1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Design Example

Shear and Torsion

Objective: To examine the adequacy of given cross section based on shear and torsion capacities.

Problem: At a section of a beam, the internal forces are V_u = 45 kips, M_u = 300 kips-ft, and T_u = 120 kips-ft. The material strengths are f'_c = 4 ksi and f_y = 60 ksi. Assume that the distance from beam faces to the center of stirrups is 2 in and d is 21 in.

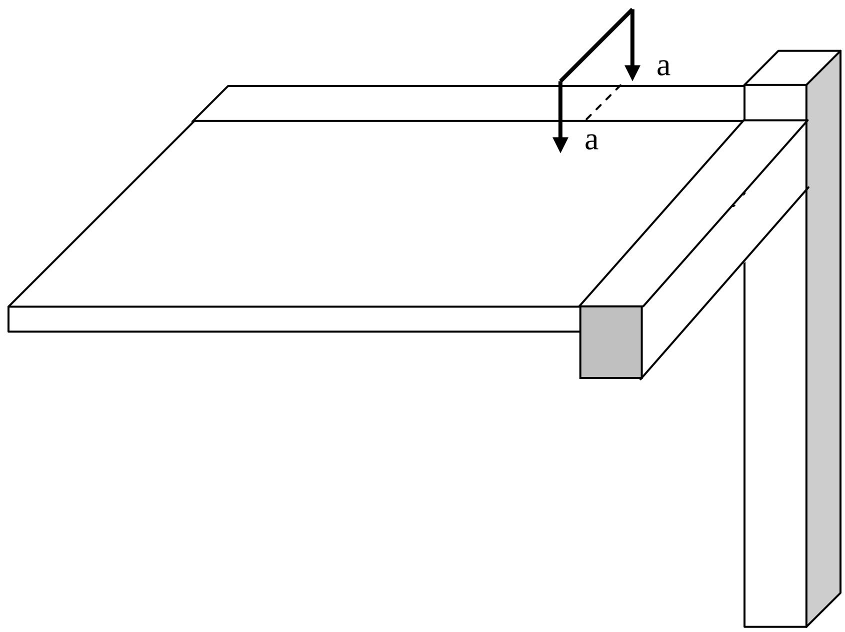

Figure 1. Plan view of overpass

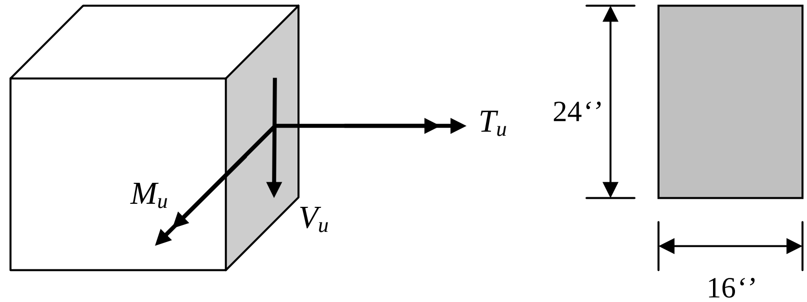

Figure 2. Cross section a-a

Task: Answer or accomplish the following questions and tasks based on given assumption.

- Will this given cross section (16 in wide and 24 in deep) be adequate for shear and torsion requirements? If not, what width is required?
- Assume that the depth is to be held at 24 in.
- Select the required torsion, shear, and bending reinforcement for the minimum required width (to nearest inch).
- Summarize reinforcement on a sketch of the cross section.

[Design Procedures]

1. Check maximum torsion capacity **(ACI)**

For the given section: Section/(Equation)

$$\sum x^2 y = (16)^2(24) = 6144\,\text{in}^3$$

$$C_T = \frac{b_w d}{\sum x^2 y} = \frac{(16)(21)}{6144} = 0.05469\ \text{in}^{-1}$$

$$T_c = \frac{0.8\sqrt{f_c'}\cdot\sum x^2 y}{\sqrt{1+\left(\dfrac{0.4V_u}{C_T T_u}\right)^2}} = \frac{0.8\sqrt{4000}\cdot(6144)}{\sqrt{1+\left(\dfrac{0.4\times 45}{0.05469\times 120\times 12}\right)^2}} = 318.88\times 10^3\ \text{lbs-in}$$

Since $(T_s)_{\max} = 4T_c$ **11.6.1 &**

$$\Rightarrow (T_u)_{\max} = \phi(T_c + T_s) = \phi(T_c + 4T_c) = 4.25T_c \quad (\phi = 0.85\ \text{for torsion})$$ **11.6.2.2**

$$\Rightarrow (T_u)_{\max} = 1355\times 10^3\ \text{lbs-in}$$

However, $(T_u)_{\text{actual}} = 120\ \text{kips-ft} = 1440\times 10^3\ \text{lbs-in}$

$(T_u)_{\text{actual}} > (T_u)_{\max} \Rightarrow$ Section is NOT adequate.

2. Selection of cross section dimensions

Let h = 24 in = constant.

$$(T_u)_{\text{actual}} < 4.25T_c \Rightarrow 1440\times 10^3\ \text{lbs-in} < 4.25\frac{0.8\sqrt{4000}\cdot x^2(24)}{1.02579}$$

Assume C_T to be about the same $\Rightarrow$ same dimension.

$\Rightarrow x > 16.92$ in

Try x =17 in,

$$\sum x^2 y = (17)^2 (24) = 6936 \text{ in}^3,\ C_T = 0.05147 \text{ in}^{-1}.$$

$$T_c = \frac{0.8\sqrt{4000}\cdot(6936)}{\sqrt{1+\left(\frac{0.4\times 45}{0.05147\times 120\times 12}\right)^2}} = 341\times 10^3 \text{ lbs-in}$$

$$\Rightarrow (T_u)_{\max} = 4.25T_c = 1449\times 10^3 \text{ lbs-in} > T_u \qquad \textbf{(O.K.)}$$

$\therefore$ Section $17'' \times 24''$ is OK. (Although predicting heavy reinforcement.)

3. Selection of stirrups

$$T_s = \frac{T_u}{\phi} - T_c = \left(\frac{1440}{0.85} - 341\right)\times 10^3 = 1353\times 10^3 \text{ lbs-in}$$

$$x_1 = 17 - 2(2) = 13 \text{ in}$$

$$y_1 = 24 - 2(2) = 20 \text{ in}$$

$$\alpha_T = 0.66 + 0.33\frac{y_1}{x_1} = 1.168$$

$$\frac{A_t}{s} = \frac{T_s}{\alpha_T x_1 y_1 f_y} = \frac{1353\times 10^3}{1.168(13)(20)(60\times 10^3)} = 0.0743 \text{ in}^2/\text{in}$$

$$\left(\frac{A_t}{s}\right)_{\min} = \frac{25b_w}{f_{yv}} = \frac{(25)(17)}{60000} = 0.00708 \text{ in}^2/\text{in} \qquad \textbf{(O.K.)}$$ **11.6.5.3**

$$V_c = \frac{2\sqrt{f_c'}b_w d}{\sqrt{1+\left(2.5C_T\frac{T_u}{V_u}\right)^2}} = \frac{2\sqrt{4000}(17)(21)}{\sqrt{1+\left(2.5(0.05147)\frac{120\times 12}{45}\right)^2}} = 10657 \text{ lbs}$$

$$V_s = \frac{V_u}{\phi} - V_c = \left(\frac{45}{0.85} - 10.66\right)\times 10^3 = 42.28\times 10^3 \text{ lbs}$$

$$\frac{A_v}{s} = \frac{V_s}{f_y d} = \frac{42.28\times 10^3}{(60\times 10^3)21} = 0.0336 \text{ in}^2/\text{in}$$ **(11-15)**

Since $\left(\frac{A}{s}\right)_{\text{stirrup}} = \frac{A_t}{s} + \frac{1}{2}\frac{A_v}{s} = 0.0743 + \frac{0.0336}{2} = 0.0911 \text{ in}^2/\text{in}$

$$s_{\max} = \frac{x_1 + y_1}{4} = \frac{13+20}{4} = 8.25 \text{ in}$$

$4\sqrt{f_c^{'}}b_w d = 4\sqrt{4000}(17)(21) = 90314 \text{ lbs} > V_s$ **(Not applicable)** **11.5.4.3**

$s_{\max} = \frac{d}{2} = \frac{21}{2} = 10.5 \text{ in}$ **11.5.4.1**

$\phi\left(0.5\sqrt{f_c^{'}}\sum x^2 y\right) = 0.85(0.5)\sqrt{4000}(6936) = 186\times10^3$ lbs-in < T_u **(Applicable)**

$\left(A_v + 2A_t\right)_{\min} = \frac{50 b_w s}{f_y} = \frac{50(17)s}{60\times10^3} = 0.0142s$ **(11-23)**

$$\Rightarrow \left(A_{\text{stirrup}}\right)_{\min} = \frac{0.0142s}{2} = 0.0071s \text{ in}^2$$

Thus, choose A_{stirrup} and *s* to satisfy:

(i) $s > \frac{A}{0.0911}$ in;

(ii) $s < 8.25$ in;

(iii) $A > 0.0071$s in^2.

Try #5 stirrups @ 3.5in, $A = 0.31$ in^2.

$s_{\min} = \frac{A}{0.0911} = 3.4 \text{ in}$ **(O.K.)**

$s_{\max} = 8.25 \text{ in}$ **(O.K.)**

$A_{\min} = 0.0071s = 0.0071(3.5) = 0.0249 \text{ in}^2$ **(O.K.)**

$\Rightarrow$ #5 stirrups @ 3.5in are O.K.

4. Selection of longitudinal reinforcement

$$A_l = 2A_t\left(\frac{x_1 + y_1}{s}\right) = 2\frac{A_t}{s}\left(x_1 + y_1\right) = 2(0.0743)(13+20) = 4.9 \text{ in}^2$$

$A_{l,\min} = \frac{5\sqrt{f_c^{'}}A_{cp}}{f_{yl}} - \frac{A_t}{s} p_h \frac{f_{yv}}{f_{yl}}$ **(11-24)**

A_{cp} is the area enclosed by outside perimeter of concrete cross section, in^2, and p_h is the perimeter of centerline of outermost closed transverse torsional reinforcement, in.

Assuming $f_{yv} = f_{yl}$ (f_{yv}: yield strength of stirrups/ f_{yl}: yield strength of longitudinal steels) and substituting all known numbers ($A_{cp} = 17\times 24 = 408\text{ in}^2$, $p_h = 2(17-4+24-4) = 66\text{ in}$) provides $A_{l,\min} < 0$, therefore (11-24) is disregarded.

Use $\dfrac{50 b_w s}{f_y} < 2A_t$ or $\dfrac{50 b_w}{2 f_y} < \dfrac{A_t}{s}$ to find $2A_t$. **(11-23)**

$$\Rightarrow \frac{50 b_w}{2 f_y} = \frac{50(17)}{2(60\times 10^3)} = 0.0071 < \frac{A_t}{s} = 0.0743$$

Substitute for $2A_t$ in **(11-23)**.

$$A_l = \left[\frac{400x}{f_y}\left(\frac{T_u}{T_u + \dfrac{V_u}{3C_T}}\right) - 2\frac{A_t}{s}\right](x_1 + y_1)$$

$$= \left[\frac{400(17)}{60\times 10^3}\left(\frac{1440\times 10^3}{1440\times 10^3 + \dfrac{45\times 10^3}{3\times 0.05147}}\right) - 2(0.0743)\right](33) < 0$$

This leads to a negative value $\rightarrow$ disregard it!

$\Rightarrow$ <u>A_l = 4.9 in^2 for torsion</u>

$$M_u = \phi\left(0.85 f_c' ab\right)\left(d - \frac{a}{2}\right) \tag{1}$$

$$A_s = \frac{0.85 f_c' ab}{f_y} \tag{2}$$

Eq.(1) provides

$$300\times 12 = 0.9(0.85)(4)a(17)\left(21 - \frac{a}{2}\right) \Rightarrow 26.01a^2 - 1092.42a + 3600 = 0$$

$$\Rightarrow a = 3.605\text{ in}$$

Eq.(2) provides $A_s = 3.47\text{ in}^2 \rightarrow \rho = \dfrac{3.47}{(17)(21)} = 0.0097$

$$A_{\min} = \frac{200}{f_y} = 0.0033\text{ in}^2 < A_s = 3.47\text{ in}^2 \quad \textbf{(O.K.)}$$

In balanced state,

$$C_{\text{balanced}} = \frac{b(0.003)}{0.003+\varepsilon_y} = 12.6\text{ in} \Rightarrow a_{\text{balanced}} = 0.85C_{\text{balanced}} = 10.71\text{ in}$$

$$(A_s)_{\text{balanced}} = (A_s)_b = \frac{0.85(4)(10.71)(17)}{60} = 10.32\ \text{ in}^2$$

$$\Rightarrow \rho_b = \frac{10.32}{(17)(21)} = 0.0289$$

$\rho_{\max} = 0.75\rho_b = 0.0217$ **(O.K.)**

$\Rightarrow$ <u>A_s = 3.47 in^2 for flexure</u>

5. Summary

Use

<u>4 #10 @ the bottom (3.47 in^2 for *M*, 1.61 in^2 for *T*)</u>

<u>2 #6 @ intermediate level (1.50 in^2 for *T*)</u>

<u>3 #6 @ the top (1.79 in^2 for *T*, 0.46 in^2 excess)</u>

We have an excess of steel. In the worse case, we have a moment M_u without T_u. The 5.08in^2 of steel at the bottom can all be considered for flexural tensile reinforcement purpose. In that case,

$\rho = \frac{5.08}{(17)(21)} = 0.0142 < 0.75\rho_b$ (Check for flexural capacity as singly-reinforced section)

which is still O.K.

The results are illustrated in Fig. 3:

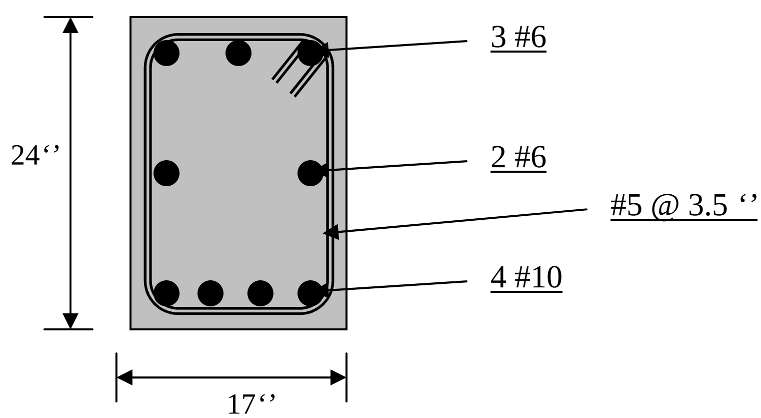

Figure 3. Configuration of the cross section

Comments on the final design:

1. Different design configurations are possible, in general. Various combinations of different sizes of steel bars can achieve same reinforcement ratio. However, relevant designs are made typically considering the convenience of construction and the spacing between any two steel bars (the concrete between two steel bars will crash undesirably if the spacing between them is not enough).
2. Considering the constructability, four corner positions are usually required to deploy longitudinal bars to fix stirrups.
3. It is preferable to use same size of steel bars on each cross section for economic reason unless it is not possible to achieve the requirement of reinforcement.