

I B.Tech Examinations, May/June 2012
MATHEMATICS - I

Common to CE, ME, CHEM, BME, IT, MECT, MEP, AE, AME, ICE,
E.COMP.E, MMT, ETM, E.CONT.E, EIE, CSE, ECE, CSSE, EEE

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) Test the series for convergence whose n^{th} term is $(3n-1)/2^n$. [5]
 (b) Examine whether the following series is absolutely convergent or conditionally convergent $1 - \frac{x^2}{2} + \frac{x^4}{3} - \frac{x^6}{4} + \dots$ ($x > 0$). [5]
 (c) Write the Maclaurin's series with Lagrange's form of remainder for $f(x) = \cos x$. [6]
2. (a) Obtain the differential equation of all the rectangular hyperbolas with asymptotes as the co-ordinate axes. [3]
 (b) Solve the differential equation: $x \frac{dy}{dx} + y = x^3 y^6$. [7]
 (c) The rate at which bacteria multiply is proportional to the number present initially. If the original number doubles in 2 hours; in how many hours will it be tripled? [6]
3. Verify Stokes theorem for $F = x^2 i - yz j + k$ integrated around the square $x=0, y=0, x=1, \text{ and } y=1$ [16]
4. (a) Solve the differential equation: $(D^4 - 4)y = x \cos 2x$
 (b) Solve the differential equation: $(x^2 D^2 + 4x D - 4)y = x \log x$. [8+8]
5. (a) Solve $(D^2 + 1)x = t \cos 2t$ using Laplace transforms given that $x(0)=0, x'(0)=0$.
 (b) Evaluate $\iint_R x^2 dx dy$ where R is the region in the first quadrant bounded by the hyperbola $xy=16$ and the lines $y=x, y=0$ and $x=8$. [8+8]
6. (a) Prove that $\text{curl}(\mathbf{A} \times \mathbf{B}) = \mathbf{A} \text{div} \mathbf{B} - \mathbf{B} \text{div} \mathbf{A} + (\mathbf{B} \cdot \nabla) \mathbf{A} - (\mathbf{A} \cdot \nabla) \mathbf{B}$.
 (b) Find the directional derivative of $\phi(x, y, z) = x^2 yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of the normal to the surface $f(x, y, z) = x \log z - y^2$ at $(-1, 2, -1)$. [8+8]
7. (a) Find the perimeter of the curve $3ay^2 = x^2(a - x)$
 (b) Find the volume of a spherical cap of height h cut off from a sphere of radius a . [8+8]
8. (a) If $x = r \sin \theta \cos \phi, y = r \sin \theta \sin \phi$ and $z = r \cos \theta$ prove that $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin \theta$. [6]
 (b) Find the radius of curvature at any point on the curve $y = c \cosh \frac{x}{c}$. [10]
