6.851 Advanced Data Structures (Spring'10)

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Problem 8 Sample Solutions

Cuckoo Hashing. Consider a cuckoo graph with t edges. We have the probability of any particular cuckoo graph to be $\frac{2^t}{m^{2t}}$. There are $\leq n^t m^t$ configurations of that graph that contain a cycle (there must be a selection of $\leq n$ elements with their hash values on the cycle).

Hence, the probability that a cuckoo graph with t edges contains a cycle is $\leq \frac{n^t m^t 2^t}{m^{2t}} = \frac{1}{3^t}$. Since $\sum_{t=2}^{\infty} \frac{1}{3^t} = \frac{1}{2}$, we conclude that the probability of a cuckoo graph containing no cycles is at least $1/2$.

Conditional Expectations. Consider the following greedy algorithm. We start with two empty sets A and B. Now we consider all the vertices one by one. For each vertex v_i we decide to add it to the set A if there are less edges between v_i and vertices in A then edges between v_i and vertices in B. Otherwise, we add v_i to B. This is equivalent to choosing where to put v_i by maximizing the conditional expectation on the size of the cut given $A \subseteq V'$ and $B \subseteq V \backslash V'$. This way we will end up with at least half of the edges having one vertex in A the other in B. Setting $V' = A$ we get a cut with the value $\geq |E|/2$.

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Spring 2010