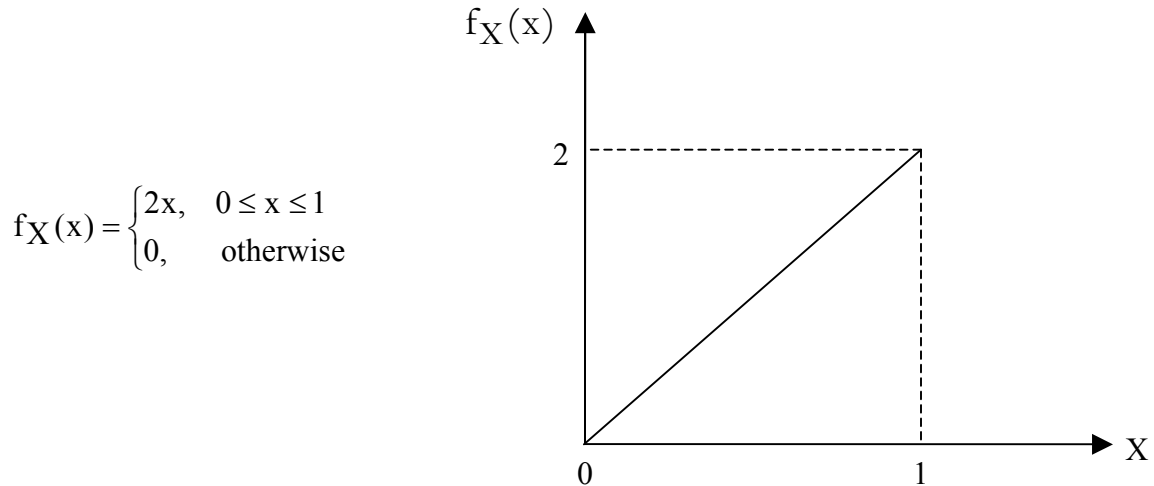


Homework Set #5

Problem 1

X has probability density function as shown below.



Calculate the mean value m_X , variance σ_X^2 and second initial moment $E[X^2]$. Verify the relation $E[X^2] = m_X^2 + \sigma_X^2$.

Problem 2

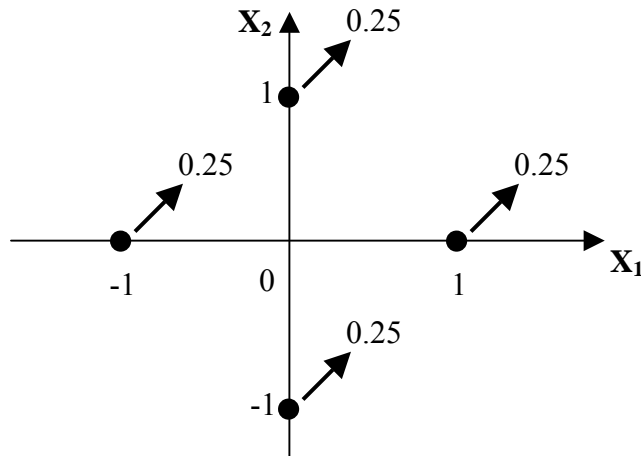
X has uniform distribution between 2 and 3. Consider a new variable $Y = X^3$.

- Sketch the function $Y(X)$.
- Find the probability density function of Y .
- Calculate the mean value and variance of X .
- Using the probability density function found in (b), calculate the mean value and variance of Y .
- Verify that m_Y and σ_Y^2 can be obtained also as

$$m_Y = \int_2^3 x^3 f_X(x) dx \quad \text{and} \quad \sigma_Y^2 = \int_2^3 (x^3 - m_Y)^2 f_X(x) dx.$$

Problem 3

Consider two discrete random variables X_1 and X_2 , with the joint probability mass function shown in the figure below. (Notice that the distribution is concentrated at four points, with equal probability 0.25 at each point).



- Are X_1 and X_2 independent? Briefly explain why or why not.
- Find the mean values m_1 and m_2 , the variances σ_1^2 and σ_2^2 , and the correlation coefficient ρ between X_1 and X_2 .