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1.010 Uncertainty in Engineering
Fall 2008

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1.010 Fall 2008
Homework Set #10
Due December 4, 2008 (in class)

1. Random variables $X_1, \ldots, X_{50}$ are independent with the following distributions:

$X_1, \ldots, X_{10} \sim$ U[0, 1]
$X_{11}, \ldots, X_{20} \sim$ N(1, 0.1)
$X_{21}, \ldots, X_{30} \sim$ LN(3, 0.2)
$X_{31}, \ldots, X_{40} \sim$ Ex[0.4]
$X_{41}, \ldots, X_{50} \sim$ Ga(2, 0.2)

where U[0, 1] is the uniform distribution in [0,1], N(m, σ^2) and LN(m, σ^2) are the normal and lognormal distributions with mean value m and variance σ^2, Ex[m] is the exponential distribution with mean value m, and Ga(n, m) is the gamma distribution that results from adding n iid exponential variables with distribution Ex[m]. Find in approximation the probability that $Y = \sum_{i=1}^{50} X_i$ exceeds 56.

2. The daily SO_2 concentration at a given location is normally distributed with mean 0.03 ppm (parts per million) and coefficient of variation 40%. Clean air standards require that: a) the daily SO_2 concentration does not exceed 0.06 ppm, and b) the weekly average SO_2 concentration does not exceed 0.045 ppm. Assuming that SO_2 concentrations in different days are statistically independent, determine which of the above criteria is more likely to be violated. Would correlation between daily concentrations lead you to a different conclusion? Explain using qualitative arguments.

3. Water flow into a reservoir is contributed by two rivers. During the spring season, the flows from the rivers, Q_1 and Q_2, the water demand Y, and the stored water volume in the reservoir at the beginning of the season S have multivariate normal distribution

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Y \\ S \end{bmatrix} \sim \mathrm{N}\left(\begin{bmatrix} 12 \\ 8 \\ 25 \\ 10 \end{bmatrix}, \begin{bmatrix} 5^2 & 8 & 5 & 3 \\ 8 & 4^2 & 5 & 2.5 \\ 5 & 5 & 5^2 & 0 \\ 3 & 2.5 & 0 & 2^2 \end{bmatrix} \right)$$

where Q_1, Q_2, Y and S are in million cubic feet. At the beginning of one spring season S=8. Find the probability of not meeting demand at the end of the season, i.e. find $P[(Q_1+Q_2-Y|S=8)<-8]$.