# Logistical and Transportation Planning Methods

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Quiz #2
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OPEN BOOK
TWO HOURS
5 PAGES, 3 PROBLEMS

PLEASE SHOW ALL YOUR WORK!

INSPIRATIONAL QUOTE:

*"It's tough to make predictions, especially about the future."*
Yogi Berra, once a baseball catcher for the New York Yankees

**<u>Problem 1 (35 points)</u>**

Consider a single-server queueing system which operates as follows:

1. There are two types of customers, Type 1 and Type 2, each type arriving at the system in a Poisson manner at rates $\lambda_1$ and $\lambda_2$ respectively.

2. Customers of type 1 have negative exponential service times with expected value of $1/\mu_1$ per service time. Similarly, customers of type 2 have negative exponential service times with expected value of $1/\mu_2$ per service time. Successive service times are mutually independent.

3. The queueing system has a total capacity of three customers, including the one receiving service. In other words, whenever there are three customers in the system (the one receiving service and two waiting) any additional arriving customers are rejected and go elsewhere.

4. The next customer to be processed by the server is selected in the following way: If, after the completion of a service, there is no other customer waiting, then the next customer to arrive at the system, no matter of what type, obtains access to the server. If, after the completion of a service, there is at least one customer waiting, then customers (if any) of the same type as the customer who just completed service receive priority over the other type. If no customers of the same type are waiting, then a customer of the other type, if any is/are waiting, is admitted for service. Customers of each type are served in a FCFS way.

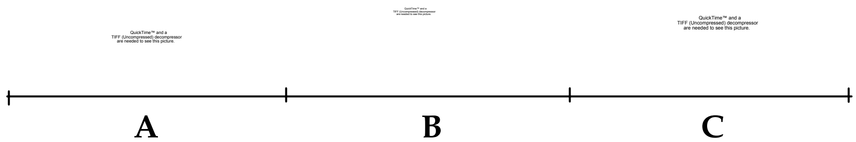
(a) (22 points) Please draw a NEAT state transition diagram for this queueing system. Make sure to define clearly the states and to indicate the transition rates for every possible transition. (Omit the "dt" to keep the picture uncluttered.)

(b) (5 points) Suppose you know that $\lambda_1 = \lambda_2$ and that $\mu_1 = \mu_2$. Let $L_1$ and $L_2$ be, respectively, the expected number of customers of Type 1 and of Type 2 in this queueing system at a random time, when the system is in steady state. How do $L_1$ and $L_2$ compare? Please justify your answer briefly and clearly.

(c) (8 points) Suppose the system is currently occupied by one Type 1 customer and one Type 2 customer, with the Type 1 customer receiving service. What is the probability that the next customer to be served by the server will be a Type 2 customer?

**<u>Problem 2 (35 points)</u>**

Consider three equal-length straight-line police patrol sectors as shown in the figure below. The length of each sector is one mile.




The pictures above the sectors are shown to remind you that we are dealing with police patrol cars, each of which patrols its own sector, **A, B** or **C**, respectively, uniformly when not responding to a call for service. The dispatch strategy (or, 'server assignment' strategy) is as follows: If a call for service arises in a given sector, dispatch that sector's police car, if it is available; otherwise dispatch the closest available police car. If all three police cars are simultaneously busy when the call arrives, the call is LOST to the system forever. Thus, police cars are dispatched only from the idle or patrolling state.
Since it will be patrolling when dispatched, the police car's location upon dispatch is drawn from a uniform distribution over its one-mile-long sector. Calls for police service arise in accordance with a homogeneous Poisson process, with rate $\lambda = 1.5$ calls/hour. The location of each respective call is drawn from a uniform distribution over the entire three-mile length of the three sectors combined. Thus, each sector generates internally 1/3 of the entire region's workload. Each dispatched police car responds at 1000 mi/hr (these are ***fast*** cars!). Service time at the scene is a random variable that is negative exponential with mean one hour. All service times are mutually independent. Upon completion of service on a call, the police car returns at 1000 mi/hr to its sector and resumes random patrolling.

(a) Determine the average workload or fraction of time busy responding to calls for service, averaged over all cars.
(b) Determine the fraction of calls for service that are LOST to the system due to all cars being simultaneously busy.
(c) Determine the fraction of dispatch assignments that are **intra-sector** assignments, i.e., the fraction of calls for service, over the entire region, that are responded to by the car assigned to the sector containing the call.
(d) Determine the fraction of time that precisely two police cars are busy on calls for service.
(e) Determine the workload or fraction of time busy of police car **B**.
(f) Determine the fraction of callers from Patrol Sector B who get patrol car **A** in response to a call for service.
(g) Suppose the system is in the empty state, that is all police cars are patrolling and there are no calls for service in the system. Then a call for service arrives from one of the three sectors and the dispatcher dispatches the car patrolling the sector containing the call. How frequently is the dispatcher dispatching other than the closest available police car in this circumstance?

**Problem 3 (30 points)**

Consider a (deterministic) tree network, whose set of nodes is denoted as N. Let the 1-median of this tree be at node $\mu$. Assume, without loss of generality that all nodes in N have a strictly positive weight and that their total weight is H(N). Let the nodes which are immediate neighbors of $\mu$ be the set V = {v1, v2, .... , vk}. Please see Figure 1. Note that each of these nodes, vj, is the root of a sub-tree. (The sub-tree may, of course, consist of the node vj only.)

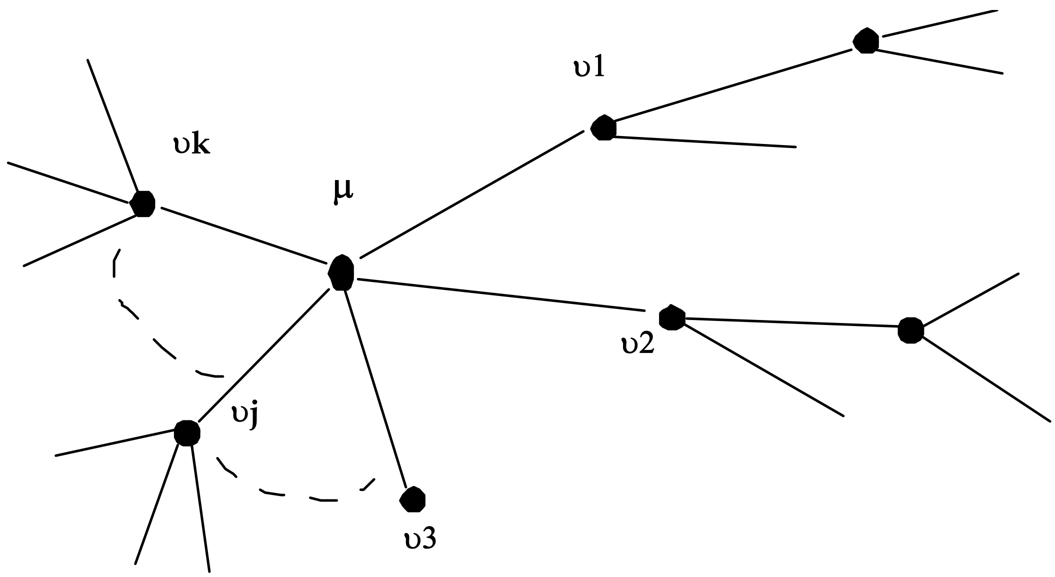



**Figure 1**

Assume now that we are looking for the 2-median of the same tree network and let the locations of the 2-median be at a pair of nodes $\mu_1$ and $\mu_2$. Suppose that the following claim is made:

"Both $\mu_1$ and $\mu_2$ are located in the same sub-tree rooted in node $vi \in V$, one of the immediate neighbors of $\mu$ (please see Figure 2). In other words, the two facilities are on two distinct nodes in the set $N_{vi}$ of the nodes in the sub-tree rooted in vi. The facility $\mu_2$ then serves (i.e., is closest to) a subset $N_2$ of the nodes in that sub-tree and the other facility, $\mu_1$, serves the remaining nodes in the <u>entire</u> network (including one or more nodes in $N_{vi}$), i.e., the set $N - N_2$ (see Figure 2)."

Argue that the above claim cannot be true, i.e. that both $\mu_1$ and $\mu_2$ cannot be located in $N_{vi}$. (Stated differently, the 1-median, $\mu$, must be "between" $\mu_1$ and $\mu_2$). [Hint: Argue that if $\mu_2$ is in $N_{vi}$, as shown in Figure 4, $\mu_1$ cannot be in $N_{vi}$.]

## Figure 2

**The claimed relative locations of μ, υi, μ1 and μ2 are shown along with the relevant subsets of the set of nodes N.**