Massachusetts Institute of Technology
Department of Mechanical Engineering

2.003J/1.053J Dynamics & Control I

Fall 2007

Homework 7

Issued: Nov. 2. 2007 **Due**: Nov.9. 2007

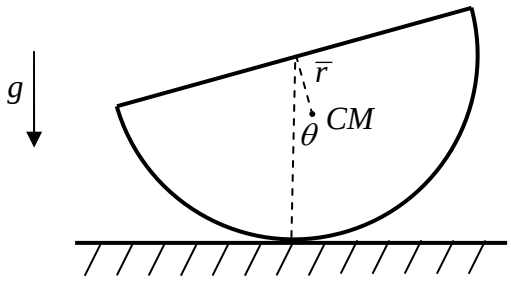
Problem 7.1 : Derivation of the equation of the motion for a rolling half-disk

Half-disk is rolling without slipping on the plane surface.

i) Derive the equation of the motion. Keep all nonlinear terms and do not linearize.

ii) Linearize the nonlinear motion in case of small angle oscillation.
Hint: use small angle approximations such as $\sin\theta \approx \theta$

iii) Solve the linearized equation of motion obtained in ii) analytically with following initial conditions: $\theta(0) = \theta_0, \dot{\theta}(0) = \dot{\theta}_0$.



Radius $= r$, $\bar{r} = \frac{4r}{3\pi}$, and $I_{CM} = \frac{1}{2}mr^2 - m\bar{r}^2$

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Problem 7.2 : Generate simulation codes for motion for rolling half-disk

Generate functions to simulate the trajectory of θ for rolling half-disk based on following instructions. Simulation time is 10 seconds. Set r =1m.

i) Use the nonlinearized equation of motion obtained in problem 7.1. i). Use "ode45" for simulation. **Function name (and m-file name) should be 'RockerRK_*your_kerberos_name*' and upload it to 2.003 MIT Server site. You also submit the hardcopy of your code with appropriate comments.** Function has following structure.

```
function [t,theta]= RockerRK_your_kerberos_name(theta0)
t: time matrix (N×1)
theta: angle matrix (N×1)
theta0: initial condition matrix (1×2)
```

ii) Use analytical solution obtained in 7.1. iii). Trajectory can be obtained by simply evaluating the analytical solution as a function of time. **Function name (and m-file name) should be 'RockerAN_*your_kerberos_name*' and upload it to 2.003 MIT Server site. You also submit the hardcopy of your code with appropriate comments.** Function has following structure.

```
function [t,theta]= RockerAN_your_kerberos_name(theta0)
t: time matrix (N×1)
theta: angle matrix (N×1)
theta0: initial condition matrix (1×2)
```

Problem 7.3 : Trajectory of $\theta(t)$ for both small and large angle oscillations

For the initial conditions given below, simulate the nonlinear motion and the linearized motion for rolling half-disk up to 10 seconds (use results of Problem 7.2). Compare these results by plotting both of them in the same figure with the appropriate legends. Are they identical? Otherwise, explain why not. You should hand in hardcopy of the plots.

i) Small angle oscillation: $\theta(0) = 5^o$ and $\dot{\theta}(0) = 0$.

ii) Large angle oscillation: $\theta(0) = 30^o$ and $\dot{\theta}(0) = 0$.

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