1.020 Ecology II: Engineering for Sustainability
Spring 2008

Lectures 08_6 & 08_7 Outline: Networks, Traffic Modeling

Motivation/Objective

Develop a model to compute atmospheric (CO) emissions rates from vehicles on a road network

Approach

1. Formulate mathematical description of system (road network: nodes, directed links, paths).
2. Select variables to describe steady-state vehicle movement through the network. Identify unknowns (link flow rates x_l and link travel times t_l).
3. Formulate coupled equations that use linear travel-time functions, mass balance conditions, and a user equilibrium condition to relate unknown flows and travel times.
4. Specify network properties, solve equations for unknowns (MATLAB)
5. Relate emissions rate to network variables, examine effect of network properties on emissions

Concepts and Definitions:

Networks: Represent with nodes, directed links, and paths. Describe connectivity, specify link lengths.
Steady-state condition: Flow into network = flow out of network.
Conditions used to relate link travel times t_l (hr) and vehicle flow rates x_l (vehicles hr^{-1}):

- Link travel-time functions: $t_l = f(x_l) = t_0 + \alpha_l x_l^{\beta_l}$ for link l. Asume $t_l = f(x_l)$ is linear ($\beta_l = 1$) for the example.
- Mass balance at nodes: sum of x_l for links entering node = sum of x_l for links leaving node.
- User equilibrium condition: path travel times are equal for all possible paths

Each conditions generates linear equations in the unknown link times and flows.

Emissions: Used to compute source rate of pollutant to atmosphere (g km^{-1} hr^{-1}) from each link $E_l(v_l) = x_l c_l(v_l)\gamma_l(v_l)$, $c_l(v_l)$ = consumption, $\gamma_l(v_l)$ = production rate, $v_l = x_l / t_l$ = vehicle velocity

Network balance eqs:

Assemble the three types of linear equations in a matrix form and solve with MATLAB:

$$\begin{bmatrix} A_{travel,t} & A_{travel,x} \\ A_{mass,t} & A_{mass,x} \\ A_{equil,t} & A_{equil,x} \end{bmatrix} \begin{bmatrix} t \\ \\ x \end{bmatrix} = \begin{bmatrix} b_{travel} \\ b_{mass} \\ 0 \end{bmatrix}$$

, t and x are vectors of link travel times and flows

Model Results

Note effect of fuel consumption and CO production rate on CO emissions pattern. Examine changes in network geometry and Braess' paradox.