First let $P_0 \in \Pi$ a certain nominal plant in Π. For any other plant $P \in \Pi$ we have,

$$P(j\omega) = P_0(j\omega)(1 + W(j\omega)\Delta(j\omega)).$$

The weight $|W(j\omega)|$ satisfies

$$\begin{aligned}
|W(j\omega)| &\geq |W(j\omega)\Delta(j\omega)| = \left|\frac{P(j\omega) - P_0(j\omega)}{P_0(j\omega)}\right| \\
|W(j\omega)| &\geq \max_{P\in\Pi}\left|\frac{P(j\omega) - P_0(j\omega)}{P_0(j\omega)}\right| = \ell_m(j\omega).
\end{aligned}$$

With the knowledge of the envelope $\ell_m(j\omega)$, we find a stable system $W(s)$ such that $|W(j\omega)| \geq \ell_m(j\omega)$

Example 19.1 Uncertain Gain

Suppose we have a plant $P = k\bar{P}(s)$ with an uncertain gain k that lies in the interval $k_1 \leq k \leq k_2$. We can write $k = \alpha(1 + \beta x)$ such that

$$\begin{aligned}
k_1 &= \alpha(1 - \beta) \\
k_2 &= \alpha(1 + \beta).
\end{aligned}$$

Therefore $\alpha = \frac{k_1+k_2}{2}$, $\beta = \frac{k_2-k_1}{k_2+k_1}$, and we can express the set of plants as

$$\Pi = \left\{P(s)|P(s) = \frac{k_1 + k_2}{2}\bar{P}(s)\left(1 + \frac{k_2 - k_1}{k_2 + k_1}x\right), -1 \leq x \leq 1\right\}.$$

We can embed this Π in a multiplicative structure by enlarging the uncertain elements x which are real numbers to complex $\Delta(j\omega)$ representing dynamic perturbations. This results in the following set

$$\Omega_m = \left\{P(s)|P(s) = \frac{k_1 + k_2}{2}\bar{P}(s)\left(1 + \frac{k_2 - k_1}{k_2 + k_1}\Delta\right), \|\Delta\|_\infty \leq 1\right\}.$$

Note that in this representation $P_0 = \frac{k_1+k_2}{2}\bar{P}$, and $W = \frac{k_2-k_1}{k_2+k_1}$.

Example 19.2 Uncertain Delay

Suppose we have a plant $P = e^{-ks}P_0(s)$ with an uncertain delay $0 \leq k \leq k_1$. We want to represent this family of plants in a multiplicative perturbation structure. The weight $W(s)$ should satisfy

$$
\begin{aligned}
|W(j\omega)| &\geq \max_{0\leq k\leq k_1} \left| \frac{e^{-j\omega k}P_0(j\omega) - P_0(j\omega)}{P_0(j\omega)} \right| \\
&= \max_{0\leq k\leq k_1} |e^{-j\omega k} - 1| \\
&= \begin{cases} |1 - e^{-j\omega k_1}| & \omega < \frac{\pi}{k_1} \\ 0 & \omega \geq \frac{\pi}{k_1} \end{cases} \\
&= \ell_m(\omega).
\end{aligned}
$$

A stable weight that satisfies the above relation can be taken as

$$W(s) = \alpha \frac{2\pi k_1 s}{\pi k_1 s + 1}.$$

where $\alpha > 1$. The reader should verify that this weight will work by ploting $|W(j\omega)|$ and $\ell_m(\omega)$, and showing that $\ell_m(\omega)$ is below the curve $|W(j\omega)|$ for all ω.

19.4 The Nyquist Criterion

Before we analyze the stability of feedback loops where the plant is uncertain, we will review the Nyquist criterion. Consider the feedback structure in Figure 19.3. The transfer function

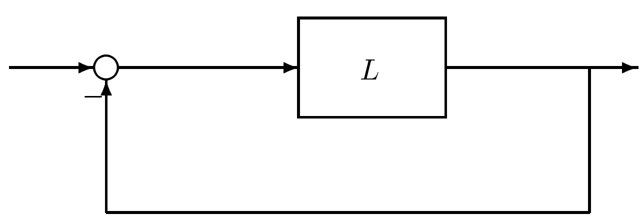

Figure 19.3: Unity Feedback Confuguration.

L is called the open-loop transfer function. The condition for the stability of the system in 19.3 is assured if the zeros of $1 + L$ are all in the left half of the complex plane. The agrument principle from complex analysis gives a criterion to calculate the difference between the number of zeros and the number of poles of an analytic function in a certain domain, $\mathcal{D}$ in the complex plane. Suppose the domain is as shown in Figure 19.4, and the boundary of $\mathcal{D}$, denoted by $\delta\mathcal{D}$, is oriented clockwise. We call this oriented boundary of $\mathcal{D}$ the Nyquist contour.

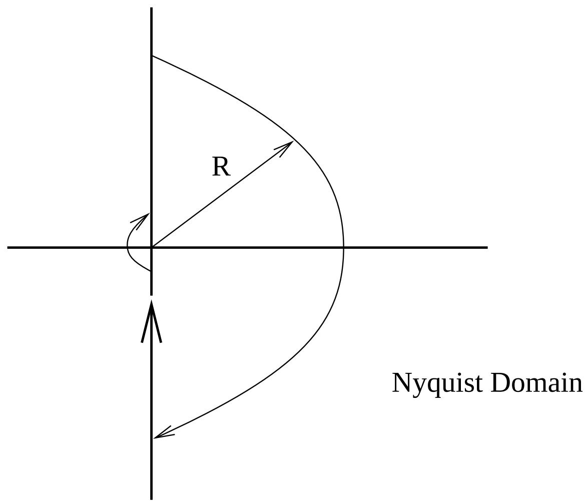

Figure 19.4: Nyquist Domain.

As the radius of the semicircle in Figure 19.4 goes to infinty the domain covers the right half of the complex plane. The image of $\delta\mathcal{D}$ under L is called a Nyquist plot, see Figure 19.5. Note that if L has poles at the $j\omega$ axis then we indent the Nyquist contour to avoid these poles, as shown in Figure 19.4. Define

$$\begin{aligned} \pi_{ol} &= \text{Open}-\text{loop poles} = \text{Number of poles of } L \text{ in } \mathcal{D} = \text{Number of poles of } 1+L \text{ in } \mathcal{D} \\ \pi_{cl} &= \text{Closed}-\text{loop poles} = \text{Number of zeros of } 1+L \text{ in } \mathcal{D}. \end{aligned}$$

From the argument principle it follows that

$$\pi_{cl}-\pi_{ol} = \text{The number of clockwise encirclements that the Nyquist Plot makes of the point } -1.$$

Using this characterization of the difference of the number of the closed-loop poles and the open-loop poles we arrive at the following theorem for the stability of Figure 19.3

Theorem 19.1 *The closed-loop system in Figure 19.3 is stable if and only if the Nyquist plot*

- *does not pass through the origin,*
- *makes π_{ol} counter-clockwise encirclements of -1.*

19.5 Robust Stability

In this section we will show how we can analyze the stability of a feedback system when the plant is uncertain and is known to belong to a set of the form that we described earlier. We will start with the case of additive pertubations. Consider the unity feedback configuration in Figure 19.1. The open-loop transfer function is $L(s) = (P_0(s) + W(s)\Delta(s))K(s)$, and the

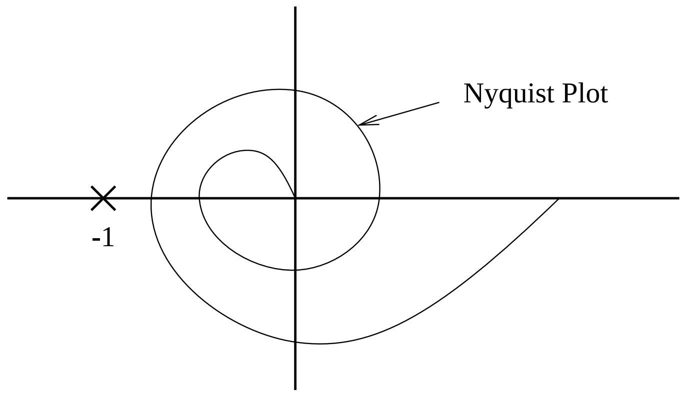

Figure 19.5: Nyquist Plot.

nominal open-loop transfer function is $L_0(s) = P_0(s)K(s)$. The nominal feedback system with the nominal open-loop transfer function L_0 is stable, and we want to know whether the feeback system remains stable for all $\Delta(s)$ satisfying $|\Delta(j\omega)| \leq 1$ for all $\omega \in \mathbb{R}$. We will assume that the nominal open-loop system is stable. This causes no loss of generality and the result holds in the general case. From the Nyquist criterion, we have that the Nyquist plot of L_0 does not encircle the point -1. For the perturbed system, we have that

$$\begin{aligned} 1 + L(j\omega) &= 1 + P(j\omega)K(j\omega) \\ &= 1 + (P_0(j\omega) + W(j\omega)\Delta(j\omega))K(j\omega) \\ &= 1 + L_0(j\omega) + W(j\omega)\Delta(j\omega)K(j\omega) \end{aligned}$$

From the Figure 19.6, it is clear that $L(\jmath\omega)$ will not encircle the point -1 if the following condition is satisfied,

$$|W(j\omega)K(j\omega)| < |1 + L_0(j\omega)|,$$

which can be written as

$$\left| \frac{W(j\omega)K(j\omega)}{1 + L_0(j\omega)} \right| < 1. \tag{19.6}$$

A Small Gain Argument

Next we will present a different derivation of the above result that does not rely on the Nyquist criterion, and will be the basis for the multivariable generalizations of the robust stability results. Since the nominal feedback system is stable, the zeros of $1 + L_0(s)$ are all in the left half of the complex plane. Therefore, by the continuity of zeros, the perturbed system

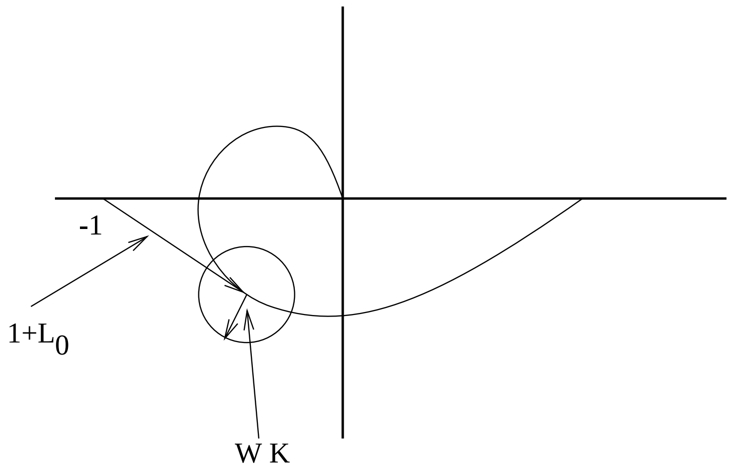

Figure 19.6: Nyquist Plot Illustrating Robust Stability.

will be stable if and only if

$$|1 + (P_0(j\omega) + W(j\omega)\Delta(j\omega))K(j\omega)| > 0$$

for all $\omega \in \mathbb{R}$, $\|\Delta\|_\infty \leq 1$. By rearranging the terms, the perturbed system is stable if and only if

$$\min_{|\Delta(j\omega)|\leq 1} \left|1 + \frac{W(j\omega)K(j\omega)}{1 + P_0(j\omega)K(j\omega)}\Delta(j\omega)\right| > 0 \quad \text{for all } \omega \in \mathbb{R}$$

The following lemma will help us to transform this condition to the one given earlier.

Lemma 19.1 *The following are equivalent*

1.

$$\min_{|\Delta(j\omega)|\leq 1} \left|1 + \frac{W(j\omega)K(j\omega)}{1 + P_0(j\omega)K(j\omega)}\Delta(j\omega)\right| > 0 \quad \textit{for all } \omega \in \mathbb{R}$$

2.

$$1 - \left|\frac{W(j\omega)K(j\omega)}{1 + P_0(j\omega)K(j\omega)}\right| > 0 \quad \textit{for all } \omega \in \mathbb{R}$$

Proof. First we show that 2) implies 1), which is a consequence of the following inequalities

$$\begin{aligned} \left|1 + \frac{W(j\omega)K(j\omega)}{1 + P_0(j\omega)K(j\omega)}\Delta(j\omega)\right| &\geq 1 - \left|\frac{W(j\omega)K(j\omega)}{1 + P_0(j\omega)K(j\omega)}\Delta(j\omega)\right| \\ &\geq 1 - \left|\frac{W(j\omega)K(j\omega)}{1 + P_0(j\omega)K(j\omega)}\right|. \end{aligned}$$

For the converse suppose 2) is violated, that is there exists ω_0 such that

$$\left|\frac{W(j\omega_0)K(j\omega_0)}{1+P_0(j\omega_0)K(j\omega_0)}\right| \geq 1.$$

Write

$$\frac{W(j\omega_0)K(j\omega_0)}{1+P_0(j\omega_0)K(j\omega_0)} = ae^{j\phi},$$

and let $\bar{\Delta}(j\omega_0) = \frac{1}{a}e^{-j\phi-j\pi}$. Clearly, $|\bar{\Delta}(j\omega_0)| \leq 1$ and

$$1 + \frac{W(j\omega_0)K(j\omega_0)}{1+P_0(j\omega_0)K(j\omega_0)}\bar{\Delta}(j\omega_0) = 0.$$

Now select a real rational perturbation $\bar{\Delta}(s)$ as

$$\bar{\Delta}(s) = \pm\frac{1}{a}\frac{s-\alpha}{s+\alpha},$$

such that $\pm\frac{j\omega_0-\alpha}{\omega_0+\alpha} = e^{-j\phi-j\pi}$.

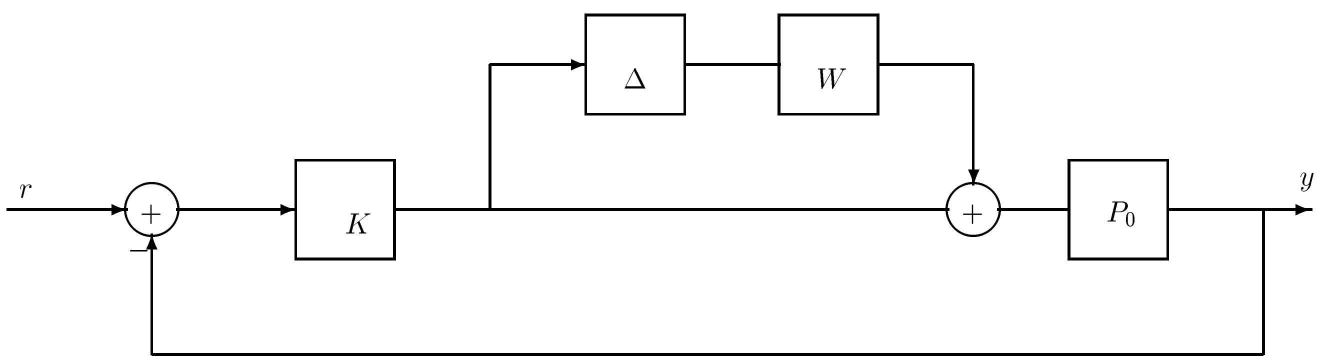

Figure 19.7: Representation of the actual plant in a servo loop via a multiplicative perturbation of the nominal plant.

A similar set of results can be obtained for the case of multiplicative perturbations. In particular, a robust stability of the configuration in Figure 19.7 can be guaranteed if the system is stable for the nominal plant P_0 and

$$\left|\frac{W(j\omega)P_0(j\omega)K(j\omega)}{1+P_0(j\omega)K(j\omega)}\right| < 1. \quad \text{for all } \omega \in \mathbb{R}. \tag{19.7}$$

Example 19.3 Stabilizing a Beam

We are interested in deriving a controller that stabilizes the beam in Figure 19.8 and tracks a step input (with good properties). The rigid body model from torque input to the tip deflection is given by

$$P_0(s) = \frac{6.28}{s^2}$$

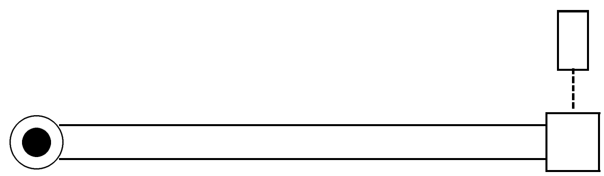

Figure 19.8: Flexible Beam.

Consider the controller

$$K_0(s) = \frac{500(s+10)}{s+100}$$

The loop gain is given by

$$P_0(s)K_0(s) = \frac{3140(s+10)}{s^2(s+100)}$$

and is shown in Figure 19.9. The closed loop poles are located at -49.0, -28.6, -22.4, and the nominal Sensitivity function is given by

$$S_0(s) = \frac{1}{1+P_0(s)K_0(s)} = \frac{s^2(s+100)}{s^3+100s^2+3140s+31400}$$

and is shown in Figure 19.10. It is evident from this that the system has good disturbance rejection and tracking properties. The closed loop step response is show in Figure 19.11

While this controller seems to be an excellent design, it turns out that it performs quite poorly in practice. The bandwidth of this controller (which was never constrained) is large enough to excite the flexible modes of the beam, which were not taken into account in the model. A more complicated model of the beam is given by

$$P_1(s) = \underbrace{\frac{6.28}{s^2}}_{\text{nominal plant}} + \underbrace{\frac{12.56}{s^2+0.707s+28}}_{\text{flexible mode}}$$

If K_0 is connected to this plant, then the closed loop poles are -1.24, 0.29, 0.06, -0.06, which implies instability.

Instead of using the new model to redesign the controller, we would like to use the nominal model P_0, and account for the flexible modes as unmodeled dynamics with a certain frequency concentration. There are several advantages in this. For

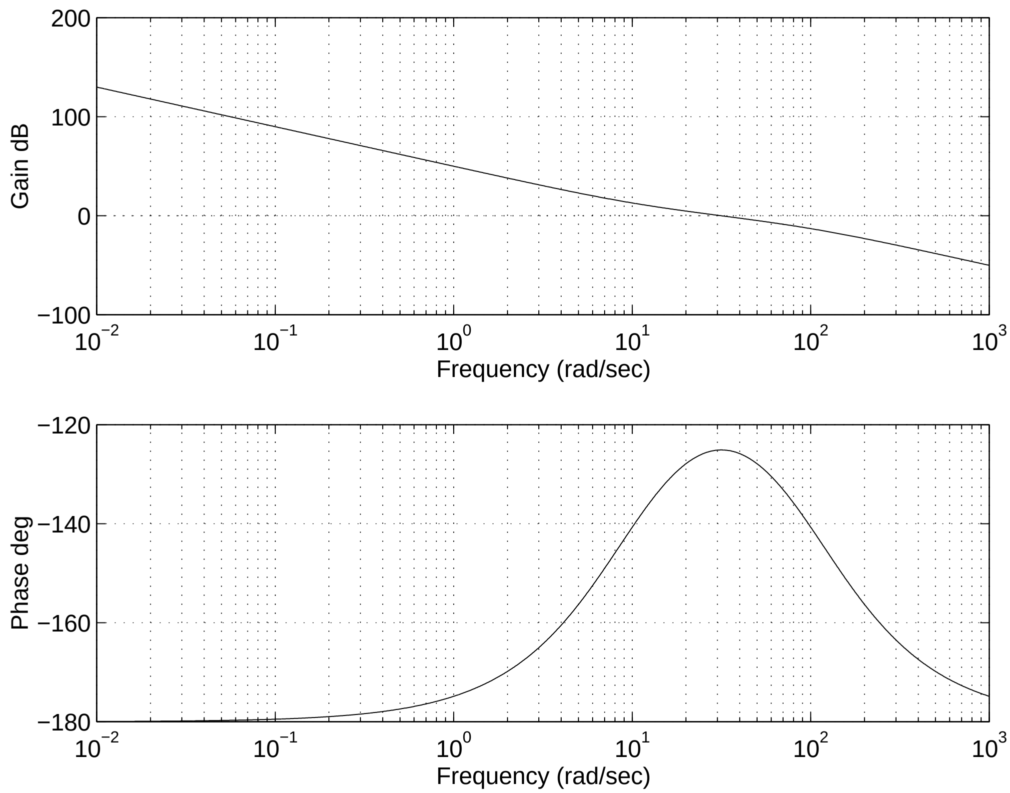

Figure 19.9: Open-loop Bode Plot

one, the design is based on a simpler nominal model and hence may result in a simpler controller. This approach also allows us to acomodate additional flexible modes without increasing the complexity of the description. And finally, it enables us to tradeoff performance for robustness.

Consider the set of plants:

$$\Omega = \{P = P_0(1+\Delta);\ \ |\Delta(j\omega)| \leq \ell(\omega), \Delta \text{ is stable}\}$$

where

$$\ell(\omega) \leq 2\left|\frac{\omega^2}{28-\omega^2+0.707j\omega}\right|$$

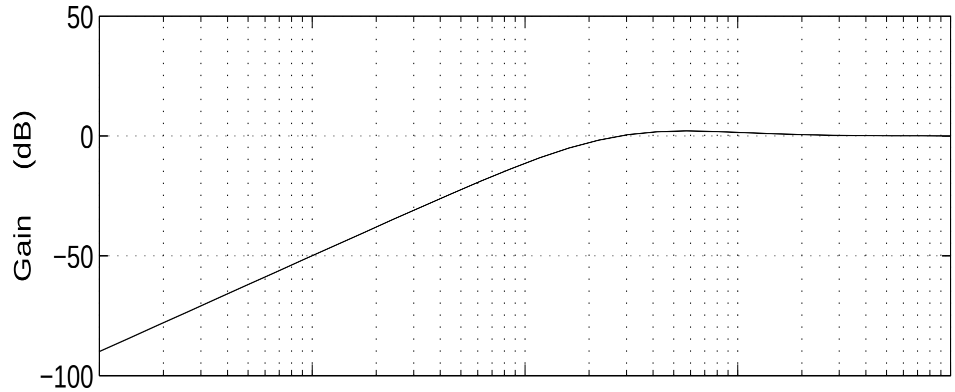

Figure 19.10: Nominal Sensitivity

This set includes the model P_1. The stability Robustness Condition is given by:

$$|T(j\omega)| < \frac{1}{\ell(\omega)}$$

Where T is the nominal closed loop map with any controller K. First, consider the stability analysis of the initial controller $K_0(s)$. Figure 19.12 shows both the frequency response for $|T_0(j\omega)|$ and $[\ell(\omega)]^{-1}$. It is evident that the Stability robustness condition is violated since

$$|T_0(j\omega)| \not< \frac{1}{\ell(\omega)}, \quad 3 \leq \omega \leq 70 \text{ rad/sec}$$

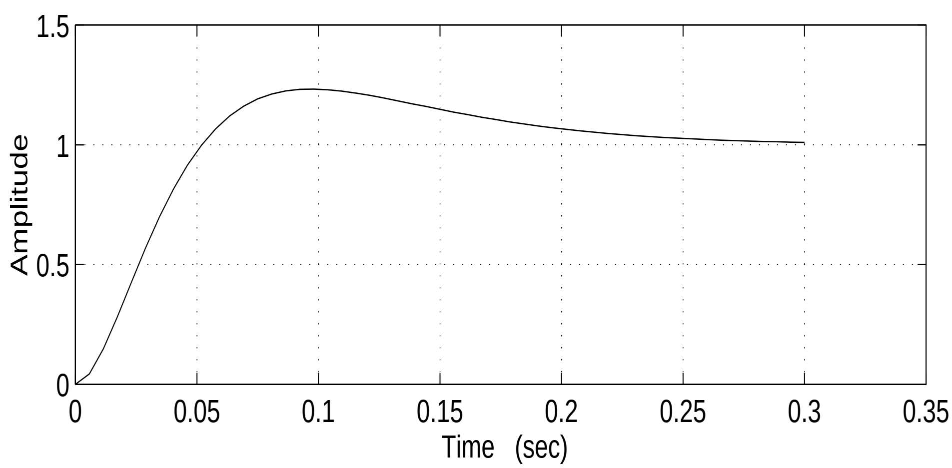

Figure 19.11: Step Response

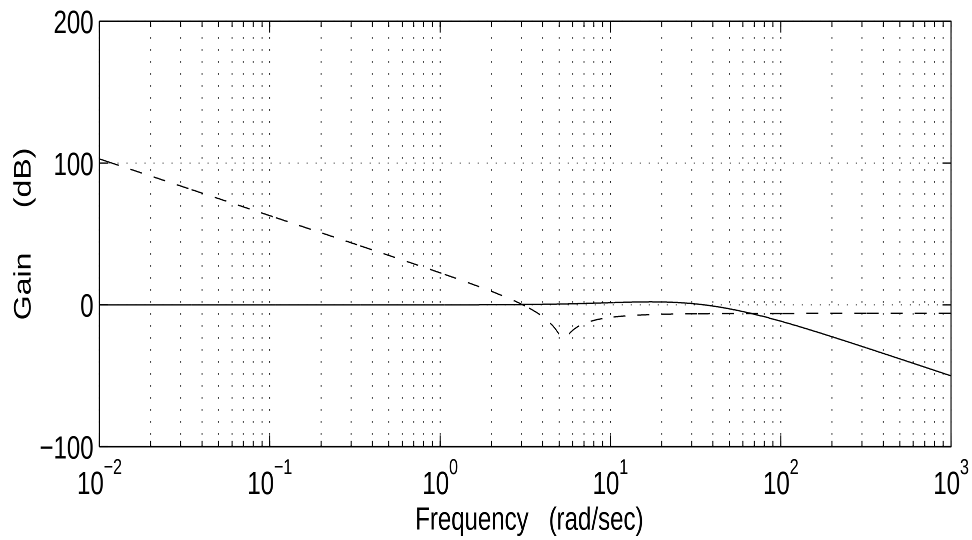

Figure 19.12: $|T_0(j\omega)|$ and $[\ell(\omega)]^{-1}$

Let's try a new design with a different controller

$$K_1(s) = \frac{(5 \times 10^{-4})(s + 0.01)}{s + 0.1}$$

The new loop-gain is

$$P_0(s)K_1(s) = \frac{(3.14 \times 10^{-3})(s + 0.01)}{s^2(s + 0.1)}$$

which is shown in the Figure 19.13 We first check the robustness condition with the new controller. T_1 is given by

$$T_1(s) = \frac{P_0(s)K_1(s)}{1 + P_0(s)K_1(s)}$$

Figure 19.14 depicts both $|T_1(j\omega)|$ and $[\ell(\omega)]^{-1}$. It is clear that the condition is satisfied. Figure 19.15 shows the new nominal step response of the system. Observe that the response is much slower than the one derived by the controller K_0. This is essentially due to the limited bandwidth of the new controller, which was necessary to prevent instability.

Exercises

Exercise 19.1 Suppose $P(s) = \frac{a}{s}$ is connected with a controller $K(s)$ in a unity feedback configuration. Does there exists a K such that the system is stable for both $a = 1$ and $a = -1$.

Exercise 19.2 For $P(s)$ and $K(s)$ given by

$$P(s) = \frac{1}{(s+2)(s+a)}, \qquad K(s) = \frac{1}{s},$$

find the range of a such that the closed loop system with P and K is stable.

Exercise 19.3 Let P be given by:

$$P(s) = (1 + W(s)\Delta(s))P_0,$$

where

$$P_0(s) = \frac{1}{s-1}, \qquad W(s) = \frac{2}{s+10},$$

and Δ is arbitrary stable with $\|\Delta\|_\infty \leq 2$. Find a controller $K(s) = k$ (constant) gain such that the system is stable. Compute all possible such gains.

Exercise 19.4 Find the stability robustness condition for the set of plant described by:

$$P = \{\frac{P_0}{1 + \Delta W P_0}, \qquad \|\Delta\|_\infty \leq 1\}.$$

Assume WP_0 is strictly proper for well posedness.

Exercise 19.5 Suppose

$$P(s) = \frac{1}{s-a} \text{and} \qquad \mathrm{K(s) = 10},$$

are connected in standard feedback configuration. While it is easy in this case to compute the exact stability margin as a changes, in general, such problems are hard to solve when there are many parameters. One approach is to embed the problem in a robust stabilization problem with unmodeled dynamics and derive the appropriate stability robustness condition. Clearly, the later provides a conservative bound on a for which the system remains stable.

(a) Find the exact range of a for which the system is stable.

(b) Assume the nominal plant is $P_0 = \frac{1}{s}$. Show that P belongs to the set of plants:

$$\Omega = \{P = \frac{P_0}{1 + W\Delta P_0}, \|\Delta\|_\infty \leq 1\}$$

and $W = -a$.

(c) Derive a condition on the closed loop system that guarantees the stability of the set Ω. How does this condition constrain a? Is this different than part (a)?

(d) Repeat with nominal plant $P_0 = \frac{1}{s+100}$.

Exercise 19.6 Let a model be given by the stable plant:

$$P_0(z) = \frac{1}{z^{-1} - (1 + a_0)}, \quad 1 >> a_0 > 0.$$

Consider the class of plants given by:

$$\Omega = \left\{ (z) = \frac{1}{z^{-1} - (1 + b)} | \ -2a_0 \leq b \leq 2a_0 \right\}.$$

1. Can the set Ω be embedded in a set of additive or multiplicative norm bounded perturbations, with nominal plant P_0? Show how or explain your answer.
2. If your answer to the previous part is NO, show that the class Ω can be embedded in some other larger set characterized by norm-bounded perturbations. Give a sufficient condition for stability using the small gain theorem.
3. Improve your earlier condition so that it captures the fact that the unknown is a real parameter. (The condition does not have to be necessary, but should still take into consideration the phase information!).

Exercise 19.7 Consider Exercise 17.4. Suppose that due to implementation problems (e.g. quantization effects), the actual controller can be modeled as:

$$K_a = (I - KW\Delta)^{-1} K$$

where W is a fixed stable filter, and Δ is a stable perturbation of $\mathcal{H}_\infty$-norm less than 1, but otherwise arbitrary. Provide a non-conservative condition for the stability robustness of the closed loop system. Use the parametrization of K in terms of Q to express your condition as a function of P and Q.

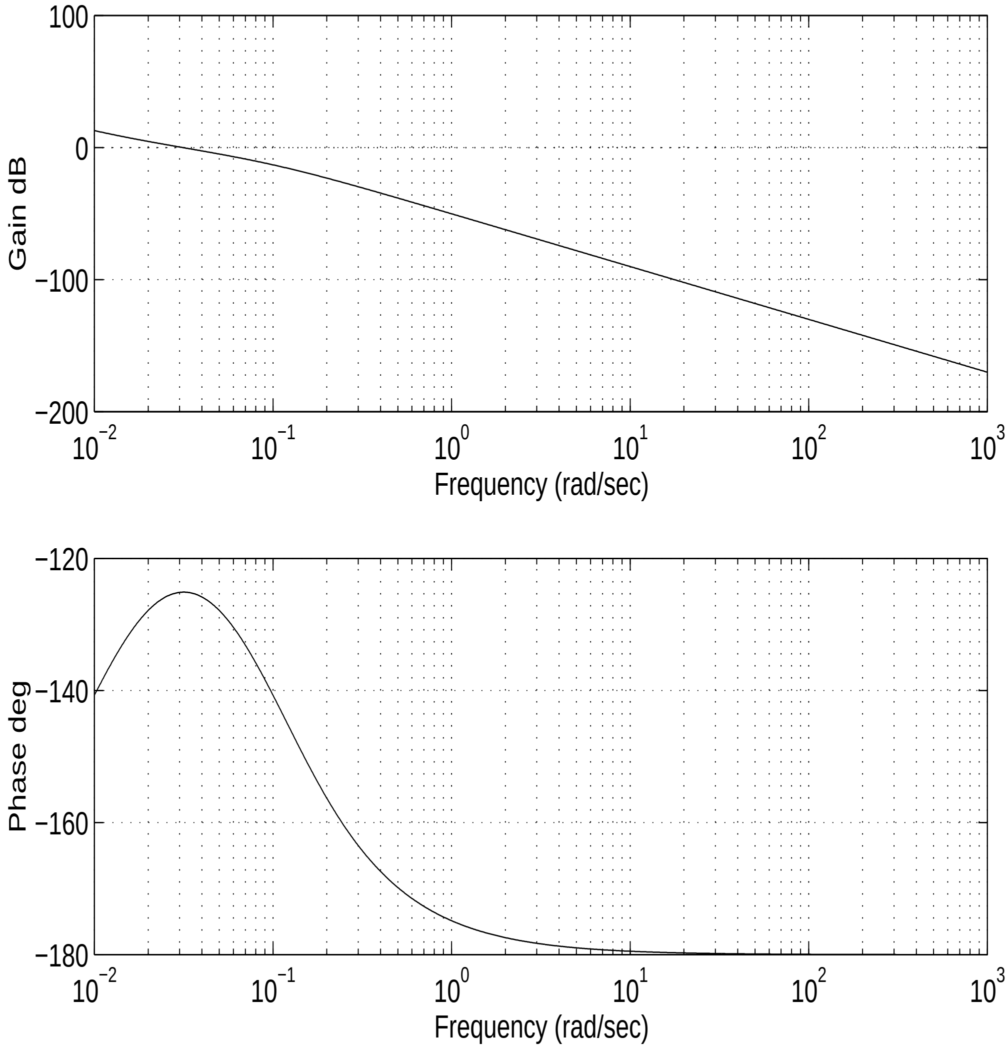

Figure 19.13: Loop Gain P_0K_1

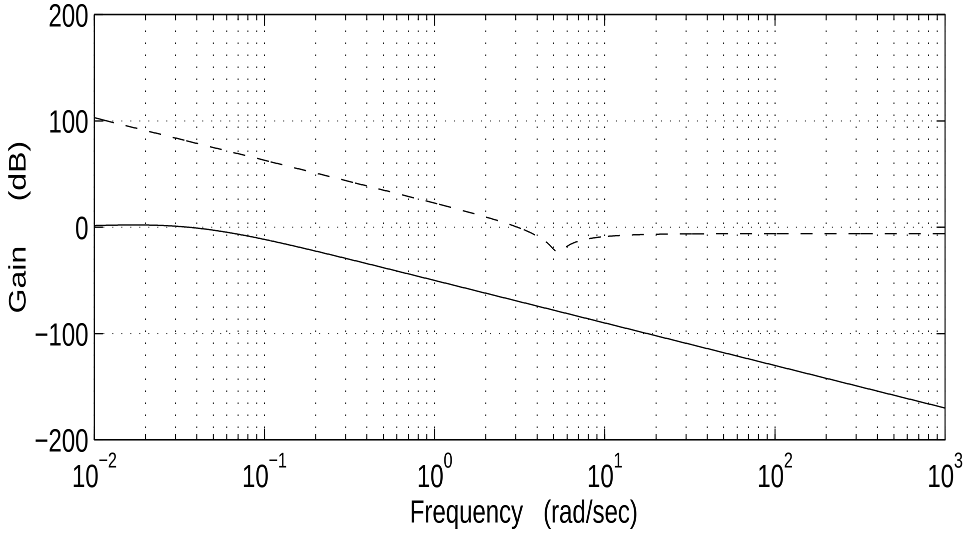

Figure 19.14: $|T_1(j\omega)|$ and $[\ell(\omega)]^{-1}$.

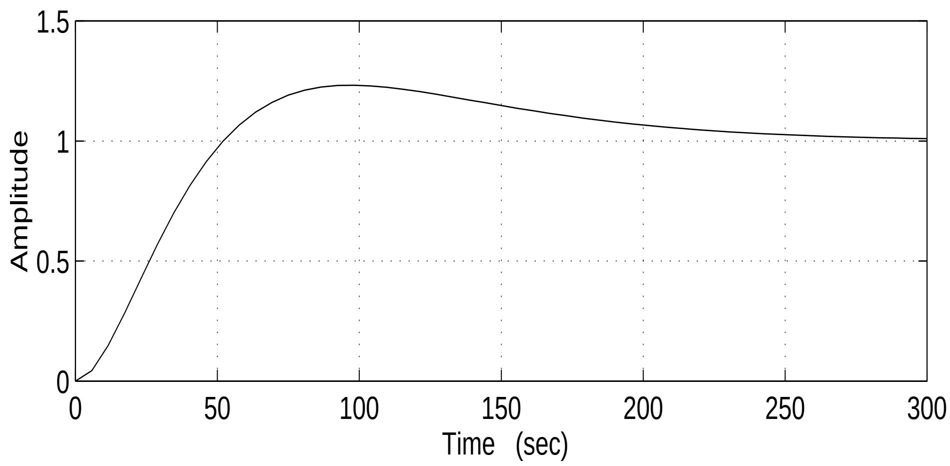

Figure 19.15: New Nominal Closed-loop Step Response

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