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Massachusetts Institute of Technology
Logistical and Transportation Planning Methods

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Quiz #2 OPEN BOOK
December 4, 2002

1. Please do Problems 2 and 3 in a booklet separate from Problem 1.
2. We have given you extra copies of the figures, in case you would like to use the drawings in your answers. You are obviously also welcome to write all answers directly in your exam book. If you want the grader to refer to answers or work shown on these extra figure sheets, please note this in your exam book (e.g. "See enclosed figure") and label this figure as "Answer to Question <blank>".
3. Put your name on each booklet, this exam, and the extra copies of the figures we have given you. At the end of the exam, turn in all of these items, even if you did not write on them.
4. Remember to please explain all of your work! We like to give deserved partial credit.

Good luck!

Problem 1 (35 points)
Consider a square one kilometer on a side, as shown in Figure 1. Geometrically, this is the same square that appeared in Problem 1 of Quiz #1. That is, emergency incidents can only occur on the perimeter of the square and travel can occur only along the perimeter of the square. There is no travel within the square. There are no emergency incidents within the square. U-turns are allowed and travel always occurs along the shortest path.

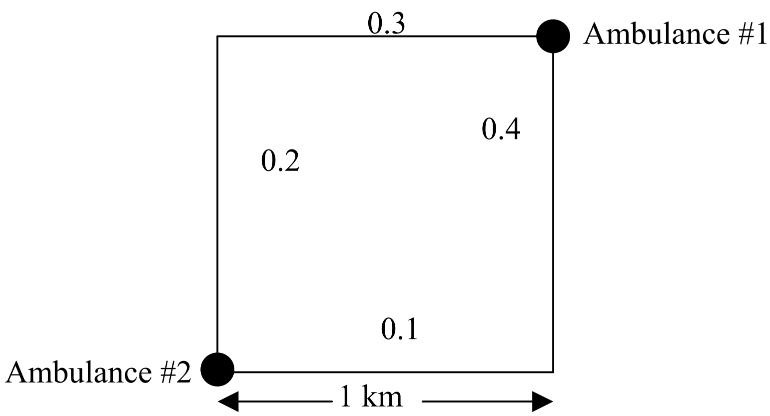


Figure 1

The square is served by two ambulances, ambulance #1 garaged in the northeast corner of the square and ambulance #2 garaged in the southwest corner of the square, as shown in Figure 1. Ambulances always return to their home garage locations after answering emergencies. So, an ambulance will never be dispatched directly from one call to another without returning to the home garage location.

Emergency incidents are ***not*** uniformly distributed over the square. The number adjacent to each link of the square is the probability that a random emergency incident will be generated on that link. Once the link of the incident is known, the conditional pdf of its location on the link is uniform over the link.

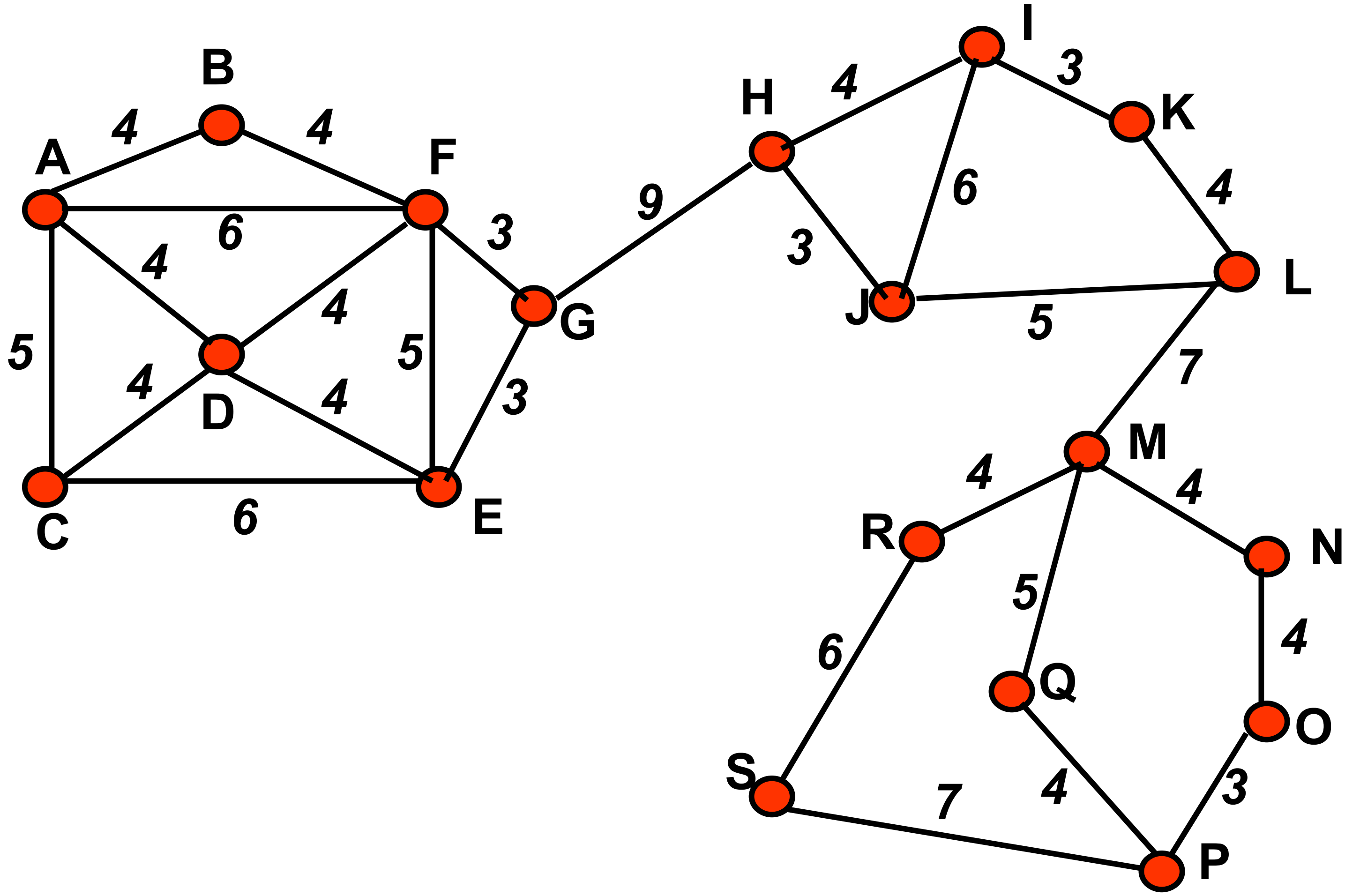
We model this system as an $N = 2$ server hypercube queueing model with μ^{-1} = mean service time per incident = 1 hour, λ = Poisson arrival rate of emergency incidents from entire square = 1/hour, and response travel speed = 100 km/hour. The usual assumptions related to negative exponential service times apply, as does the assumption that on scene time dominates the very small travel time component of the service time. Assume that the dispatcher dispatches the closest available ambulance (i.e. of the available ambulances, the one whose home garage location is closest to the emergency). Emergency incidents that occur while both ambulances are simultaneously busy are lost.

(a) (10 points) Given that an emergency incident occurs while ambulance #1 is busy and ambulance # 2 is available, find and plot the conditional pdf of the travel distance for ambulance #2 to travel to the scene of the emergency incident.
(b) (12 points) Find the workload (fraction of time busy) of each of the two ambulances.
(c) (13 points) If the dispatcher moves to an optimal dispatch strategy, i.e., one that minimizes mean travel time to a random incident, determine the optimal boundaries for response areas 1 and 2.

Problem 2 (30 points)
The undirected network shown in Figure 2 has 19 nodes, denoted as A through S. ALL NODES HAVE WEIGHT EQUAL TO 1 UNIT. The link lengths are shown next to each link. The total length of the links is 130.

(a) (12 points) What is the length of the optimal Chinese Postman tour of this network? Please justify your answer briefly.
(b) (18 points) Find the absolute 1-median of the original undirected network. Please explain your work clearly.

Figure 2



Problem 3 (35 points)
Consider the set of 3m+2 points shown in Figure 3. Note that these points are arranged in essentially two rows. The bottom row (see Figure) consists of 2m+1 points. These, are arranged so that the leftmost point is the one labeled "*a*" in the Figure and the point immediately to *a*'s right is ε units away from *a*, where ε is very small. This point is followed on the right by another point, (1- ε) units away (and 1 unit away from point *a*). This pattern is repeated, as shown in the Figure. Note that the last point [i.e., point (2m+1) from the left] on the bottom row does not have another point a distance ε away.

Turning to the top row, note that it consists of m+1 points. The leftmost of these points is located perpendicularly above and 1 unit away from the leftmost point on the bottom row (i.e., point *a*); the second point on the top row is located perpendicularly above and 1-ε units away from the third point on the bottom row; the third point on the top row is located perpendicularly above and 1-ε units away from the fifth point on the bottom row; and so on, until the rightmost point (m+1 point) on the top row, which is located perpendicularly above and 1 unit away from the rightmost point on the bottom row. Note that the vertical distances are all equal to 1-ε, except for the leftmost and rightmost ones, which are equal to 1 unit, as shown in Figure 3.

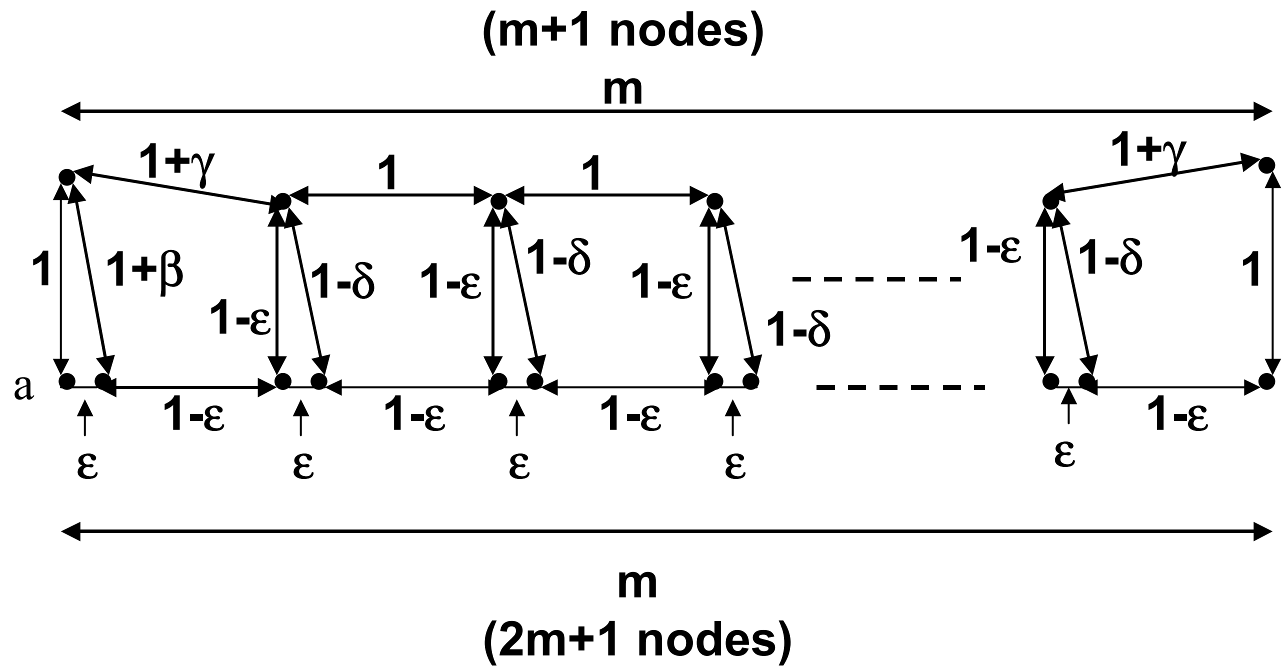
Note also the "diagonal distances" equal to 1+β or 1-δ, as shown in Figure 3. From the construction it is clear that 1-δ>1-ε, or that $\delta < \varepsilon$.

(a) (10 points) Beginning at point *a*, find and draw schematically a minimum spanning tree (MST) of the 3m+2 points of Figure 3.

(b) (10 points) Find *by inspection* and draw schematically the optimal Traveling Salesman tour that begins and ends in node *a*. Denote the length of this optimal tour as *L(TSP)*.

(c) (15 points) To apply to this problem the "MST heuristic" for the TSP that was described in the lectures, all we have to do is duplicate the MST you found in part (a). We would then find a solution to the TSP by beginning at *a* and tracing the graph obtained from the two MST's (the original and its copy) visiting every point, skipping points already visited and eventually returning to *a*. Suppose the length of the solution to the TSP which is obtained in this way is denoted as *L(2-MST HEURISTIC)*.

Compute the limiting value of the ratio $\dfrac{L(2-MST\ \ HEURISTIC)}{L(TSP)}$ as δ, β, ε and γ all tend to zero (i.e., assume δ, β, ε, γ are infinitesimally small compared to 1; also assume that δ is smaller than ε). What is the limiting value of this ratio as m tends to infinity? In one sentence, the construction of Figure 3 provides an example of what?



[β,γ, δ, ε **are all "arbitrarily small" and much smaller than 1;** δ **is also smaller than** ε]

Figure 3