6.002 CIRCUITS AND ELECTRONICS

Basic Circuit Analysis Method
(KVL and KCL method)

Review

Lumped Matter Discipline LMD:

Constraints we impose on ourselves to simplify our analysis

$$\frac{\partial \phi_B}{\partial t} = 0$$

Outside elements

$$\frac{\partial q}{\partial t} = 0$$

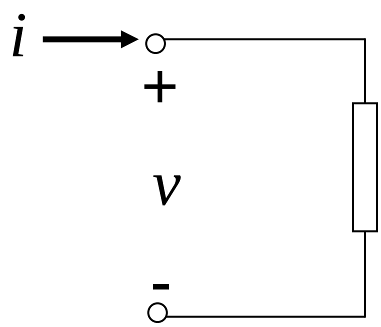
Inside elements

wires resistors sources

Allows us to create the lumped circuit abstraction

Review

LMD allows us to create the lumped circuit abstraction



Lumped circuit element

power consumed by element = vi

Review

Maxwell's equations simplify to algebraic KVL and KCL under LMD!

KVL:

$$\sum_j v_j = 0$$

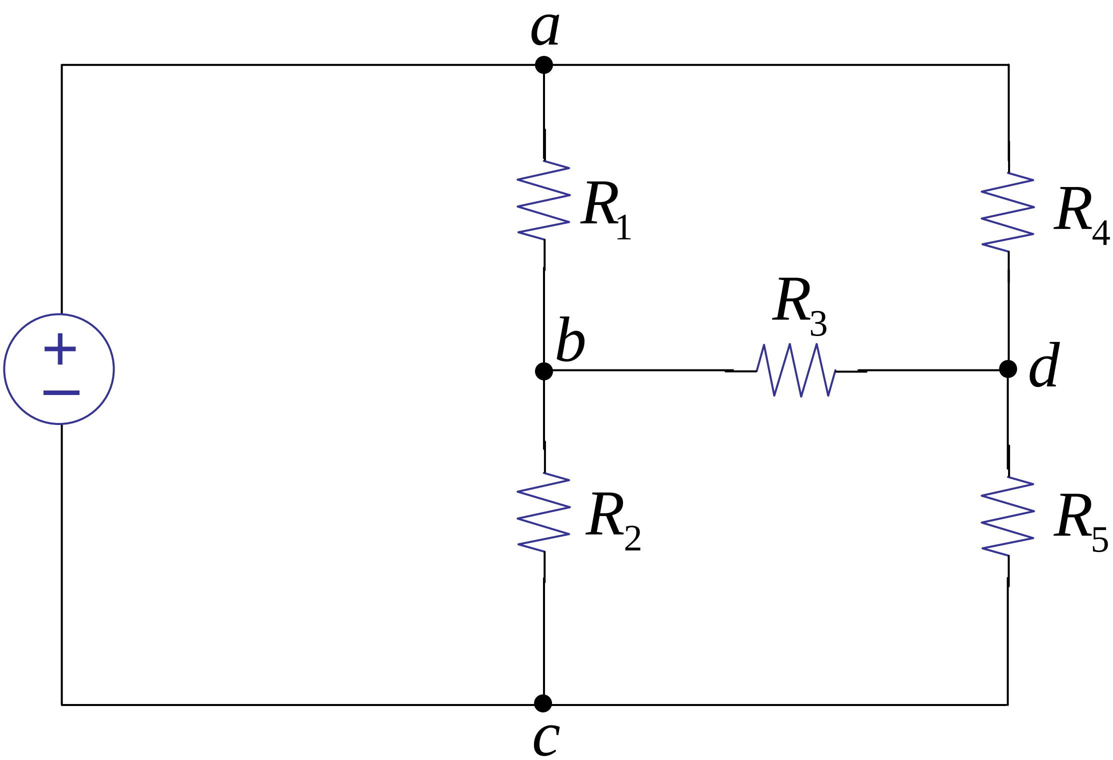
loop

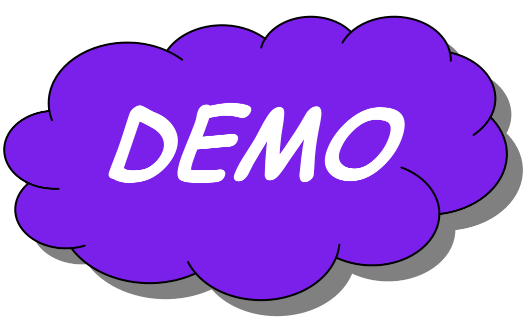
KCL:

$$\sum_j i_j = 0$$

node

Review





$$v_{ca} + v_{ab} + v_{bc} = 0 \quad \text{KVL}$$

$$i_{ca} + i_{da} + i_{ba} = 0 \quad \text{KCL}$$

Method 1: Basic KVL, KCL method of Circuit analysis

Goal: Find all element v's and i's

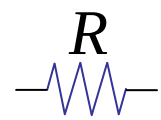
1. write element v-i relationships (from lumped circuit abstraction)
2. write KCL for all nodes
3. write KVL for all loops

lots of unknowns
lots of equations
lots of fun
solve

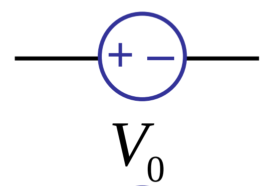
Method 1: Basic KVL, KCL method of Circuit analysis

Element Relationships

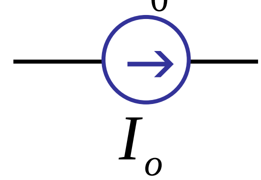
For R, $V = IR$



For voltage source, $V = V_0$

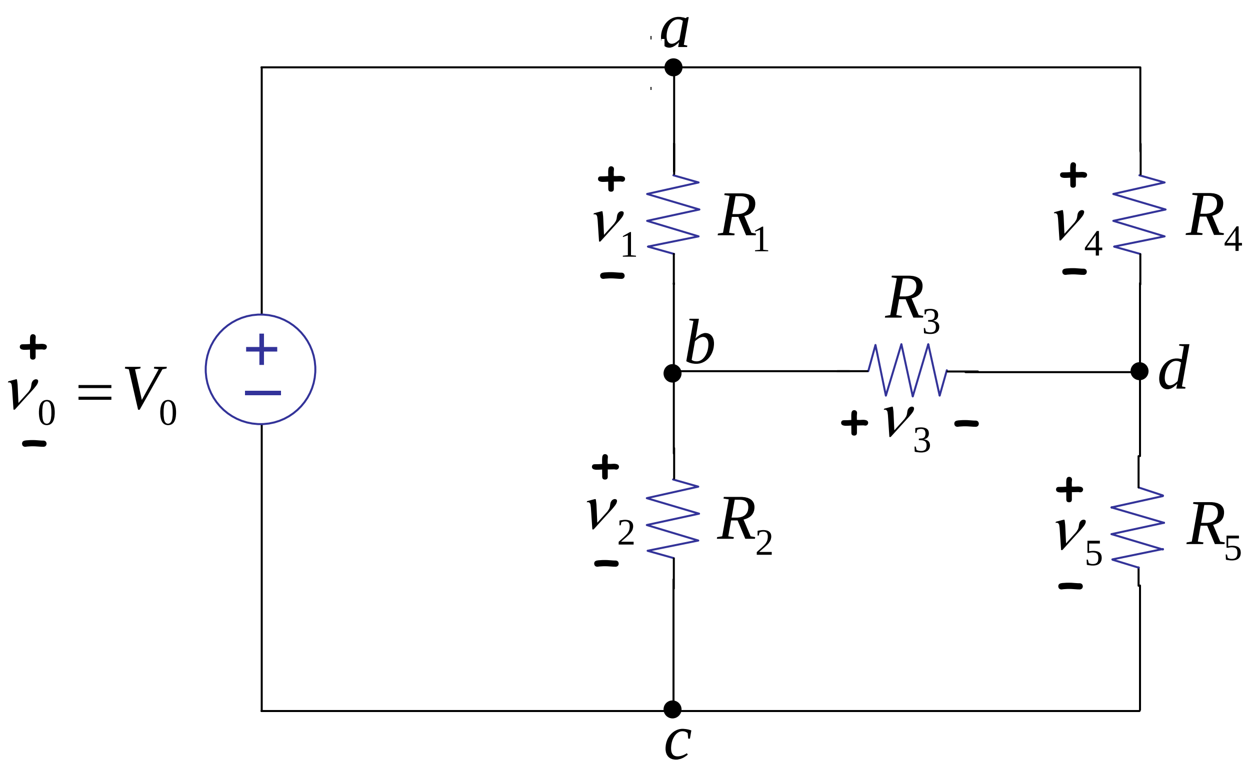


For current source, $I = I_0$



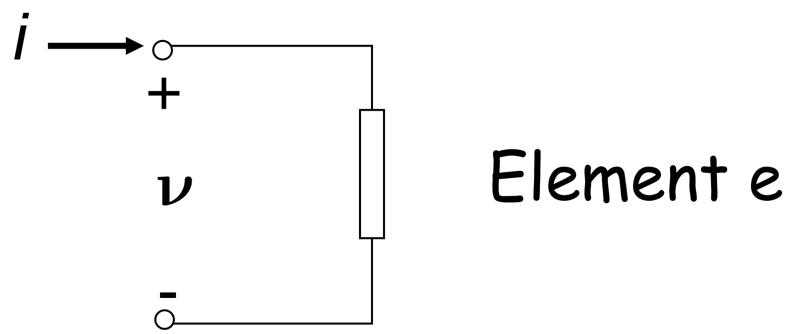
3 lumped circuit elements

KVL, KCL Example



The Demo Circuit

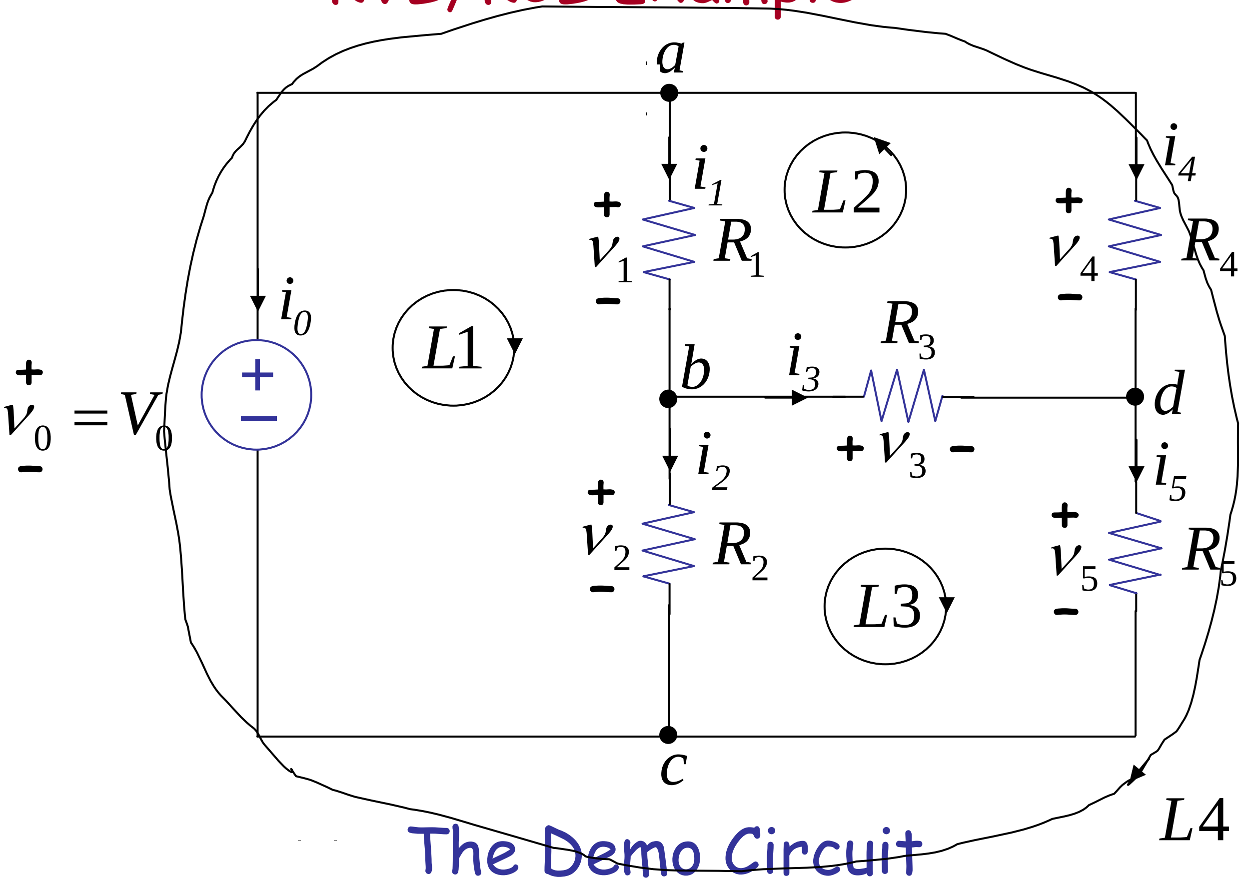
Associated variables discipline



Current is taken to be positive going into the positive voltage terminal

$$\left.\begin{array}{l}\text{Then power consumed}\\ \text{by element e}\end{array}\right\} = vi \text{ is positive}$$

KVL, KCL Example



The Demo Circuit

Analyze

$v_0 \ldots v_5, i_0 \ldots i_5$ — 12 unknowns

1. Element relationships (v,i)

$v_0 = V_0$ ⟵ given — $v_3 = i_3 R_3$

$v_1 = i_1 R_1$ — $v_4 = i_4 R_4$

$v_2 = i_2 R_2$ — $v_5 = i_5 R_5$

6 equations

2. KCL at the nodes

a: $i_0 + i_1 + i_4 = 0$

b: $i_2 + i_3 - i_1 = 0$

d: $i_5 - i_3 - i_4 = 0$

e: $-i_0 - i_2 - i_5 = 0$ redundant

3 independent equations

3. KVL for loops

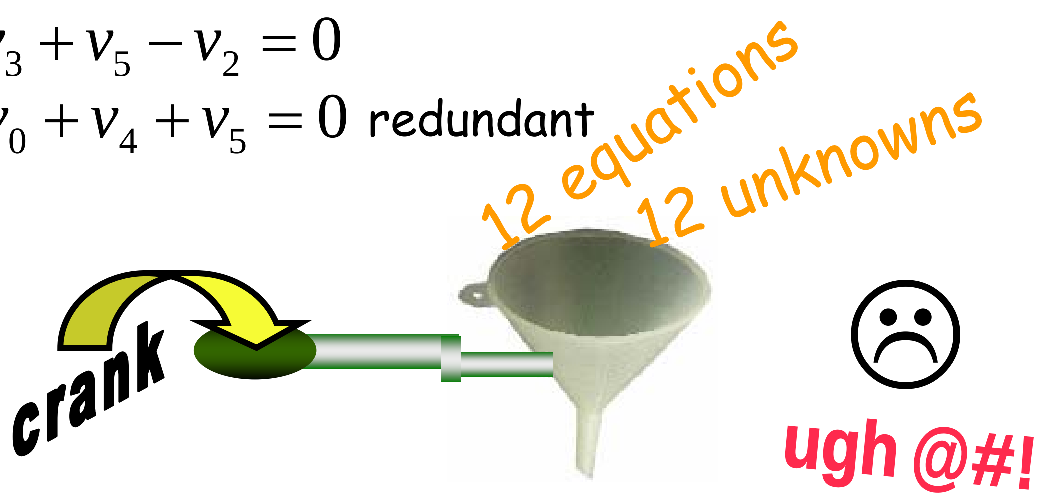
L1: $-v_0 + v_1 + v_2 = 0$

L2: $v_1 + v_3 - v_4 = 0$

L3: $v_3 + v_5 - v_2 = 0$

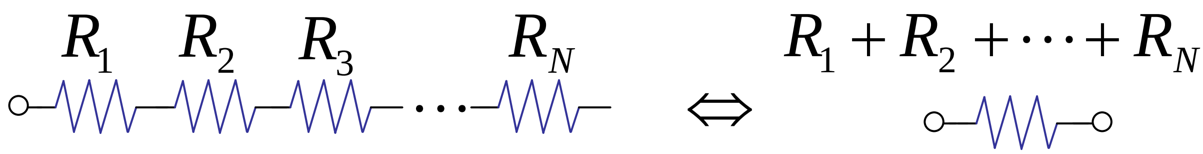
L4: $-v_0 + v_4 + v_5 = 0$ redundant

3 independent equations

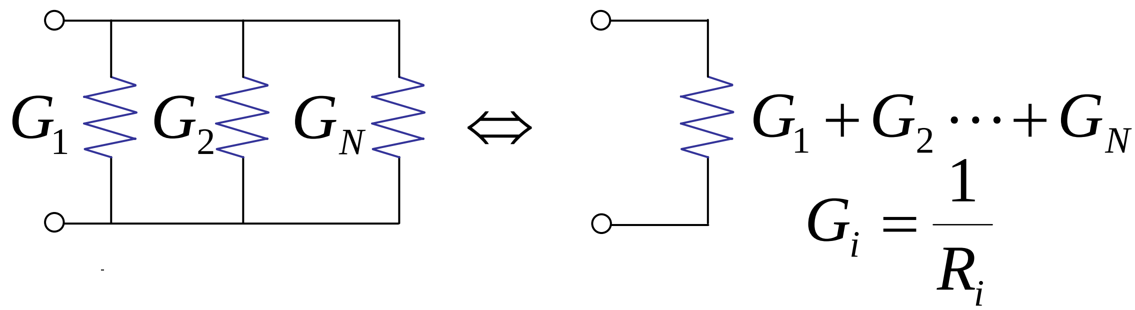


Other Analysis Methods
Method 2— Apply element combination rules

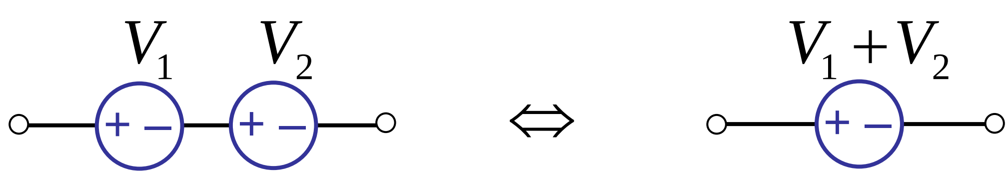
A



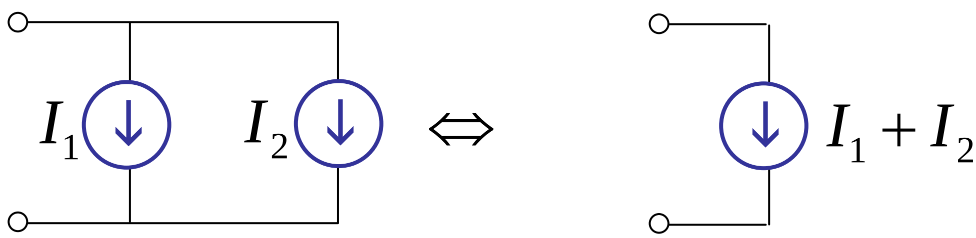
B



C



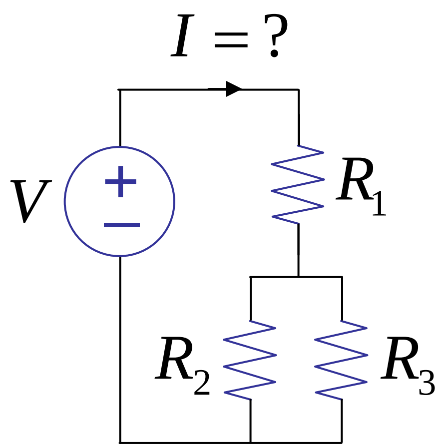
D

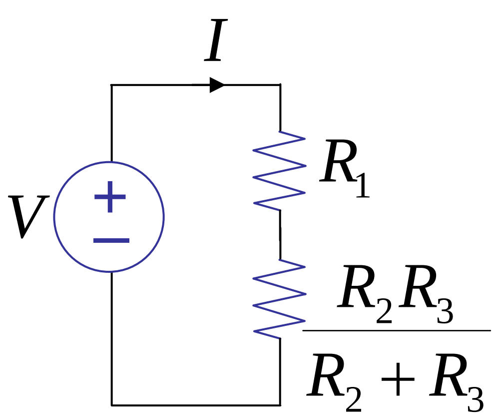


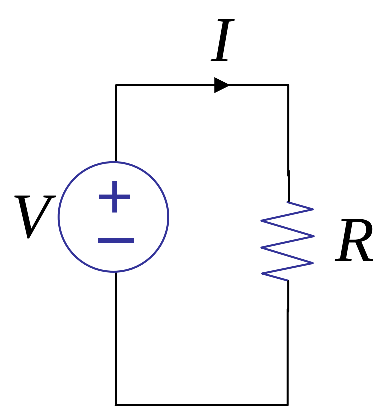
Surprisingly, these rules (along with superposition, which you will learn about later) can solve the circuit on page 8

Other Analysis Methods
Method 2— Apply element combination rules

Example







$$R = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

$$I = \frac{V}{R}$$

Method 3—Node analysis
Particular application of KVL, KCL method

1. Select reference node (⊥ ground) from which voltages are measured.

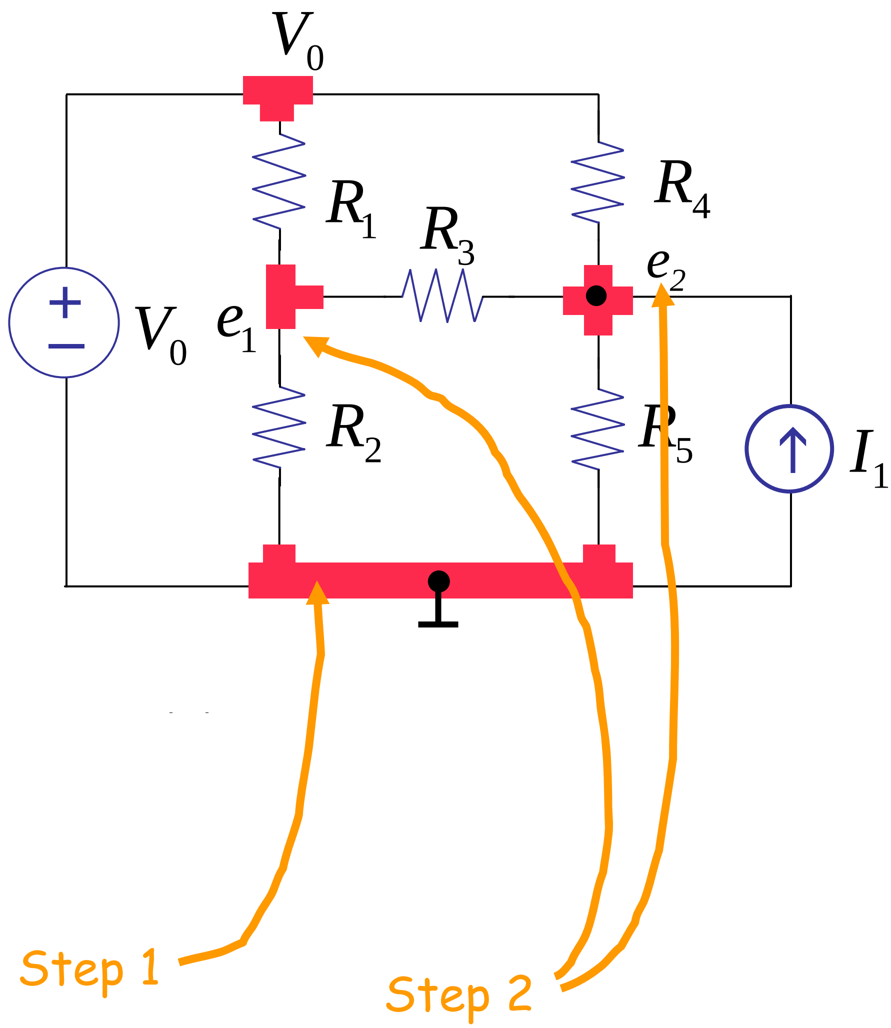
2. Label voltages of remaining nodes with respect to ground.
   These are the primary unknowns.

3. Write KCL for all but the ground node, substituting device laws and KVL.

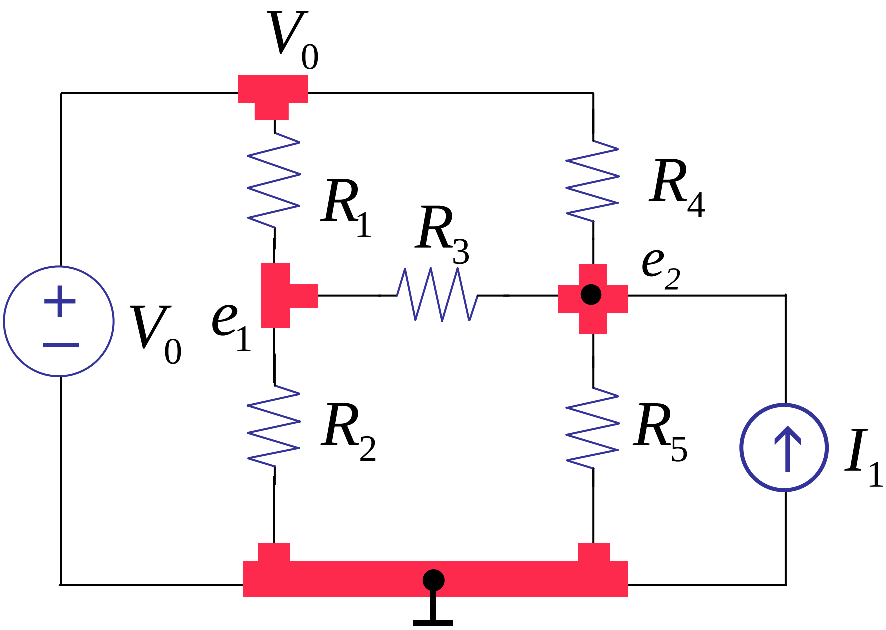
4. Solve for node voltages.

5. Back solve for branch voltages and currents (i.e., the secondary unknowns)

Example: Old Faithful plus current source



Example: Old Faithful plus current source



for convenience, write

$$G_i = \frac{1}{R_i}$$

KCL at e_1

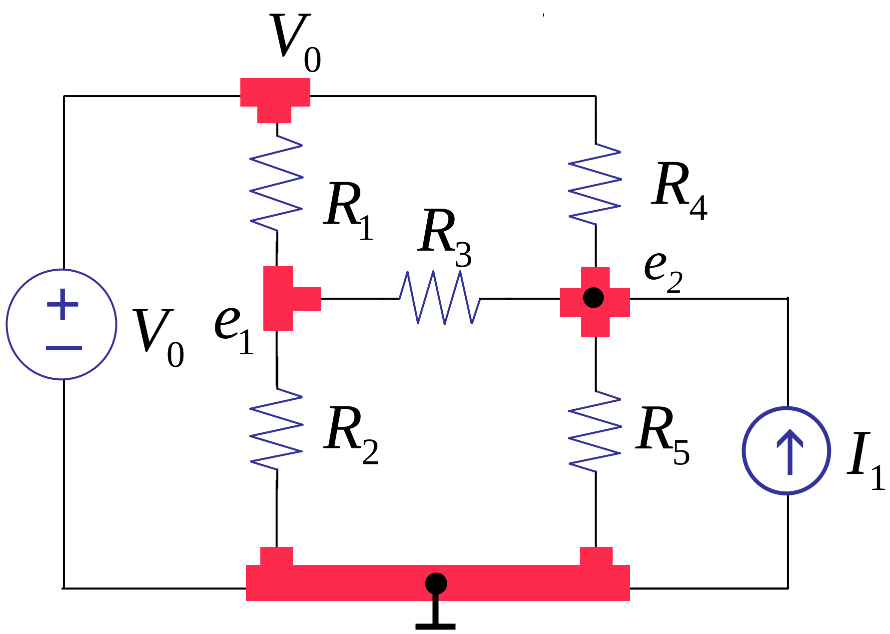
$$(e_1 - V_0)G_1 + (e_1 - e_2)G_3 + (e_1)G_2 = 0$$

KCL at e_2

$$(e_2 - e_1)G_3 + (e_2 - V_0)G_4 + (e_2)G_5 - I_1 = 0$$

Step 3

Example: Old Faithful plus current source



$$G_i = \frac{1}{R_i}$$

KCL at e_1

$$(e_1 - V_0)G_1 + (e_1 - e_2)G_3 + (e_1)G_2 = 0$$

KCL at l_2

$$(e_2 - e_1)G_3 + (e_2 - V_0)G_4 + (e_2)G_5 - I_1 = 0$$

move constant terms to RHS & collect unknowns

$$e_1(G_1 + G_2 + G_3) + e_2(-G_3) = V_0(G_1)$$

$$e_1(-G_3) + e_2(G_3 + G_4 + G_5) = V_0(G_4) + I_1$$

2 equations, 2 unknowns $\longrightarrow$ Solve for e's

(compare units)

Step 4

In matrix form:

$$\left[\begin{array}{c|c} G_1+G_2+G_3 & -G_3 \\ \hline -G_3 & G_3+G_4+G_5 \end{array}\right] \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} G_1V_0 \\ G_4V_0+I_1 \end{bmatrix}$$

conductivity matrix

unknown node voltages

sources

Solve

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \frac{\left[\begin{array}{c|c} G_3+G_4+G_5 & G_3 \\ \hline G_3 & G_1+G_2+G_3 \end{array}\right] \begin{bmatrix} G_1V_0 \\ G_4V_0+I_1 \end{bmatrix}}{(G_1+G_2+G_3)(G_3+G_4+G_5)-G_3^{\ 2}}$$

$$e_1 = \frac{(G_3+G_4+G_5)(G_1V_0)+(G_3)(G_4V_0+I_1)}{G_1G_3+G_1G_4+G_1G_5+G_2G_3+G_2G_4+G_2G_5+G_3^{\ 2}+G_3G_4+G_3G_5}$$

$$e_2 = \frac{(G_3)(G_1V_0)+(G_1+G_2+G_3)(G_4V_0+I_1)}{G_1G_3+G_1G_4+G_1G_5+G_2G_3+G_2G_4+G_2G_5+G_3^{\ 2}+G_3G_4+G_3G_5}$$

(same denominator)

Notice: linear in V_0, I_1, no negatives in denominator

Solve, given

$$\left.\begin{matrix} G_1 \\ G_5 \end{matrix}\right\} = \frac{1}{8.2K} \qquad \left.\begin{matrix} G_2 \\ G_4 \end{matrix}\right\} = \frac{1}{3.9K} \qquad G_3 = \frac{1}{1.5K}$$

$$I_1 = 0$$

$$e_2 = \frac{G_3 G_1 V_0 + (G_1 + G_2 + G_3)(G_4 V_0 + I_1)}{(G_1 + G_2 + G_3) + (G_3 + G_4 + G_5) - G_3^2}$$

$$G_1 + G_2 + G_3 = \frac{1}{8.2} + \frac{1}{3.9} + \frac{1}{1.5} = 1$$

$$G_3 + G_4 + G_5 = \frac{1}{1.5} + \frac{1}{3.9} + \frac{1}{8.2} = 1$$

$$e_2 = \frac{\frac{1}{8.2} \times \frac{1}{1.5} + 1 \times \frac{1}{3.9}}{1 - \frac{1}{1.5^2}} V_0$$

$$e_2 = 0.6 V_0$$

Check out the DEMO

If $V_0 = 3V$, **then** $e_2 = 1.8 V_0$