Problem Set #4 Solution 1.050 Solid Mechanics
Fall 2004

Problem 4.1

An automobile tire normally requires internal pressure from 24-36 psi. This pressure might be higher under certain situations. We will take an internal pressure as 32 psi for this problem. Let's say that a soda can have 3 inches diameter. We get that:

$$\sigma_a = p_i\left(\frac{R}{2t}\right)$$

$$\sigma_a = 32\left(\frac{1.5}{2\cdot 0.0025}\right) = 9600\ psi$$

$$\sigma_\theta = p_i\left(\frac{R}{t}\right)$$

$$\sigma_\theta = 32\left(\frac{1.5}{0.0025}\right) = 19200\ psi$$

Problem 4.2

In this problem, you can solve it either using mathematic equation or drawing a Mohr's circle. I will provide the mathematic solution here (drawing a Mohr's circle is an acceptable method and relatively easy and quick to do as well).

The stress transformation equations

$$\sigma'_x = \left[\frac{\sigma_x+\sigma_y}{2}\right]+\left[\frac{\sigma_x-\sigma_y}{2}\right]\cos 2\phi+\sigma_{xy}(\sin 2\phi)$$

$$\sigma'_y = \left[\frac{\sigma_x+\sigma_y}{2}\right]-\left[\frac{\sigma_x-\sigma_y}{2}\right]\cos 2\phi-\sigma_{xy}(\sin 2\phi)$$

$$\sigma'_{xy} = -\left[\frac{\sigma_x-\sigma_y}{2}\right]\sin 2\phi+\sigma_{xy}(\cos 2\phi)$$

We get that, when ϕ = 30 degree,

$$\sigma'_x = \left[\frac{6+(-2)}{2}\right]+\left[\frac{6-(-2)}{2}\right]\cos 60+4(\sin 60) = 7.464$$

$$\sigma'_y = \left[\frac{6+(-2)}{2}\right]-\left[\frac{6-(-2)}{2}\right]\cos 60-4(\sin 60) = -3.464$$

$$\sigma'_{xy} = -\left[\frac{6-(-2)}{2}\right]\sin 60+4(\cos 60) = -1.464$$

Knowing that the shear stress component will vanish on planes that yield maximum and minimum normal stress components, we get that:

$$\sigma'_{xy} = -\left[\frac{6-(-2)}{2}\right]\sin 2\phi+4(\cos 2\phi) = 0$$

$$-4(\sin 2\phi)+4(\cos 2\phi) = 0$$

$$\tan 2\phi = 1$$

So ϕ = 22.5 degree.

Problem 4.3

A thin walled glass tube of radius R = 1 inch, and wall thickness t= 0.05 inches, is closed at both ends and contains a fluid under pressure, p = 80 psi. A torque, M_t, of 300 inch-lbs, is applied about the axis of the tube.

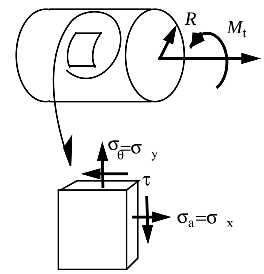
Compute the stress components relative to a coordinate frame with its x axis in the direction of the tube's axis, its y axis circumferentially directed and tangent to the surface.

Determine the maximum tensile stress and the orientation of the plane upon which it acts.

For this problem, we know that $R = 1$ inch, $t = 0.05$ inches, $p_i = 80\ psi$, and $M_t = 300$ *inch-lbs*. Figure shows the sketch of the tube and an element subjected to the stresses caused by the internal pressure and the applied torque.

If we assume the torque produces a force per unit length, f_R, uniformly distributed around the circumference, we have, from moment equilibrium about the axis of the can: $M_t = 2\pi R\cdot f_R\cdot R$

Now if we also assume the force per unit length of the circumference is uniformly distributed across the thickness of the can we have

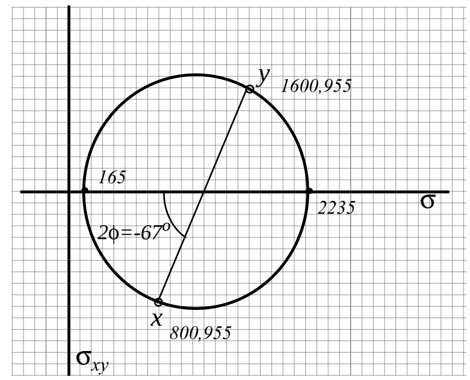


$$\tau = (f_R/t) = \frac{M_t}{2\pi R^2 \cdot t} = \frac{300}{2\pi 1^2 \cdot 0.05} = 955\,psi$$

The axial stress and the hoop stress components are:

$$\sigma_a = p_i \cdot \left(\frac{R}{2t}\right) = 80(10) = 800\,psi$$

$$\sigma_\theta = p_i \cdot \left(\frac{R}{t}\right) = 80(20) = 1600\,psi$$



From the stress transformation equations and the fact that the shear stress at the planes which have maximum and minimum normal stress is zero, we get that

$$0 = -\left(\frac{800 - 1600}{2}\right)\sin 2\phi + 955\cos 2\phi$$

so $2\phi = -67^o$, $\phi = -33.6^o$

$$so \qquad \tan 2\phi = \frac{-955}{400} = -2.4$$

The extreme values for the tensile stress is , substituting into the transformation relationship for σ_x' and σ_y'

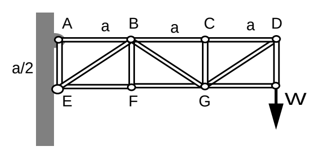
$$\left|\sigma'_x\right|_{extreme} = \left[\frac{(800+1600)}{2}\right] + \left[\frac{(800-1600)}{2}\right] \cdot \cos 67 + 955\sin(-67) = 165\,psi$$

$$\left|\sigma'_y\right|_{extreme} = \left[\frac{(800+1600)}{2}\right] - \left[\frac{(800-1600)}{2}\right] \cdot \cos 67 - 955\sin(-67) = 2235\,psi$$

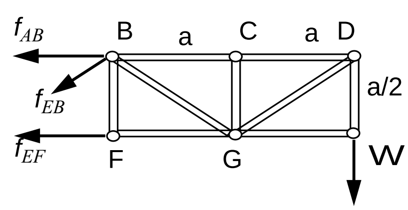
Note the invariance of the sum of the normal stress components (their sum = 2400 psi).

Problem 4.4 (Potential Quiz Question).

Find the axial stress acting in member EF of the end-loaded truss if its cross-sectional area is 0.1 in^2 and W= 1500 lb.



We can solve this problem with but one isolation, as shown at the right. We want the force in member EF so we take moments about pt. B and require the resultant moment to be zero. This gives (cw positive):



$$f_{EF} \cdot (a/2) + W \cdot (2a) = 0$$

So

$$f_{EF} = 4W$$

and the axial stress is then: 4(1500)/(0.1) = 60,000 psi.