QC1: Quantum Computing with Superconductors

Quantum Computing

Qubits are two level systems
a) Spin states can be true two level systems, or
b) Any two quantum energy levels can also be used

We will call the lower energy state $|0\rangle$ and the higher energy state $|1\rangle$

In general, the wave function can be in a superposition of these two states

$$|\psi\rangle = a|0\rangle + b|1\rangle$$

Computing with Quantum States

- Consider two qubits, each in superposition states

$$|\psi\rangle_A = |0\rangle_A + |1\rangle_A \qquad |\psi\rangle_B = |0\rangle_B + |1\rangle_B$$

- We can re-write these states a single state of the 2-e$^-$ system

$$\begin{aligned} |\psi\rangle &= |\psi\rangle_A |\psi\rangle_B \\ &= \left(|0\rangle_A + |1\rangle_A\right) \otimes \left(|0\rangle_B + |1\rangle_B\right) \\ &= |0\rangle_A |0\rangle_B + |0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B \end{aligned}$$

- All four “numbers” exist simultaneously
- Algorithm designed so that states interfere to give one “number” with high probability

The Promise of a Quantum Computer

A Quantum Computer …

- Offers exponential improvement in ***speed*** and ***memory*** over existing computers
- Capable of ***reversible computation***
- e.g. Can factorize a 250-digit number in seconds while an ordinary computer will take 800 000 years!

Quantum Information Science and Technology Roadmapping Project

1. Quantum Computing Roadmap Overview
2. Nuclear Magnetic Resonance Approaches
3. Ion Trap Approaches
4. Neutral Atom Approaches
5. Optical Approaches
6. Solid State Approaches
7. Superconducting Approaches
8. "Unique" Qubit Approaches
9. The Theory Component of the Quantum Information Processing and Quantum Computing Roadmap

http://qist.lanl.gov

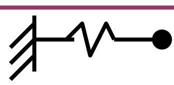
Outline

1. **Introduction to Quantum Computation**
   1.The Unparallel Power of a Quantum Computer
   2.Two state systems: qubits
   3.Types of Qubits
2. **Quantum Circuits**
3. **Building a Quantum Computer with Superconductors**
   1.Quantizing Superconducting Josephson Circuits
   2.Dynamics of Two-Level Quantum Systems
   3.Types of Superconducting qubits
   4.Experiments on superconducting qubits
      1. Charge qubits
      2. Phase/Flux qubits
      3. Hybrid qubits
      4. Advantages of superconductors as qubits

Circuits for Qubits

- Need to find circuits (dissipationless) which have two "good" energy levels
- Need to be able to "manipulate" qubits and couple them together

Harmonic Oscillator



$$H = \frac{1}{2} m v^2 + \frac{1}{2} m \omega^2 x^2$$

$$v = \frac{dx}{dt}$$

$$H = \frac{1}{2} m \left(\frac{dx}{dt} \right)^2 + \frac{1}{2} m \omega^2 x^2$$

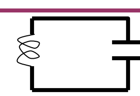
$$p = m \frac{dx}{dt}$$

Quantum Mechanically

$$\Delta x \Delta p \geq \hbar / 2$$

$$E = \hbar \omega \left(n + \frac{1}{2} \right)$$

LC Circuit



$$H = \frac{1}{2} C V^2 + \frac{1}{2} L I^2$$

$$v = \frac{d\Phi}{dt} \quad and \quad I = \frac{\Phi}{L}$$

$$H = \frac{1}{2} \underbrace{C}_{M} \left(\frac{d\Phi}{dt} \right)^2 + \frac{1}{2} \underbrace{C}_{M} \underbrace{\frac{1}{LC}}_{\omega^2} \Phi^2$$

$$p = C \frac{d\Phi}{dt} = C V$$

Quantum Mechanically

$$L C \Delta I \Delta V \geq \hbar / 2$$

$$E = \hbar \omega \left(n + \frac{1}{2} \right)$$

Outline

Quantization of Circuits

1. Find the energy of the circuit
2. Change the energy into the Hamiltonian of the circuit by identifying the canonical variables
3. Quantize the Hamiltonian
   - Usually we can make it look like a familiar quantum system

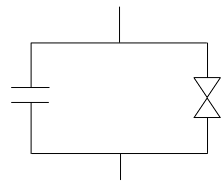
Quantization of a Josephson Junction

Charging Energy

$$U_C = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$

$$= \frac{1}{2}\left(\frac{\Phi_0}{2\pi}\right)^2 C\left(\frac{\partial\varphi}{\partial t}\right)^2$$

$$E_C = \frac{e^2}{2C}$$



Josephson Energy

$$U_J = \frac{\Phi_0 I_c}{2\pi}(1-\cos\varphi)$$

$$E_J = \frac{\Phi_0 I_c}{2\pi}$$

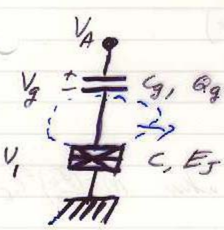
Hamiltonian: $$H = \frac{1}{2}\left(\frac{\Phi_0}{2\pi}\right)^2 C\left(\frac{\partial\varphi}{\partial t}\right)^2 + \frac{\Phi_0 I_c}{2\pi}(1-\cos\varphi)$$

Circuit behaves just like a physical pendulum.

For Al-Al_2O_3-Al junction with an area of 100x100 nm^2
C = 1fF and I_c=300 nA, which gives E_C=10μeV and E_J=600μeV

To see quantization, Temperature < 300 mK

QUANTUM BOX



$$V_A = V_g + V_1$$

$$Q_g = V_g C_g = C_g(V_A - V_1)$$

$$V_1 = \frac{\Phi_0}{2\pi}\dot{\varphi}_1$$

Electric Energy $\quad T = \frac{1}{2}\sum_k C_k V_k^2 - Q_g V_A$

Magnetic Energy $\quad U = E_J(1-\cos\varphi)$

$$T = \frac{1}{2}C\left(\frac{\Phi_0}{2\pi}\right)\dot{\varphi}^2 + \frac{1}{2}C_g\left(V_A - \frac{\Phi_0}{2\pi}\dot{\varphi}\right)^2 - C_g\left[V_A - \frac{\Phi_0}{2\pi}\dot{\varphi}\right]V_A$$

$$= \frac{1}{2}\left(\frac{\Phi_0}{2\pi}\right)^2 \underbrace{(C+C_g)}_{C_\Sigma}\dot{\varphi}^2 \underbrace{-\frac{1}{2}C_g V_A^2}_{\text{constant}}$$

Proceed with φ as the coordinate:

$$\mathcal{L} = T - V \qquad \text{Lagrangian}$$

$$p = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} \qquad \text{canonical momentum}$$

$$\mathcal{H} = p\dot{\varphi} - \mathcal{L} \qquad \text{"Hamiltonian"}$$

$$\mathcal{L} = \frac{1}{2}\left(\frac{\Phi_0}{2\pi}\right)^2 C_\Sigma \dot{\varphi}^2 - E_J(1-\cos\varphi)$$

$$p = \left(\frac{\Phi_0}{2\pi}\right)^2 C_\Sigma \dot{\varphi}$$

$$\boxed{\mathcal{H} = \frac{1}{2}\frac{p^2}{m} + E_J(1-\cos\varphi)} \quad \text{where } m = \left(\frac{\Phi_0}{2\pi}\right)^2 C_\Sigma$$

$$Q_{Dot} = CV_1 - C_g V_g = \frac{2\pi}{\Phi_0} p - C_g V_A$$

IT IS CONVENIENT for physics, but not calculation, to do

$$\mathcal{L}' = \mathcal{L} - \left(\frac{\Phi_0}{2\pi}\right)\dot{\varphi} C_g V_A$$

$$p' = p - \left(\frac{\Phi_0}{2\pi}\right) C_g V_A$$

$$Q_{Dot} = \frac{2\pi}{\Phi_0} p'$$

$$\boxed{\mathcal{H}' = \frac{1}{2m}\left(p' + \frac{\Phi_0}{2\pi} C_g V_A\right)^2 + E_J(1-\cos\varphi)}$$

$$= \frac{1}{2}\frac{(2e)^2}{C_\Sigma}(n - n_g)^2 + E_J(1-\cos\varphi)$$

Both $\mathcal{H}$ & $\mathcal{H}'$ describe the system

QUANTUM Desciption

$$\hat{H} = \frac{\hat{p}^2}{2M} + E_J(1-\cos\hat{\varphi}) + \frac{2\pi}{\Phi_0}\frac{C}{C_\Sigma}V\hat{p} \quad + \text{constant} \to 0$$

Phase Picture $\hat{\varphi} = \varphi \qquad \hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial\varphi} \qquad |\psi\rangle = \psi(\varphi)$

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial\varphi^2} + E_J(1-\cos\varphi) + \frac{2\pi}{\Phi_0}\frac{C}{C_\Sigma}V\left(\frac{\hbar}{i}\frac{\partial}{\partial\varphi}\right)$$

Charge Picture $\hat{p} = \hbar q \qquad \varphi = -\frac{1}{i}\frac{\partial}{\partial q} \qquad |\psi\rangle = \psi(q)$

$$\hat{H} = \frac{\hbar^2 q^2}{2m} + E_J\left[1 - \frac{1}{2}\left(e^{\frac{\partial}{\partial q}} + e^{-\frac{\partial}{\partial q}}\right)\right] + \frac{2\pi}{\Phi_0}\frac{C}{C_\Sigma}V\hbar q$$

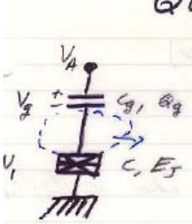
Note: $e^{\frac{\partial}{\partial q}}\psi(q) = \psi(q+1)$

In either picture, we can write

$$|\psi\rangle = \sum_i c_i \phi_i$$

	Phase	Charge
ϕ_i = states of definite p_0 "Charge states"	$e^{i\frac{p_0}{\hbar}\varphi}$	$\delta(\hbar q - p_0)$
ϕ_i = states of definite φ_0 "Phase states"	$\delta(\varphi - \varphi_0)$	$e^{-iq\varphi_0}$

QUANTUM BOX



$V_A = V_g + V_1$

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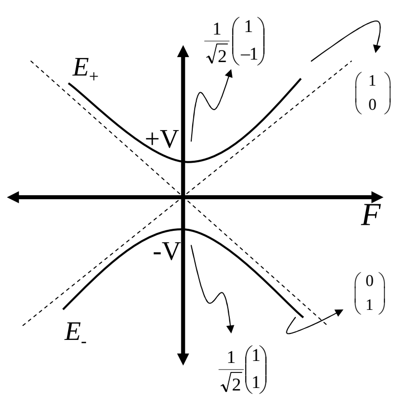
Dynamics Two-Level Quantum Systems

$$H = \begin{pmatrix} -F & -V \\ -V^* & F \end{pmatrix} \quad , \quad |\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$



Eigenenergies $E = \sqrt{F^2 + V^2}$

At F=0, let $|\psi(t=0)\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$|\psi(t)\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{i\frac{V}{\hbar}t} + \frac{1}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-i\frac{V}{\hbar}t}$$

$$= \cos\frac{Vt}{\hbar}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + i\sin\frac{Vt}{\hbar}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

System oscillates between $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ with period $T = \hbar / 2V$

Rabi Oscillations

Drive the system with $V(t)=V_0\, e^{i\omega t}$ at the resonant frequency $\omega = E_+ - E_-$

If $|\psi(t=0)\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, then

$$|\psi(t)\rangle = \cos\frac{V_0 t}{\hbar}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + i\sin\frac{V_o t}{\hbar} e^{i\omega t}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Oscillations between states can be controlled by V_0 and the time of AC drive, with period

$$\mathrm{T} = \frac{h}{2V_0}$$

Charge-State Superconducting Qubit

Images removed for copyright reasons.

Please see: Nakamura, Y., Yu A. Pashkin, and J. S. Tsai. *Nature* **398**, 786 (1999).

Massachusetts Institute of Technology
6.763 2005 Lecture QC1

Charge qubit

a Cooper-pair box $E_J / E_C \sim 0.3$

Coherence up to ~ 5 ns, presently limited by background charge noise (dephasing) and by readout process (relaxation)

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Please see: Nakamura, Y., Yu A. Pashkin, and J. S. Tsai. *Nature* 398, 786 (1999).

Types of Superconducting Qubits

- **Charge-state Qubits** (voltage-controlled)
  - Cooper pair boxes
- **Flux/Phase-state qubits (**flux-current control)
  - Persistent Current Qubits
  - RF SQUID Qubits
  - Phase Qubits (single junction)
- **Hybrid Charge-Phase Qubits**

Three-Junction Loop Measurements

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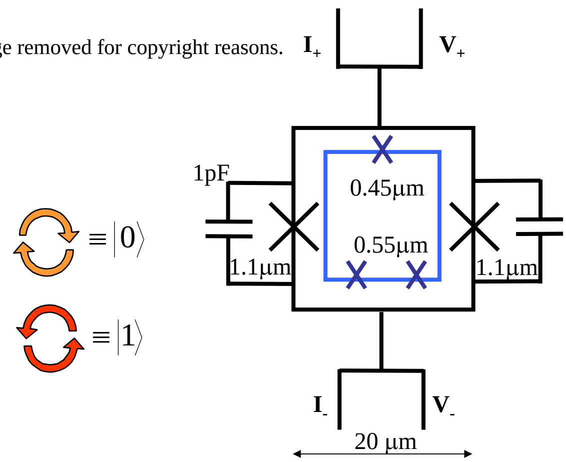


Three-junction Loop
Jct. Size ~ 0.45µm, 0.55µm
Loop size ~16x16µm²
$L_{3\text{-junction}}$ ~ 30pH
I_c~1 & 2µA
E_J/E_c ~ 350 & 550

DC SQUID
Shunt capacitors ~ 1pF
Jct. Size ~ 1.1µm
Loop size ~20x20µm²
L_{SQUID} ~ 50pH
I_c~10 & 20µA
M~35pH
J_c~350 & 730A/cm²

Persistent current qubits require high-quality sub-micron junctions with low current density, and only MIT Lincoln has demonstrated this capability in Nb.

Energy Band Diagram of MIT-LL PC-Qubit: 1-20 GHz

Images removed for copyright reasons.

Observation of Coherent Superposition of Macroscopic States

Jonathan Friedman, Vijay Patel, Wei Chen, Sergey Tolpygo and James Lukens

Images removed for copyright reasons.

Please see: Friedman, J., et al. *Nature* **406**, 43 (July, 2000); and
Van der Wal, C., et al. *Science* 290, 773 (Oct. 2000).

Advantages of Superconductors for Quantum Computing

- Employs lithographic technology
- Scalable to large circuits
- Combined with on-chip, ultra-fast control electronics
  - Microwave Oscillators
  - Single Flux Quantum classical electronics

Circuits Fabricated at Lincoln Laboratory

Images removed for copyright reasons.

SQUID on-chip oscillator and qubit

Images removed for copyright reasons.

On-chip oscillator couples to qubit: No spectroscopy yet due to high temperature

Feasibility of Superconductive Control Electronics Fabrication

Image removed for copyright reasons.

On-chip Control for an RF-SQUID

M.J. Feldman, M.F. Bocko, Univ. of Rochester

www.ece.rochester.edu/~sde/

Images removed for copyright reasons.

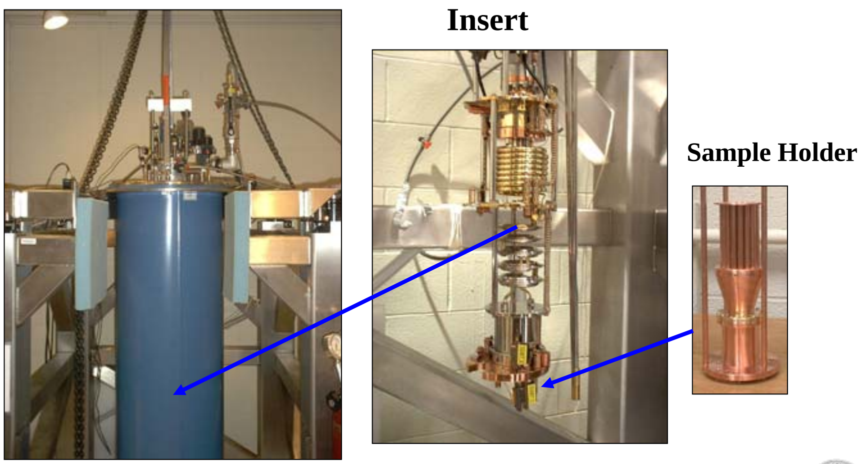
Quantum Computation with Superconducting Quantum Devices

T.P. Orlando, Yang Yu, D.Nakada, B. Singh, J. Lee, D. Berns,
Ken Segall, D. Crankshaw, B Cord- MIT

ARDA

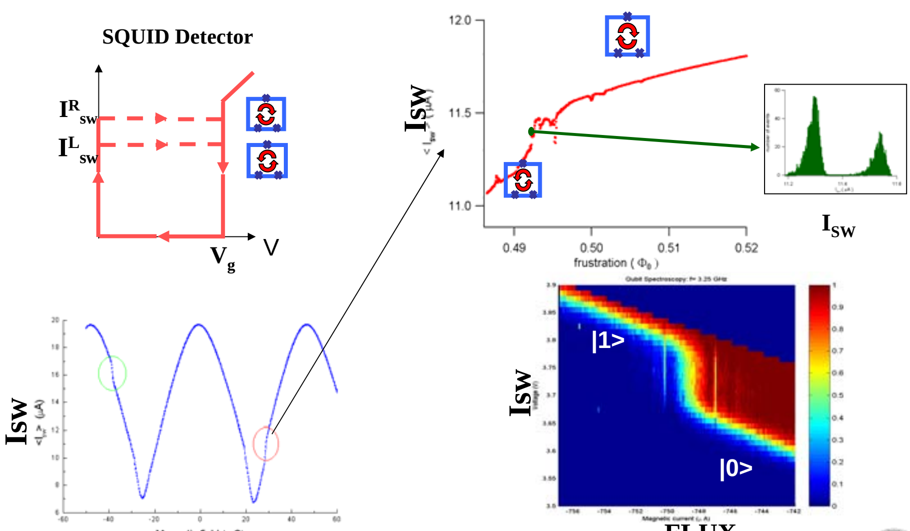
1/27/03

Dilution Refrigerator

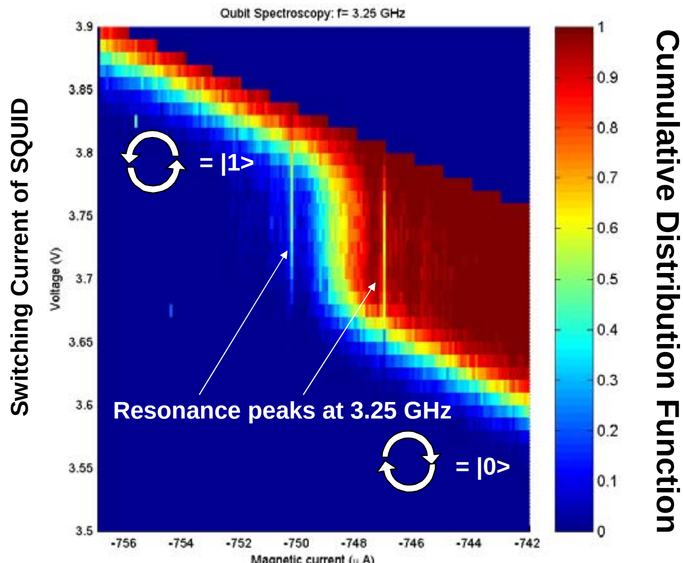


Installed and to begin dc data taking in February and ac data taking in April

DC Measurement on Nb Persistent Current Qubit



Cumulative Distribution Function: Weakly-Driven limit, 3.25 GHz



Yang Yu (MIT) and
Will Oliver (MIT-LL)

Magnetic Flux

T= 25 mK

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CHARGE-FLUX QUBIT

Quantronics Group
CEA-Saclay
France

M. Devoret (now at Yale)
D. Esteve, C. Urbina
D. Vion, H. Pothier
P. Joyez, A. Cottet

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Coherence time measured by Ramsey fringes : 500ns
Qubit transition frequency: 16.5 GHz; coherence quality factor: 25 000

RAMSEY FRINGES

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