# Massachusetts Institute of Technology
## Department of Electrical Engineering and Computer Science

## 6.002 – Circuits & Electronics
## Spring 2007

## Homework #4
## Handout S07-023

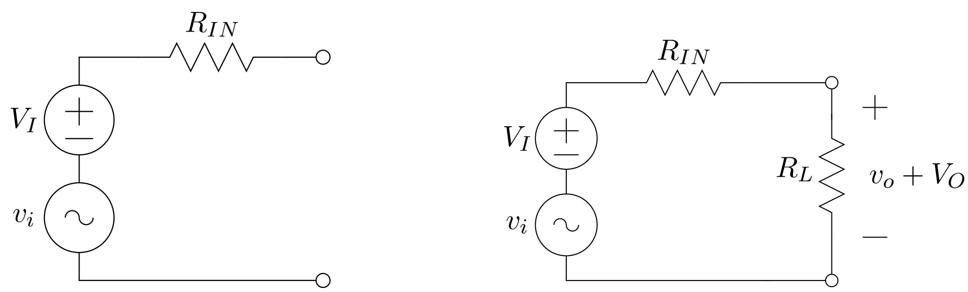
**Issued 03/01/2007 – Due 03/09/2007**

Helpful readings for this homework: Chapter 4, Chapter 6.1-6.6, 6.10.

**Exercise 4.1:** Exercise 4.3 from Chapter 4 of A&L (page 231).

**Exercise 4.2:** Exercise 6.5 from Chapter 6 of A&L (page 323). Use Figure 6.59(c) instead and assume that $R_6 = R_7 = 10k\Omega$.

**Problem 4.1:** A voltage source can be represented as the series connection of an ideal DC voltage source and a resistance $R_{IN} = 1k\Omega$, as illustrated in Figure 1a. The model of the voltage source also includes a small signal voltage source $v_i$ to represent the noise generated by the source. Assume $V_I = 10\,V$ and $v_i = 50\,mV$.




(a) Unloaded voltage source. (b) Loaded voltage source.

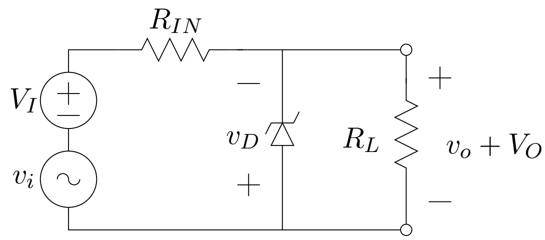
Figure 1: Simple model of a non-ideal voltage source.

In practice, connecting the non-ideal voltage source to a load resistance $R_L$, as in Figure 1b, may result in undesirable effects. This problem studies such effects and how to correct them by introducing a Zener diode into the circuit (Figure 2a).

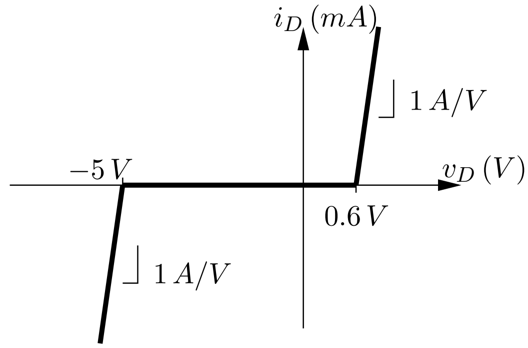
(a). In Figure 1b, calculate $v_o$ (i.e. output noise) and $V_O$ (i.e. DC output voltage) for $R_L = 2\,k\Omega$ and $R_L = 4\,k\Omega$. What can you say about $v_o$ and $V_O$ as a function of $R_L$?

(b). Repeat part (a) for the voltage source in Figure 2a. In this setting, how do $v_o$ and $V_O$ change with $R_L$?

(c). In Figure 2a, what is the minimum value of $R_L$ that would guarantee that the circuit operates as in part (b)?




(a) Voltage source connected to a Zener diode.




(b) $i$–$v$ characteristic for a Zener diode.

Figure 2: The use of a Zener diode can correct some of the undesirable effects in non-ideal voltage sources.

**Problem 4.2:** Consider the circuit containing the nonlinear element $N$ as shown below in Figure 3. The $i$–$v$ relation for the element $N$ is given by

$$i_A = (10\,A)\left(1 - \mathrm{e}^{-v_A/5V}\right) \tag{1}$$

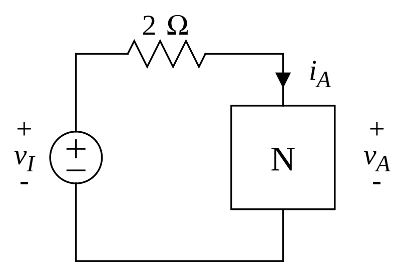



Figure 3: Circuit used in Problem 4.2.

(a). Write an equation which relates the voltage $v_A$ to the input voltage $v_I$.

(b). Solve for the voltage $v_A$ when $v_I = 10\,V$. Note that this requires that you solve the equation in part (a) iteratively for $v_A$. (*Hint: Use the exponential term to solve for $v_A$ as a function of the assumed value of $v_A$, and then iterate. Taking logs on both sides may facilitate convergence.*)

(c). Find the incremental change in $v_A$ for a 2% increase in $v_I$ and calculate the ratio $\Delta v_A/\Delta v_I$.

(d). Find the value for the incremental resistance of the nonlinear element $N$ by linearizing the expression for $i_A$ about the operating point when $v_I = 10\,V$.

(e). Draw the incremental circuit model for the circuit shown in Figure 3.

(f). Find the ratio $\Delta v_A/\Delta v_I$ from the incremental circuit model and compare it with your exact model from part (c).

**Problem 4.3 (OPTIONAL):** Problem 4.3 from Chapter 4 of A&L (page 233-234). (Hint: You may want to read section 4.4 of A&L on Piecewise Linear Analysis of non-linear devices).