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2.830J / 6.780J / ESD.63J Control of Manufacturing Processes (SMA 6303)  
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2.830/6.780J Control of Manufacturing Processes

Quiz 2  
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Open Book and Notes

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**Problem 1 (45 %)**

You are faced with the need to model a system with three inputs but time and cost constraints limit the number of experiments that can be performed to four. Since you know that a full factorial experiment would take eight independent experiments, you have some choices to make.

Starting from the following  $2^2$  design matrix (our  $\mathbf{X}$  matrix in the equation  $y=\mathbf{X}\beta$ )

	Input 1	Input 2	
1	A	B	AB
1	<b>-1</b>	<b>-1</b>	1
1	<b>-1</b>	<b>1</b>	-1
1	<b>1</b>	<b>-1</b>	-1
1	<b>1</b>	<b>1</b>	1

Consider the problem of using this same design for the three input problem. Notice that all row entries are unique combinations of the inputs. Thus, we could assign the AB column to the third input and run the experiment.

If that were done, we would have the matrix:

	Input 1	Input 2	Input 3
1	A	B	C
1	<b>-1</b>	<b>-1</b>	<b>1</b>
1	<b>-1</b>	<b>1</b>	<b>-1</b>
1	<b>1</b>	<b>-1</b>	<b>-1</b>
1	<b>1</b>	<b>1</b>	<b>1</b>

Based on this approach:

- Write out the full interaction matrix for the  $2^3$  problem given that you have only 4 trials available. (You should have a 4 row - 8 column matrix design.)
- Identify the factors that will be confounded or confused with each other, based on this matrix.

- c) If you want to be sure you get the input 1 and 2 **main effects** and the input 1-3 interaction, how would you reassign the columns to the factors? Show this on your 4 x 8 matrix.
- d) What is the experiment design matrix that you would give someone running this experiment? (i.e. a table of settings for the three inputs for all of the experiments.)

Now assume that you want to have non-aliased experiments for all the main effects *and* the 1-3 interaction.

- e) What is the minimum number of new rows (new experiments) that would have to be added and what would they be? (Remember, the only columns that are independent are the ones for the inputs; i.e. the A, B, C columns)
- f) A good experimental design is one that is balanced, meaning all inputs see the same number of levels over all experiments. Is your design for part e) balanced? If not describe what you could do to make it so.

### Problem 2 (20%)

A designed experiment has been performed to determine optimal surface finish ( $y$ ) as a function of feedrate ( $x_1$ ) and tool nose radius ( $x_2$ ). The resulting model is:

$$\hat{y} = 90 + 10x_1 - 15x_2 + 5x_1x_2$$

- a) What experimental design must have been used to get this model?
- b) What is the average surface finish for the experiment
- c) Sketch the response surface for this model over the scaled input range of  $\pm 1$  for the inputs
- d) If a surface finish of 85 is desired, what are the optimal values of feedrate and nose radius assuming that the operating point has no effect on tool wear. (Be sure to define what you are optimizing!)
- e) As it turns out, the operating point has an effect on the resulting variance. Using the variance model  $\sigma^2 = 1 + x_1 - 1.12x_2$ , find the optimum operating point if a surface finish of 85 is desired. How does this interact with your answer from part d)?

**Problem 3 (35%)**

Our company has recently begun production of diagnostic chips for drug screening. Because of reported product quality problems, we have decided to find out whether the curing temperature for the polymer has an effect on the critical channel dimensions of our product. Below we have the results of an experiment on this product with four temperature levels and 15 replicates.

Temp	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Y10	Y11	Y12	Y13	Y14	Y15	ybar	s <sup>2</sup>
500	41.8	41.9	41.7	41.6	41.5	41.7	41.5	41.5	41.4	41.4	41.3	41.3	41.2	41.2	41.1	41.5	0.05
525	41.4	41.3	41.7	41.6	41.7	41.8	41.9	42	42.1	42.2	42.2	42.3	42.4	42.5	42.6	42.0	0.16
550	41.2	41	41.6	41.9	41.7	41.3	41.7	41.8	41.9	42	42.1	42.2	42.2	42.3	42.4	41.8	0.17
575	41	40.6	41.8	41.2	41.9	41.5	41.9	42.1	42.2	42.4	42.6	42.7	42.9	43.1	43.2	42.1	0.60

ANOVA				
	SS	dof	MS	F
Between	0.18			
Within				
Total				

- Fill in the missing values in the ANOVA table. (NB you should not have to calculate any large sums or averages given this data.)
- At 95% confidence, is the effect of manufacturing temperature significant?
- A fellow engineer is fuming over the wasted time on 4 different temperature levels. He claims you could have gotten the same results with only the low and high temperature experiments. Is he right? (Be sure to include a complete justification for this answer by showing the new ANOVA table.)
- Now a 2.830 graduate looks at all this, and calls the entire testing procedure into question based on the state of the process during the test. She suggest we plot any one of the fixed temperature replicates as a 15 point run chart and give it a look. Pick a temperature and make such a run chart plot.

What problem did she see and what impact does it have on the conclusions?